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The pulsar kick velocity distribution

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ABSTRACT

We analyse the sample of pulsar proper motions, taking detailed account of the selection effects of the original surveys. We treat censored data using survival statistics. From a comparison of our results with Monte Carlo simulations, we find that the mean birth speed of a pulsar is $\sim$250–300 km s^{-1}, rather than the 450 km s^{-1} found by Lyne & Lorimer. The resultant distribution is consistent with a Maxwellian with dispersion $\sigma_v = 190\,\mathrm{km\,s^{-1}}$. Despite the large birth velocities, we find that the pulsars with long characteristic ages show the asymmetric drift, indicating that they are dynamically old. These pulsars may result from the low-velocity tail of the younger population, although modified by their origin in binaries and by evolution in the galactic potential.

Key words: methods: statistical – stars: kinematics – pulsars: general.

1 INTRODUCTION

The fact that pulsars have velocities much in excess of those of ordinary stars (a subset of which are presumably the pulsar progenitors) has been known almost since their discovery (Minkowski 1970; Trimble 1971; Lyne, Anderson & Salter 1982). The origin of these velocities is not so clear. One possibility is that they result from the disruption of a binary population (Gott, Gunn & Ostriker 1970; Iben & Tutukov 1996), leaving the pulsar with a velocity characteristic of the orbital velocity of the progenitor in the binary. The problem with this scenario is that it has trouble explaining the largest observed velocities (e.g. Phinney & Kulkarni 1994; see, however, Iben & Tutukov 1996). Another possibility is that the pulsar acquired its velocity from an asymmetric supernova collapse. This hypothesis has recently been bolstered by observations of binary pulsar PSR J0045−7319, in which the observed spin–orbit precession and orbital period decay are strong arguments for a natal kick (Lai, Bildsten & Kaspi 1995; Lai 1996).

Lyne & Lorimer (1994) have analysed the known sample of pulsar velocities in the light of recent proper motion studies (Harrison, Lyne & Anderson 1993) as well as the new pulsar distance scale due to Taylor & Cordes (1993). They conclude that pulsars are born with a mean speed of $\sim$450 km s^{-1}. Although Lyne & Lorimer restricted their sample to those younger than 4×10^6 yr to avoid the 'vertical leakage' selection effect pointed out by Helfand & Tademaru (1977) and Cordes (1986), they did not treat the selection effects that result from the flux limits of the pulsar surveys or the limiting accuracy of proper motion determinations. We shall attempt to do that here. Recently, Iben & Tutukov (1996) have also addressed this question, but in a less systematic fashion than that we propose in this paper.

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We restrict our analysis to only those pulsars with velocities determined by proper motion measurements. While scintillation data have been used to estimate velocities (Cordes 1986; Harrison & Lyne 1993) to within a factor of 2, we prefer to keep our sample as homogeneous as possible, and so we exclude these data. We use the properties of well-known pulsar surveys to estimate the $V/V_{\rm max}$ correction (Schmidt 1968) for each pulsar, making use of survival statistics (Feigelson & Nelson 1985) to treat those data with upper bounds (Section 2.2). This allows us to estimate the two-dimensional velocity distribution of the observed pulsar population, and thus the kick velocity distribution taking into account the differential galactic rotation in Section 3.

2 THE PROPER MOTION DISTRIBUTION

2.1 Selection effects

The Princeton pulsar data base (Taylor, Manchester & Lyne 1993) now contains $\sim$ 800 pulsars, 101 of which have measured proper motions or upper limits. The number of surveys responsible for this profusion is also gradually increasing in size (in excess of 15). To do a proper treatment of the selection effects for the full proper motion sample would then require modelling the selection effects of a significant number of these surveys. Luckily, we note that most of the proper motion pulsars were detected in the earlier surveys. By restricting ourselves to those pulsars detected in the Molonglo 2 (Manchester et al. 1978) and Green Bank/NRAO 1, 2 and 3 surveys (Damashek, Taylor & Hulse 1978; Dewey et al. 1985; Stokes et al. 1985, 1986), we are left with 86 out of 101 pulsars. We note that 12 of the 15 pulsars left out have $P < 0.1$ s (and only one of our restricted sample satisfies this criterion), which is not surprising, since many of the later surveys focused on finding faster spinning pulsars. In performing this cut, we lose one young pulsar and all but

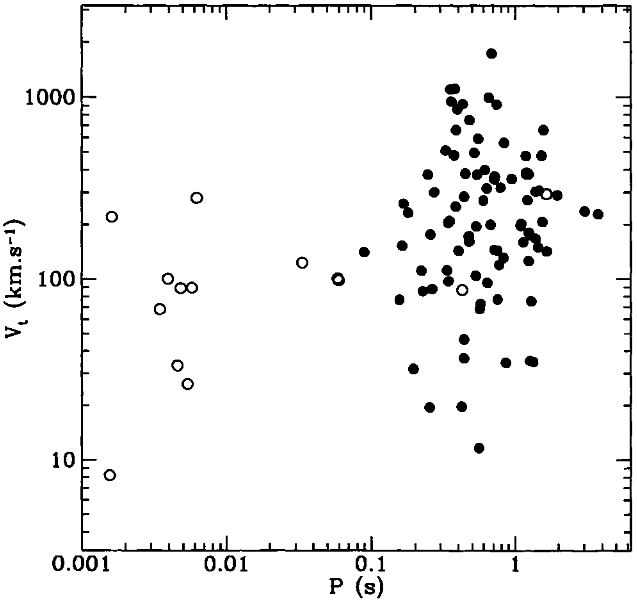

Figure 1. Sample definition. The filled circles indicate the pulsars that we include in our analysis. The open circles denote those that are excluded. Since the purpose here is simply to demonstrate which pulsars are in the sample discussed, we omit any error bars. Of the three excluded pulsars with $P > 0.1$ s, two are recycled binaries: B0655+64 and B0820+02.

