ON A DIFFERENTIAL EQUATION OCCURRING IN PAGE'S THEORY OF ELECTROMAGNETISM

By H. Bateman

California Institute of Technology, Pasadena

Communicated by R. A. Millikan, June 7, 1920

In a recent article Mr. Leigh Page[1] has generalized the electromagnetic equations by introducing the idea of a rotation of the field round a moving electric pole. A line of electric force is supposed to be the locus of points traveling with constant velocity, c, in directions which vary according to a law giving rise to three differential equations of type

$$k\frac{dl}{d\tau} = c(f + vr - wq) - m(c^2r - gu + fv) + n(c^2q - fw + hu) - cl[l(f + vr - wq) + m(g + wp - ur) + n(h + uq - vp)], \quad (1)$$

where (cl, cm, cn) are the component velocities of the point which leaves the electric pole at time τ; (u, v, w), (f, g, h) and (p, q, r) are the components of the velocity, acceleration and angular velocity of the pole at this instant. The quantity, k, is defined by the equation $k = c^2 - u^2 - v^2 - w^2$. These equations are of a type considered in my book on *Differential Equations* (Ex. 7, p. 71) and can be reduced to two Riccatian equations by means of the substitutions

$$s = \frac{l + im}{1 + n},\ \sigma = \frac{l - im}{1 + n},\ i = \sqrt{-1}. \quad (2)$$

The lines of force can consequently be found by solving a Riccatian equation just as in the usual electromagnetic theory.[2]

It should be noticed that Page's expressions for the electric and magnetic vectors may be written in the forms

$$\mathbf{E}_x = \frac{1}{c}\frac{\partial(\tau, \sigma)}{\partial(x, t)} - \frac{\partial(\tau, \theta)}{\partial(y, z)},$$

$$\text{------------} \quad (3)$$

$$\mathbf{H}_x = \frac{\partial(\tau, \sigma)}{\partial(y, z)} + \frac{1}{c}\frac{\partial(\tau, \theta)}{\partial(x, t)},$$

where

$$\theta = \frac{ec^3}{4\pi k}\frac{(x - \xi)p + (y - \eta)q + (z - \zeta)r}{c^2(t - \tau) - u(x - \xi) - v(y - \eta) - w(z - \zeta)},$$

$$\sigma = \frac{e}{4\pi}\frac{(x - \xi)f + (y - \eta)g + (z - \zeta)h + k}{c^2(t - \tau) - u(x - \xi) - v(y - \eta) - w(z - \zeta)}, \quad (4)$$

e is the electric charge associated with the moving electric pole, (ξ, η, ζ) are the coördinates of the pole at time τ and τ is defined in terms of (x, y, z, t) by the equation

$$[x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t - \tau)^2. \quad \tau < t.$$

The electromagnetic field is thus obtained by superposing on the usual electromagnetic field of a moving electric pole a field derived from a type of field, which I have considered elsewhere,[3] by writing **H** for **E** and $-$**E** for **H**. The properties of this additional field may be derived at once from the properties of the field described previously by simply interchanging the words magnetism and electricity.

If (f, g, h) are no longer restricted to be the components of the acceleration and k is no longer restricted to be $c^2-u^2-v^2-w^2$, the formulae (3), (4) still specify an electromagnetic field in which a line of force is the locus of points moving with velocity c in directions specified by a set of equations of type (1) and these equations can still be reduced to Riccatian equations by means of the substitution (2).

In a field of this more general type both electricity and magnetism travel away from the electric pole with the velocity of light while in the case of a field of Page's type there is an emission of magnetism but no emission of electricity. There is, however, no magnetic charge associate with the moving electric pole.

In a field of this general type it is possible when $f = g = h = p = q = r = 0$ for the points which trace out a line of force to move in one constant direction which is the same for all and for there to be no radiation of energy even through the electric pole has an acceleration.[3] It is possible, then, for an electric pole of this more general type to describe a circular orbit without radiating energy, as in Bohr's theory of the hydrogen atom. In the transition from one circular orbit to another the angular velocity (p, q, r) may be different from zero and there may be radiation of the type described by Page.

The weak point of the present theory is that it requires the presence of electric and magnetic currents in the space surrounding the electric pole. This space in fact contains a continuous distribution of electric and magnetic particles which have been emitted from the moving pole and are travelling along straight lines with the velocity of light.

[1] *Proc. Nat. Acad. Sci.*, **6**, March, 1920 (115).

[2] *Amer. J. Math.*, **37**, 1915 (192); see also Murnaghan, F. D., *Ibid.*, **39**, 1917; and *Johns Hopkins Circular*, July, 1915.

[3] *Proc. London Math. Soc.*, (Ser. 2) **18**, 1919 (95), paragraph 10.

A NEW PROOF OF A THEOREM DUE TO SCHOENFLIES

By J. R. Kline

Department of Mathematics, University of Pennsylvania

Communicated by E. H. Moore, July 7, 1920

In a paper recently published in the Transactions of the American Mathematical Society, Professor Robert L. Moore[1] proved the following theorem: If $ABCDA$ is a closed curve, then there exist two sets of arcs α_1 and α_2 such that (1) each arc of α_1 lies wholly within $ABCDA$ except