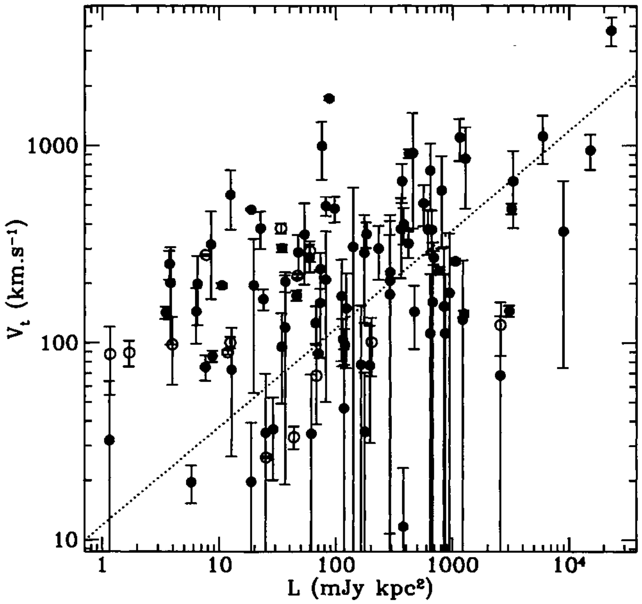

Figure 2. Luminosity and velocity for the proper motion pulsars. We again include all the pulsars with proper motions on this plot. The filled circles will be the ones to which our analysis applies. The dotted line indicates a proper motion of 5 mas yr^{-1} and a flux of 4 mJy. This line does not represent a cut-off over most of this diagram because making a pulsar brighter at a given distance will move it to the right and making a pulsar faster at a given distance will move it up, thus one can populate both sides of the line with observable pulsars. Nevertheless, at the high-luminosity/high-velocity end, it should represent the limiting case. It is also noticeable that, on average, higher velocity pulsars have higher luminosities, and so will be over-represented in an unweighted sample.

one of the pulsars with characteristic ages greater than 1 Gyr. This also means that we are not affected by the possibly different evolutionary histories of millisecond pulsars (in particular the influence of binaries). We shall omit the one old binary pulsar (PSR 0655+64) that does fall into our sample as well. Fig. 1 shows the distribution of the included and excluded pulsars as a function of period and velocity.

The observed pulsar sample suffers from two obvious selection effects, due to flux limits and proper motion limits respectively. Fig. 2 shows the distribution of inferred luminosities and transverse velocities. The lack of faint, fast pulsars (upper left corner) and bright, slow pulsars (lower right corner) is evident.

To correct for this bias, we need to weight the pulsars according to the maximum volume in which they could have been detected, i.e. using a $V_{\rm max}$ weighting. To do this we need to consider the detection efficiency of the various pulsar surveys. After Narayan (1987) (see also Dewey et al. 1984), the minimum flux detectable is

$$S_{\rm min} = S_0 \frac{(T_{\rm rec} + T_{\rm sky})}{T_0} \sqrt{\frac{PW}{W_{\rm e}(P - W)}}, \qquad (1)$$

where $T_{\rm rec}$ and $T_{\rm sky}$ are the system and sky noise temperatures, S_0 is the flux normalization, and W is the measured pulse width. $W_{\rm e}$ is the intrinsic pulse width, which is increased because of sampling, dispersion and scattering so that

$$W^2 = W_{\rm e}^2 + \tau_{\rm samp}^2 + \tau_{DM}^2 + \tau_{\rm scatt}^2. \qquad (2)$$

Thus we calculate the minimum flux for a given survey in a particular direction. The parameters describing each survey are taken from Narayan (1987)[1] and Stokes et al. (1986). We use the updated electron distribution model of Taylor & Cordes (1993) to calculate the dispersion and scatter broadening along a given line of sight. This accounts for the bias due to the flux limits. No such simple model exists for treating the proper motion limits. This is because the accuracy of a given proper motion measurement depends on the vagaries of the distribution of background radio sources near the pulsar position on the sky (Harrison et al. 1993). As a crude model of this, we fit the distribution of proper motion errors in Harrison et al. using the distribution $p(\mu) = \exp(-\mu/10.5\,{\rm mas\,yr^{-1}})$. In the $V_{\rm max}$ calculations to follow, the proper motion cut-off is randomly selected from this distribution for each line of sight. We also use simple limits of 5 and 2 mas yr^{-1} as a test of the importance of this selection effect. This introduces a variation of $\sim 10\,{\rm km\,s^{-1}}$ in the mean velocity of the young sample and about $30\,{\rm km\,s^{-1}}$ in the old sample, indicating that most of the $V_{\rm max}$ values are flux-limited rather than proper-motion-limited. Iben & Tutukov (1996) have also taken account of this selection effect.

For each pulsar, we randomly place it in different directions and at different distances with respect to the observer and calculate whether or not it would be detectable, with its known luminosity and transverse velocity (uncorrected for the local standard of rest), in any of the surveys that we consider. Thus, using this Monte Carlo integration procedure, we determine the volume within which each pulsar could have been detected. These $V_{\rm max}$ values determine the relative weights of each of the pulsars in the corrected sample.

A possible source of concern with this procedure is illustrated by Fig. 3. The analytic flux limits do not describe the complications of

[1]Narayan's formula for the sky background temperature in equation (3.5) contains a typographical error. The factor $[1 + (b/3)^2]$ should be in the denominator.

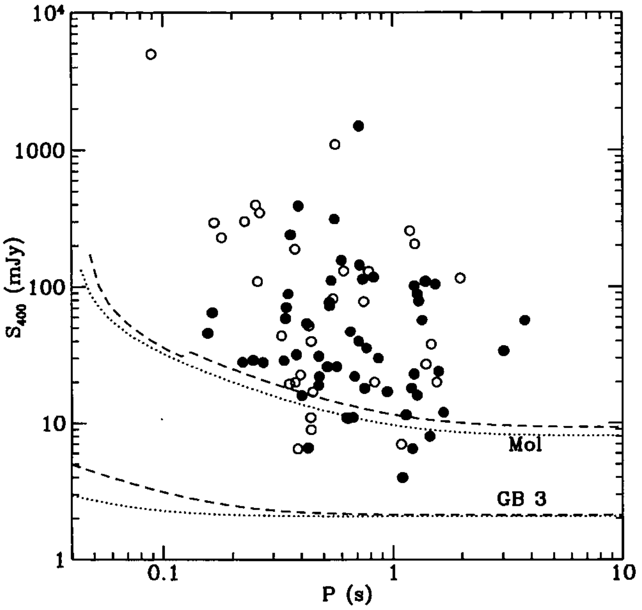

Figure 3. Selection effects. The solid circles represent those pulsars detected in the Green Bank surveys. The open circles are the ones detected in the Molonglo surveys. The dotted lines indicate the approximate limiting flux as a function of P for $DM = 50$ cm^{-3} pc and the most sensitive Molonglo and Green Bank surveys. The dashed lines are for $DM = 200$ cm^{-3} pc.

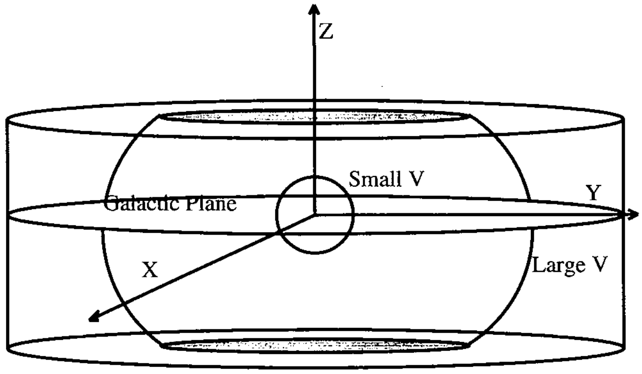

Figure 4. Detectable volume for each pulsar. Pulsars with small luminosities will only be observable near the galactic plane, so that their $V_{\max}$ will be spherical (neglecting other selection effects for the moment). Pulsars with large luminosities will be observable much further away, out to the limits of the disc that such a population, born in the galactic plane, would fill. In this case, the spherical $V_{\max}$ will be cut off above the limits of the disc height given by $z = V_z \times t$.

the true detection limits perfectly (interstellar scintillation makes the apparent flux vary). In Fig. 3 we find four pulsars detected by the Molonglo survey that lie below the analytically described detection threshold for that survey. This will reduce the weight accorded to these pulsars. To estimate the impact of this error on our results, we repeat the analysis with these pulsars artificially 'brightened' to meet the flux limit expression. The mean proper motion that we infer for the young pulsar sample increases by only 7 km s^{-1}, so this is not a significant source of error for our analysis here.

The $V_{\max}$ correction is not without biases of its own. In particular, weighting pulsars by their $V_{\max}$ presupposes that the real population is distributed uniformly throughout the galactic volume. However, the pulsars are born from a disc population with a scaleheight of about 150–450 pc (Narayan & Ostriker 1990). Thus, a population born with small velocities will not expand to fill as much of the spherical volume as a fast population. The above analysis then overcorrects for the slow pulsars (see Helfand & Tademaru 1977; Cordes 1986; Lyne & Lorimer 1994). In order to adjust for this, we consider the maximum detectable volume to be limited in the vertical extent by the scale $Z_{\max} = V_t t_p$, where t_p is the pulsar timing age. If this is larger than $D_{eq} = V_{\max}^{1/3}$, then there is no change in the weight assigned to that pulsar, but, if $D_{eq} > Z_{\max}$, then we assume that we see the edge of the distribution of pulsars of this velocity, and reduce the weight given to that pulsar accordingly (see Fig. 4). Furthermore, for young pulsars, we set a lower limit on $Z_{\max}$ of 450 pc, representative of the initial scaleheight.

In Fig. 5 we show the distribution of velocities with age and the relative weighting of each pulsar.

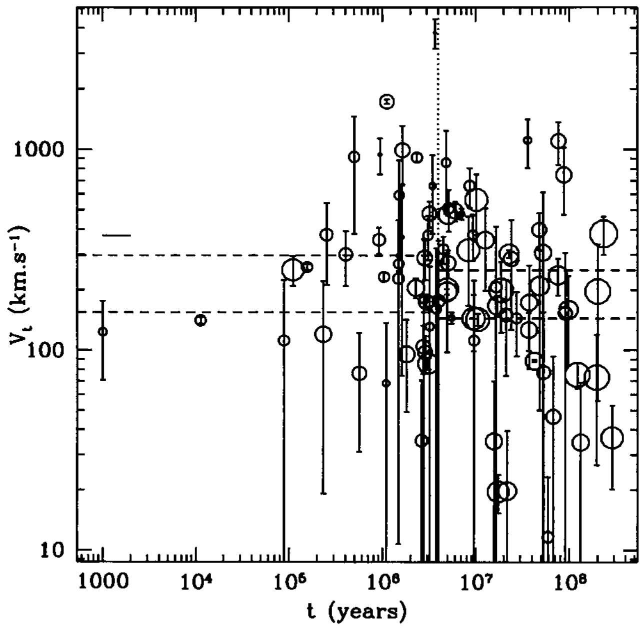

Figure 5. The weighted proper motion distribution. The size of each circle is proportional to the logarithm of the weight accorded that pulsar. The vertical dotted line indicates the dividing line between what we consider 'young' and 'old' pulsars. The horizontal dashed lines indicate the range of values that we infer for the mean proper motion of the pulsars in each sample. The short horizontal solid line at left indicates the mean proper motion value quoted by Lyne & Lorimer (1994).

2.2 Survival statistics

The $V_{\max}$ correction takes care of the selection effects, but we still need to account properly for those data that only have upper limits ('censored' data in the statistical lexicon). Of our 85 pulsars, 20 fall into this category. Using only those data with actual detections will bias our distribution to higher values, as we will see below.

Following Feigelson & Nelson (1985), we use survival statistics to treat the effects of our censored data. In particular, we use the Kaplan–Meier estimator (Kaplan & Meier 1958) to calculate the cumulative probability distribution of the transverse velocities. However, we need to modify this method slightly to take account of our $V_{\max}$ correction.

Consider our data to be the set of n distinct values $\{x_i\}_{i=1}^{n}$, where the x_i are ordered in the manner $x_1 > x_2 > \cdots > x_n$, where x_i can be either a detected value or an upper bound. Now, for a given value t, let

$$P_i = P[t \le x_{i+1} | t \le x_i],$$

i.e. the conditional probability that t is less than x_{i+1} given that it is less than x_i.

Using this, we can calculate the probability that t is less than any given x_i ($i > 1$) by

$$P[t \le x_i] = \prod_{j=1}^{i-1} P[t \le x_{j+1} | t \le x_j].$$

To calculate this, we need to estimate the P_j. To start, $P_1 = 1$, since all values are at most as big as x_1. For any other x_j, if it is a detection, then there are $n-j+1$ values at most as large as x_j, and all except x_j are also at least as large as x_{j+1}. Thus, we estimate

$$P_j = \frac{n-j}{n-j+1} = 1 - \frac{1}{n-j+1}.$$

If x_j is an upper bound, then $P_j = 1$ by following similar reasoning as above. Thus, we have that

$$P[t \le x_i] = \prod_{j=1}^{i-1} \left(1 - \frac{1}{n-j+1}\right)^{\delta_j},$$

where $\delta_j = 1$ if x_j is a detection, and $\delta(j) = 0$ if x_j is an upper limit. This is the Kaplan–Meier (1958) estimator of the distribution function. In the case where we have ties in our data (i.e. more than one measurement at the same value), this becomes

$$P[t] = \prod_{j, x_j > t} \left(1 - \frac{d_j}{n_j}\right)^{\delta_j},$$

where n_j = number of measurements $\le x_j$, and d_j is the number of measurements at the value x_j.

We also want to calculate the mean of velocities. Since $p(x)\mathrm{d}x = \Delta P$, the mean of a quantity x is given by

$$< x > = \int_0^1 x\,\mathrm{d}P = \int_0^\infty P\,\mathrm{d}x,$$

where we have integrated by parts. Thus we estimate the mean by

$$< V_t > = \sum_{i=1}^{N} P[x_i](x_i - x_{i-1}),$$

and we take $x_0 = 0$.

To include a $V/V_{\max}$ weighting, we adjust the number of 'counts' at each value according to the weights $w(j)$, and thus we have

$$\frac{d_j}{n_j} = \sum_{i \le j} \frac{w_j}{w_i}.$$

In calculating the error on this new estimator, we note that we have artificially increased the total number of observations fed into the sum and thus have reduced the error by a factor $\sqrt{N_\mathrm{W}}$, where N_W is the number of pulsars including the weightings. We remove this bias by multiplying the errors by $\sqrt{N_\mathrm{W}/N}$ where N is the true number of points in the sample.

2.3 The corrected distribution

Fig. 6 shows the cumulative probability distribution of observed transverse velocities taking into account different levels of adjustment. With no adjustments, the mean transverse velocity of the entire sample is 355 km s^{-1}, in agreement with the analysis of Lyne & Lorimer (1994). The mean of the entire sample including all corrections is 195 km s^{-1}. However, if we restrict the sample to pulsars with characteristic ages < 10^7 yr, we obtain a mean transverse velocity of 237 ± 49 km s^{-1} (51 pulsars). The mean velocity of the complementary sample with ages $> 10^7$ yr is 193 ± 50 km s^{-1} (35 pulsars). The similarity between this and the overall sample is due to the great weight assigned to old, faint and slow-moving pulsars (see also the discussion of the Iben & Tutukov results in Section 5). In Fig. 6 we see that the distribution for the old pulsars does have a larger low-velocity tail, as one might expect. If we place the velocity cut-off at 4×10^6 yr, we find 226 ± 71 km s^{-1} for the 36 young pulsars and 198 ± 53 km s^{-1} for the older pulsars.

Our analysis of the proper motion errors neglects the effects of the differential rotation or corrections to the pulsar local standard of rest. However, these are not important corrections for two reasons. The first is that the pulsar sample is largely restricted to distances < 3 kpc, so that these corrections are, at most, $\sim 2B \times 3$ kpc ~ 75 km s^{-1}, where B is Oort's B constant. At this level it might still have observable consequences, but the second reason is that the $V_{\max}$ values are determined largely by the flux limits, rather than the proper motion limits. Thus, these effects are only likely to become important when observers attempt to extend the proper motion sample to greater distances.

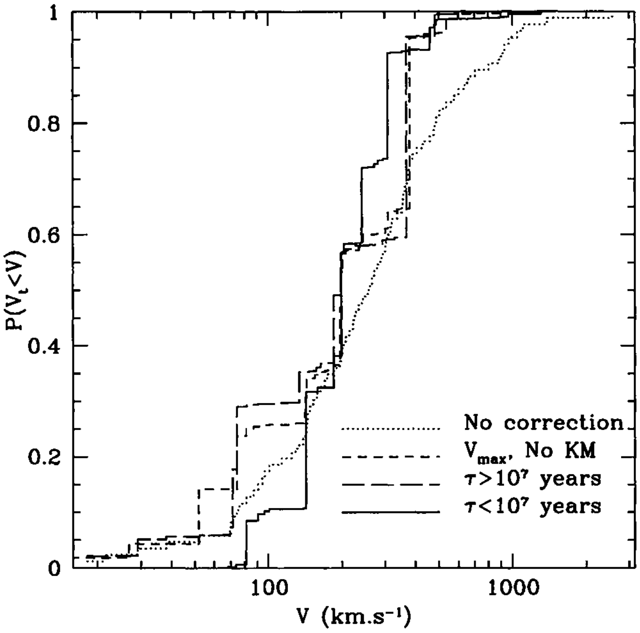

Figure 6. The inferred proper motion distribution. The cumulative probability distribution $P(V \le V_t)$ is shown for different degrees of correction. The dotted line shows the entire sample with no $V_{\max}$ weighting and no treatment of upper bounds. The short-dashed line is for only the 66 pulsars with detected proper motions corrected using our $V_{\max}$ correction (i.e. all those with only upper limits were left out). The long-dashed line is the properly corrected sample (with $V_{\max}$ corrections and Kaplan–Meier estimator) but only those with characteristic ages greater than 10^7 yr. The solid line is the complementary sample of only those pulsars with characteristic ages less than 10^7 yr. The 95 per cent confidence levels for the corrected distributions lead to an uncertainty of ~ 0.05 in P for velocities < 300 km s^{-1}. Above that, the statistics become uncertain and we cannot say much about the distribution.

2.4 The asymmetric drift

A reduction in the mean velocity for old pulsars has been noted before by several authors (Lyne & Lorimer 1994; Nice & Taylor

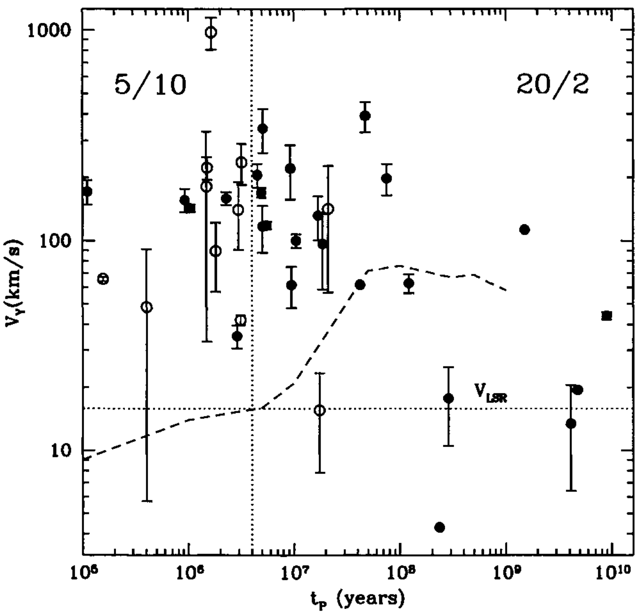

Figure 7. The asymmetric drift. The solid circles indicate a positive V_y and open circles indicate a negative V_y. The older points include binaries. The horizontal dotted line indicates the order of magnitude of the Sun's motion within its local standard of rest, and velocities below this will experience a contamination of the asymmetric drift. The vertical dotted line is at 4×10^6 yr, and represents an approximate division between 'young' and 'old' pulsars, namely those that show the asymmetric drift and those that do not. Of the young pulsars, only 33 per cent have positive V_y, while 91 per cent of the old pulsars have positive V_y. The dashed line indicates the evolution of the mean asymmetric drift velocity obtained from the calculation in Section 4.

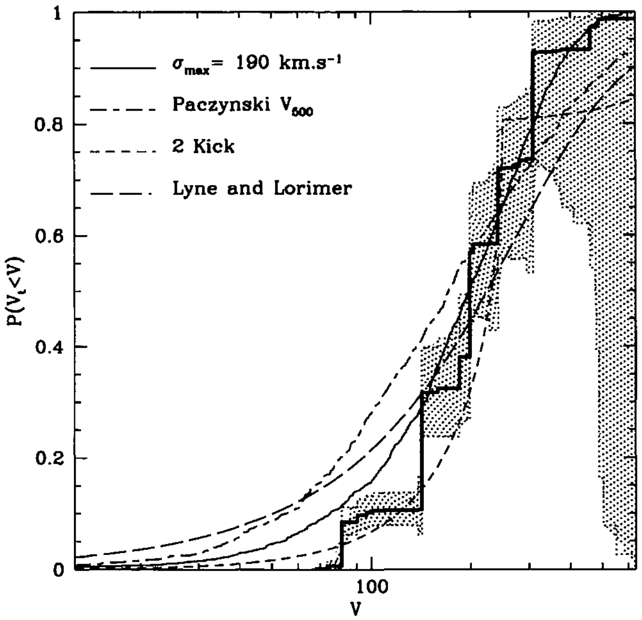

Figure 8. Determining the birth velocity distribution. The heavy solid line is the corrected proper motion distribution, with the formal uncertainty shown by the shaded region. The short dashed line is the proper motion distribution obtained for a distribution given by two delta functions at 250 and 1000 km s^{-1}, in random directions. This distribution and the Maxwellian given by (3) (thin solid line) are consistent with the data. The alternating and long dashed lines represent the results of Paczynski (1990) and Lyne & Lorimer (1994), neither of which are fully consistent with the data.

1995; Camilo, Nice & Taylor 1993). We should note, however, that when we refer to 'old' pulsars, we refer to those with characteristic ages less than 10^9 yr, i.e. we do not consider millisecond pulsars because of the increased complexity of treating their selection effects. Nevertheless, in a completely model-independent way, we can demonstrate that these pulsars are old in a dynamical sense, because they show the effects of the asymmetric drift (e.g. Mihalas & Binney 1981). Nice & Taylor (1995) have pointed out that the millisecond pulsar population might possess this property, but it appears to be true for all pulsars with spin-down ages > 10^7 yr. This is shown in Fig. 7. The effect has its origin in the fact that any virialized population with a significant radial velocity dispersion will rotate about the Galactic Centre more slowly than the local circular speed (Mihalas & Binney 1981).

To calculate this we restrict ourselves only to those pulsars with well-determined proper motions (since large enough error bars can reverse the sign of the transverse velocity). However, because we shall not treat selection effects in this case, we shall use all the proper motion pulsars that satisfy this and subsequent criteria, including millisecond pulsars. We consider a Cartesian coordinate system with origin at the Sun, with positive x pointing radially outward and positive y pointing in the direction of $\ell = 270°$. We consider the y-components of the transverse velocity, which measures the approximate azimuthal component of the pulsar velocity with respect to the Sun (strictly speaking, this should be done for each pulsar in its individual local standard of rest, but this rough approach demonstrates our result sufficiently well and remains the same if we reduce the sample radius). We also exclude pulsars with $d > 6$ kpc and within 20° of $\ell = 90°$ or 270° (where transverse velocities are primarily radial in the galactic frame) and of $b = \pm 90°$. This leaves us with 37 pulsars with characteristic ages from 10^3 to 10^{10} yr. The signature of the asymmetric drift is thus an excess of positive V_y. Indeed, this is seen to striking effect in Fig. 7, where less than 10 per cent of the pulsars with characteristic ages greater than 4×10^6 yr have negative V_y. This indicates that these pulsars must be at least 10^7 yr old, which supports the use of the age cut-off in Section 2.3. It also provides a lower limit for the magnetic field/torque decay time, consistent with that found by recent analyses (Bhattacharya et al. 1991; Hartman et al. 1996).

3 THE KICK DISTRIBUTION

In Section 2, we derived the corrected proper motion distribution appropriate to a volume-limited sample. Since pulsars receive their kicks in a rest frame rotating about the Galactic Centre and we are interested in the low-velocity tail, we need to consider the effect of differential galactic rotation.

Using different initial velocity distributions, we project the velocities along random lines of sight in a sphere of 3-kpc radius about the observer and compare the inferred proper motion distribution with what we observe. This is shown in Fig. 8. We find the distribution is consistent with a Maxwellian distribution of kick velocities with velocity dispersion of ~ 190 km s^{-1} (corresponding to a 3D mean of ~300 km s^{-1}). However, it is not consistent with the Lyne & Lorimer proper motion distribution or the earlier form suggested by Paczynski (1990).[2] Neither of these analyses

[2] $p(u) = 4/\pi(1+u^2)^2$, where $u = V/V_*$ and V_* is some normalization constant. We have chosen $V_* = 500$ km s^{-1} to be consistent with the observed mean of the distribution.

Table 1. Parameters of our galactic model.

	a (kpc)	b (kpc)	r_c (kpc)	M ($M_\odot$)
Bulge	0	0.277		1.12×10^{10}
Disc	3.7	0.20		8.07×10^{10}
Halo			6.0	5.0×10^{10}

included an extensive discussion of selection effects. Thus, our best estimate for the kick velocity distribution is

$$p(V_k) = \sqrt{\frac{2}{\pi}}\frac{v^2}{\sigma_v^3}e^{-V_k^2/2\sigma_v^2} \tag{3}$$

with $\sigma_v = 190\,\mathrm{km\,s^{-1}}$. However, there is a significant degree of uncertainty about the form of the distribution, in particular at the high end. As a demonstration of this, we propose a second extreme form which consists of two delta functions, at 250 $\mathrm{km\,s^{-1}}$ and 1000 $\mathrm{km\,s^{-1}}$, weighted such that 20 per cent of the pulsars acquire the higher velocity. Thus,

$$p(V_k) = 0.8\,\delta(V_k - 250\,\mathrm{km\,s^{-1}}) + 0.2\,\delta(V_k - 10^3\,\mathrm{km\,s^{-1}}), \tag{4}$$

where $\delta(V - V_0)$ is the Dirac delta function. Both forms are shown in Fig. 8.

4 LONG-TERM EVOLUTION

Our analysis above is concerned solely with those pulsars with characteristic ages $< 10^7$ yr. To do the same for the older pulsar population will require the incorporation of the selection effects of more pulsar surveys, as well as the effect of the death line and recycling of pulsars in binaries. Nevertheless, if we assume that all pulsars originate from a population with the velocity distribution (3), we may examine the long-term evolution of this population. We have performed Monte Carlo simulations of such a population using a galactic potential from Paczynski (1990). The potential contains two terms of the form

$$\Phi_i(R,z) = \frac{-GM_i}{\left\{R^2 + \left[a_i + (z^2+b_i^2)^{1/2}\right]^2\right\}^{1/2}}, \tag{5}$$

where $i = 1, 2$ represent the disc and bulge respectively. A third component, the halo, is represented by

$$\Phi_3(r) = -\frac{GM_c}{2r_c}\left[\ln\left(1+\frac{r^2}{r_c^2}\right) + \frac{2\arctan\left(\frac{r}{r_c}\right)}{r/r_c}\right], \tag{6}$$

and $r = (R^2+z^2)^{1/2}$. The various parameter values are given in Table 1. The birth positions of the pulsars are distributed exponentially in both galactocentric radius (R) and disc height (z), with scalelengths of 4.5 and 0.075 kpc respectively. The integration is performed using the Burlisch–Stoer integration algorithm from Press et al. (1992).

To recreate the observations we 'observe' the pulsars from a galactocentric radius of 8 kpc, using a volume of radius 3 kpc about the observer (this distance is chosen to recreate approximately the volume sampled by Harrison et al. 1993). To make more efficient use of our simulations we can use eight simultaneous observers spread equidistantly around the circle $R = 8$ kpc (see Fig. 9). As long as the observing volumes do not overlap, the observations from all volumes can be added together. We calculate the transverse velocities after subtracting the circular velocity of the observer, and derive the observed velocity distribution as a function of time.

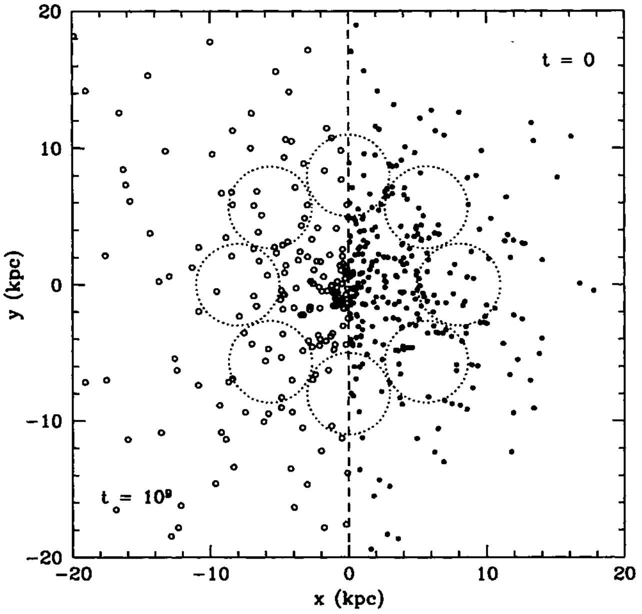

Figure 9. Monte Carlo pulsars. The right half of this projection on to the galactic plane is for the initial positions of the pulsars. The mirror images of the positions of the same pulsars after 10^9 yr are shown on the left-hand side. The circles indicate the volumes sampled by the 'observations'.

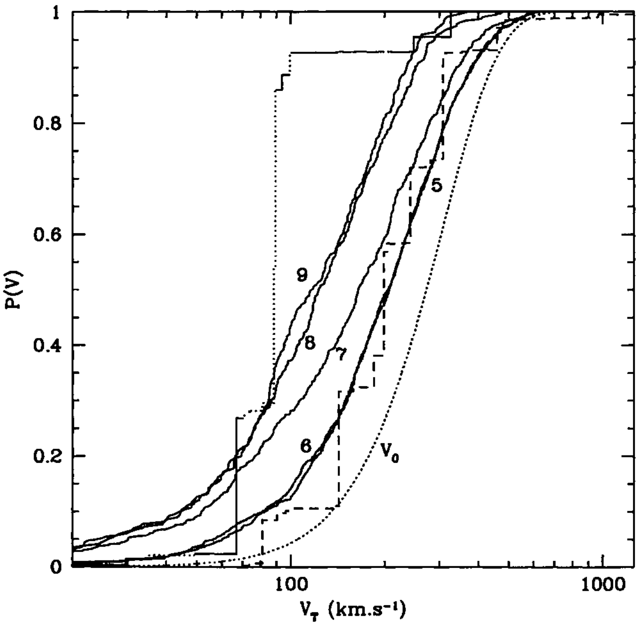

Figure 10. Velocity distribution evolution. The solid lines, labelled by log(age), are the time-dependent transverse velocity distributions produced by the initial three-dimensional distribution labelled V_0 (dotted line). The dashed line indicates the observed 'young' ($< 10^7$ yr) pulsar proper motion distribution. The heavy line indicates the observed 'old' ($> 10^8$ yr) proper motion distribution. The line is solid where the contribution is from a binary and dotted when the corresponding pulsar is isolated.

Fig. 10 shows the evolution of the transverse velocity distribution with time. We can compare it with our corrected observed distributions, both for $t < 10^7$ yr (we see that the 10^5-yr distribution

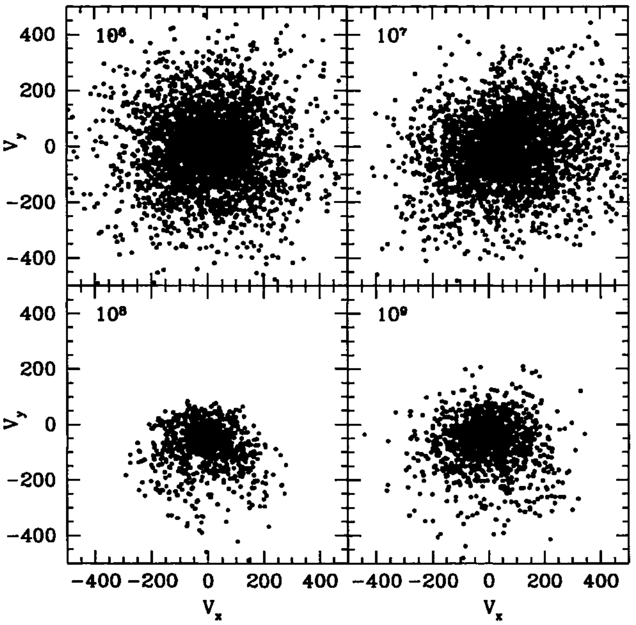

Figure 11. The four panels (all to the same scale) show the radial (V_x) and transverse (V_y) velocities in the observer frame at four different ages. After 10^6 yr the velocity distribution is isotropic because only locally born pulsars are seen. After 10^7 yr an excess of radially outward motion is seen due to the net flux of pulsars moving out from the inner parts of the Galaxy. After 10^8 yr only pulsars from the lower velocity tail are seen, and these clearly exhibit the asymmetric drift.

confirms our analysis of Section 3) and similarly for $t > 10^8$ yr (see below).

An additional constraint on the evolved distribution is the magnitude of the asymmetric drift. For the kick distribution given by equation (3) the magnitude is small ($V_\ell < 10\,\mathrm{km\,s^{-1}}$) to begin with, reaching a maximum of $\sim$80 km s^{-1} after 10^8 yr and then reaching an asymptotic value of $\sim$60 km s^{-1} after 10^9 yr. The evolution of this quantity is shown in Fig. 7. Again, we see that we have the correct order of magnitude. Fig. 11 demonstrates the magnitude of the effect (although it will be modified by the approach to the death line and larger binary fraction of the older pulsars) in our simulations, as we show V_x (radial in observer frame) and V_y (transverse) at different ages. Apart from the conspicuous appearance of the asymmetric drift at later times, we note also an excess of pulsars moving radially outwards at ages $\sim 10^7$ yr. This is because more pulsars are born at smaller galactocentric radii and the time-scale for the escaping pulsars to travel a few kpc at $\sim$300 km s^{-1} is a few $\times\,10^7$ yr. The smaller signal makes this a more difficult effect to observe in the real pulsar population.

As we have noted before, a proper comparison of our results with the older pulsars requires an analysis of the millisecond pulsar distribution. A treatment of the evolution of the entire pulsar sample is beyond the scope of this paper, but we can answer the question of whether the oldest pulsars have a velocity distribution that is a direct result of the kick distribution given by equation (3). To do this we model the effects of the surveys undertaken at Molonglo (D'Amico et al. 1988), Jodrell Bank (Biggs & Lyne 1992), Arecibo (Wolszczan 1991; Nice, Taylor & Fruchter 1993; Camilo et al. 1993; Foster, Wolszczan & Camilo 1993; Thorsett et al. 1993) and Parkes (Johnston et al. 1993; Bailes et al. 1994; Lorimer et al. 1995), although the $V_{\max}$ was dominated by the surveys from the latter two telescopes. By restricting ourselves to pulsars with characteristic ages $> 10^8$ yr, we hope to determine the velocity distribution of the oldest pulsars.[3] Fig. 10 shows the $V_{\max}$-weighted velocity distribution for this sample. We note that our derived distribution is strongly peaked between 60 and 100 km s^{-1}, with the biggest weights attributed to the pulsars B0655+64, B1952+29 and J2322+2057, although six other pulsars also lie within this range.

A striking feature of Fig. 10 is that the old pulsars do not have a velocity distribution that comes from an unadulterated evolution of the initial velocity distribution. However, this is not surprising, since many of the older pulsars are in binaries. Thus, the distribution of centre-of-mass velocities will be different from the kick velocities given to the neutron star component. Calculations of this effect have been performed by numerous authors for different kick distributions (e.g. Dewey & Cordes 1987; Brandt & Podsiadlowski 1995; Kalogera 1996) and introduce further uncertainties into the problem through the initial distribution in orbital period, companion masses and pre-supernova helium star masses. Thus we shall defer detailed discussions to future work, although we note that the effect is indeed to reduce the high-velocity tail. Another effect is that the low-velocity end is also expected to be depleted due to the gravitational scattering off giant molecular clouds (Spitzer & Schwarzschild 1951, 1953) or spiral arms (Barbanis & Woltjer 1967). This is a concern for calculations of the observability of old neutron stars accreting from the interstellar medium (because the accretion rate and thus luminosity $\propto v^{-3}$), and Madau & Blaes (1994) have recently calculated this effect, yielding a peak in the range 60–90 km s^{-1} after 10^{9-10} yr [although they used the Narayan & Ostriker (1990) kick distribution].

5 DISCUSSION

Our approach above is designed to provide a robust estimate of the characteristic birth velocities of pulsars. We have tested the procedure with simple Monte Carlo simulations using known distribution functions. We find that we can reproduce the mean velocities to within $\sim$30–50 km s^{-1}, although the exact shape of the distribution at the high-velocity end ($> 300\,\mathrm{km\,s^{-1}}$) is not well constrained. To estimate better the shape of the distribution will require more detailed modelling and the use of more of the pulsar population information, such as was done by Narayan & Ostriker (1990) or Bhattacharya et al. (1991).

Another recent analysis by Iben & Tutukov (1996) finds a large birthrate of very slow ($< 10\,\mathrm{km\,s^{-1}}$) pulsars. Their treatment neglects flux limits (although some inferences are made on the basis of a nearby sample only, their full analysis uses pulsars at all distances), and treats the proper motion limits using various ad hoc analytic cut-offs. They also use no upper age cut-offs, so that their large birthrate of slow pulsars is due to three old ($t > 10^7$ yr) pulsars, which acquired significant weight because of their small luminosities. When restricting ourselves to ages $< 10^7$ yr, we do not find any such low-velocity tail. Furthermore, we find that the flux cut-off is responsible for the detection limit of more pulsars than the proper motion limit.

Some constraints on the nature of pulsar kicks can be obtained by analysing the properties of binaries containing neutron stars. Recently, in the light of the results of Lyne & Lorimer (1994), Brandt & Podsiadlowski (1995) analysed the effect of the revised kick distribution on the post-supernova orbital parameters of

[3] Note that $\sim$40 per cent of this sample are still long-period ($P > 10$ ms) pulsars. However, if we further restrict our sample to only millisecond pulsars, we obtain a similar result.

neutron star binaries. Their analysis indicated that an isotropically distributed kick velocity of 450 km s^{-1} was inconsistent with the eccentricity–orbital period distribution of the observed binary population. However, a kick velocity of 200 km s^{-1} was perfectly consistent. Similarly, Wijers, van Paradijs & van den Heuvel (1992) obtained an upper limit of 400 km s^{-1} for a characteristic kick velocity from an analysis of the eccentricities of the known double neutron star binaries. While these results are in good agreement with our analysis, we should note that Brandt & Podsiadlowski showed that the Lyne & Lorimer distribution, despite its high characteristic velocity, was also consistent with the period–eccentricity relation, by virtue of its large low-velocity tail. Thus, the surviving binary parameters do not provide a means for distinguishing between the two distributions.

The most direct test of the pulsar kick velocities is to find the supernova remnant associated with the birth of a given pulsar. If the pulsar was born at the centre of a circularly symmetric supernova remnant, we may infer a proper motion from its displacement and thus obtain a velocity. Much work has been devoted to this (Caraveo 1993; Frail, Goss & Whiteoak 1994), and the results indicate velocities significantly larger than the mean velocity derived by other methods. In fact, the mean velocity of the Frail et al. sample is 990 km s^{-1}, with a median of 480 km s^{-1}. Whether or not this discrepancy with respect to our results is real depends on the veracity of the various assumptions used to infer a proper motion from a supernova association (see Kaspi 1996). Some of the problematic assumptions discussed by Frail et al. include the difficulty of defining the shape of a remnant and the possible displacement of the supernova blast centre with respect to the geometric centre of the remnant (perhaps caused by expansion into an inhomogeneous surrounding medium). It is illuminating, although not conclusive, to note that the two pulsars associated with supernova remnants that have measured proper motions are the Crab and Vela pulsars, with transverse velocities of 150 and 120 km s^{-1} respectively. (In particular, Frail et al. note that the methods used on other pulsar-remnant associations would imply a velocity of 800 km s^{-1} for the Vela pulsar!)

A revised velocity distribution can possibly affect the results of statistical analyses of the pulsar population as a whole. Indeed, it may even contribute to an explanation for the conflicting claims concerning magnetic field decay. Narayan & Ostriker (1990) used a Maxwellian distribution for their paper on the pulsar population as a whole. They used two different populations of pulsars and their kick distribution was a function of magnetic field. Their two populations had velocities that varied from 80 to 250 km s^{-1} for their S population and from 20 to 100 km s^{-1} for their F population. The analysis of Bhattacharya et al. (1991) also used a Maxwellian, but with a somewhat lower dispersion of 110 km s^{-1}. Our value lies a little above these characteristic values, consistent with the fact that the dispersion measure distances are now thought to be larger than those used in the above analyses. More recent analyses, such as that of Hartman (1996), attempt to constrain the velocity distribution itself using this approach. Results from such studies may differ significantly from ours because of the sundry additional assumptions required. In particular, the Crab and Vela pulsars acquire significant weight in pulsar current studies (see Phinney & Blandford 1981) because they require significant luminosity evolution to avoid a large population of high field objects near the death line. This could produce a larger low-velocity tail than in our case. Similarly, inclusion of some of the more optimistic pulsar–supernova remnant associations in the population synthesis can result in a high-velocity tail. Relative to such analyses, our results represent a minimal model, required to describe the observed proper motions under the assumption that the death line does not remove a significant fraction of pulsars before a spin-down age of 10^7 yr.

An interesting feature of our result is the lack of a pronounced low-velocity tail. This has implications for the retention fraction of neutron stars in globular clusters, as well as the existence of some obviously low-velocity pulsars with ages > 10^7 yr. Globular cluster central escape velocities are $\leq$ 50 km s^{-1}. If all kick velocities are $\sim$ 250 km s^{-1}, then no pulsars born from isolated stars are retained! If the distribution is Maxwellian, then the fraction retained is about 0.2 per cent, which is still extremely low! Yet, the retention fraction of neutron stars is claimed to be of the order of 10 per cent (Phinney 1993) or higher (Hut & Verbunt 1983). However, it is possible that binaries, either primordial or dynamically formed, could be responsible for the pulsars found in globular clusters (see Hut et al. 1992; Drukier 1996). Brandt & Podsiadlowski (1995) find that $\sim$17 per cent of binaries remain bound if the kick velocity is 200 km s^{-1}, which is the right order of magnitude to explain the required mass in dark massive remnants. However, this is likely to be an upper limit on the retention fraction because even systems that remain bound can receive significant centre-of-mass velocities. Thus, Drukier (1996), using the results of Lyne & Lorimer and Brandt & Podsiadlowski, has shown that most globular clusters would retain $\sim$1–5 per cent of their neutron stars. In the light of this, we should point out that the lack of a low-velocity tail in our distribution is not an artefact of the $V_{\max}$ weighting. Of the 51 pulsars in our sample with ages less than 10^7 yr, the lowest transverse velocity is 70 km s^{-1}. If there are young pulsars with very small velocities, then they have not been measured yet. Another possible complication is the creation of fast (P < 0.1 s) pulsars with initial timing ages > 10^7 yr. The birthrate of such pulsars is not constrained by our analysis because of both our age cut-off and the restriction of our analysis to the early Molonglo and Green Bank surveys.

In conclusion, we have shown that the distribution of young pulsar proper motions, corrected for selection effects, is consistent with a characteristic kick velocity at birth of $\sim$250–300 km s^{-1}. We find little evidence for a significant low-velocity tail to the kick distribution. Our method is robust in its reproduction of the mean velocity and low-velocity shape of the distribution. However, the shape of the distribution at velocities > 300 km s^{-1} is not well constrained by this method because of poor statistics. Our results are in good agreement with the properties of binaries containing neutron stars, and pleasantly close to the value inferred from numerical supernova simulations [e.g. Burrows, Hayes & Fryxell (1995), who inferred kicks of $\sim$300 km s^{-1} from core recoil during the collapse, but neglected asymmetries in the initial mass distribution (see Burrows & Hayes 1996) which could lead to even higher velocities]. Finally, we have shown that the pulsars with high characteristic ages show the asymmetric drift, corresponding to a dynamically old population. The velocity distribution of these pulsars is affected by their genesis in binaries as well as their subsequent motion through the Galaxy.

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