Eng

CALIFORNIA INSTITUTE OF TECHNOLOGY

ELECTRON TUBE AND MICROWAVE LABORATORY

TRAVELING WAVE COUPLERS FOR LONGITUDINAL BEAM TYPE AMPLIFIERS

by
R. W. Gould

TECHNICAL REPORT NO. 8

August 1958

A REPORT ON RESEARCH CONDUCTED UNDER

CONTRACT WITH THE OFFICE OF NAVAL RESEARCH

AND THE SPERRY RAND CORPORATION

TRAVELING WAVE COUPLERS

FOR

LONGITUDINAL BEAM TYPE AMPLIFIERS

by

Roy W. Gould

CALIFORNIA INSTITUTE OF TECHNOLOGY

Pasadena, California

A Technical Report to the Office of Naval Research

Contract Nonr 220(13)

and the Sperry Rand Corporation

August 1958

ABSTRACT

The equations governing traveling wave interaction between an electron beam and a slow wave circuit are formulated in terms of amplitudes of circuit mode and slow and fast space charge modes. The resulting equations are solved to find expressions for the matrix which relates the mode amplitudes at the output of the traveling wave coupler to the mode amplitudes at the input. The properties of this matrix are discussed and numerical values given for Kompfner Dip.

Matrices for velocity jumps and drift regions are also given and the characteristics of couplers which are preceded by or followed by a drift region and velocity jump are discussed.

It is shown that necessary and sufficient conditions for the removal of beam noise from the fast space charge wave by any lossless coupler are that, for a circuit input, there be no circuit output ($M_{11} = 0$) and no slow space charge wave output ($M_{21} = 0$).

These results are then applied to the design of fast space charge wave couplers for longitudinal beam type parametric amplifiers.

I INTRODUCTION

The prospect of obtaining a very significant decrease in the noise figure of electron beam type microwave amplifiers through the use of the parametric principle has stimulated considerable work on beam type parametric amplifiers. Conventional longitudinal beam amplifiers depend critically on the negative power flow associated with the slow space charge wave whereas parametric amplifiers can be made to use the fast space charge wave which has positive power flow. The significant distinction to be noted here is that noise can be completely removed from the fast space charge wave whereas noise on the slow space charge wave cannot. Parametric amplification in electron beams has already been analyzed and discussed by Louisell and Quate[1] and the purpose of this paper is to describe the properties of a certain class of couplers which make it possible to couple to the fast space charge wave only. While it is also possible to construct fast wave couplers using resonant cavities the possibility of using a traveling wave interaction immediately suggests itself as a method with potentially greater bandwidth.

The simplest coupler of this type, conceptually, is a large QC traveling wave structure operated at the Kompfner Dip[2]. When QC is large coupling to the slow space charge wave is negligible and a complete interchange of energy takes place between the circuit and the fast space charge wave[3]. Any circuit input is transferred completely to the fast space charge wave and any disturbance on the fast space charge wave is transferred completely to the circuit. Any noise or signal on the slow space charge wave passes through the interaction region unchanged. Used as an input coupler noise would be completely removed from the fast space charge wave. As an output coupler it would not be sensitive to the noise which remains on the slow space charge wave. Such a coupler is very attractive and it is of interest to inquire whether or not the large QC restriction is essential. Large QC may be obtained by reducing the helix to beam coupling and hence reducing C .[4] At the same time CN for Kompfner Dip increases and the coupler may become too long.

We expect that as QC is decreased coupling to the slow space charge wave becomes important and that the noise may no longer be completely removed from the fast space charge wave. The following analysis was performed in an attempt to answer the following types of questions about operation at small and intermediate values of QC :

1. How large is the coupling to the slow space charge wave?
2. How much noise remains on the fast space charge wave after passing through such a coupler?
3. Does there exist values of b and CN for which there is no coupling to the slow space charge wave?
4. Is it possible, with the aid of velocity jumps and drift regions, to achieve coupling to the fast wave only and removal of the beam noise from the fast space charge wave?

Since these questions are couched in terms of mode amplitudes it was found to be convenient to first formulate the equations of traveling wave interaction in these terms by introducing a transformation from the physical variables: circuit voltage, beam current, and beam velocity to the three mode amplitudes. The result is a derivation of the coupled mode equations[3,5]. When formulated in this way, Haus and Robinson's[6] theory of linear transducers is immediately applicable.

II TRAVELING WAVE INTERACTION IN TERMS OF MODE AMPLITUDES

In this section the theory of traveling wave interaction is formulated in terms of the amplitudes of the circuit wave, slow space charge wave and fast space charge wave. The usual small C approximation is made throughout in order to simplify the results. In terms of the physical variables:

v_1 = a.c. beam velocity

ρ_1 = a.c. beam charge density

i_1 = a.c. beam current density

V_c = a.c. circuit voltage

V_{sc} = a.c. space charge potential

the linearized equations of traveling wave interaction are:

the electronic equation of motion,

$$j\omega v_1 + \frac{\partial}{\partial z}(u_o v_1) = -\frac{e}{m}\frac{\partial}{\partial z}(V_c + V_{sc}) , \qquad (1)$$

the equation of continuity

$$j\omega\rho_1 + \frac{\partial i_1}{\partial z} = 0 , \qquad i_1 = \rho_o v_1 + \rho_1 u_o , \qquad (2)$$

the equation for the space charge voltage

$$\frac{\partial^2 V_{sc}}{\partial z^2} = R^2 \frac{\rho_1}{\epsilon_o} , \qquad R = \text{space charge reduction factor} , \qquad (3)$$

and the circuit equation

$$\left(\frac{\partial}{\partial z} + \Gamma_1\right) V_c = \pm j\beta_e \frac{K}{2} (\Sigma i_1) , \tag{4}$$

where Γ_1 is the circuit propagation constant in the absence of the beam, K is the Pierce interaction impedance, and Σ is the cross-sectional area of the beam. The upper sign applies for forward wave circuits and the lower sign applies for backward wave circuits.

Introducing the mode variables defined in the manner suggested by Haus and Robinson[6]

$$a_1 = \frac{V_c}{\sqrt{2K}} \quad \text{circuit mode amplitude} \tag{5}$$

$$a_2 = \frac{1}{2\sqrt{2W}} (V_1 - WI_1) \quad \text{slow space charge mode amplitude} \tag{6}$$

$$a_3 = \frac{1}{2\sqrt{2W}} (V_1 + WI_1) \quad \text{fast space charge mode amplitude} \tag{7}$$

where $W = \frac{V_o}{2I_o} \frac{\omega_p R}{\omega}$, the space charge wave impedance of the electron beam, V_1 is the kinetic voltage of the electron beam and $I_1 = \Sigma i_1$ is the a.c. convection current in the electron beam. These variables have the property that their absolute square gives the power flow associated with that mode (except that $|a_z|^2$ gives the <u>negative</u> of the power flow associated with the slow space charge mode). Upon substituting these new variables into equations (1) through (4) performing some straightforward algebraic manipulations, and making the small C approximation consistently we obtain the equations for the mode amplitudes,

$$\left(\frac{\partial}{\partial z} + \Gamma_1\right)a_1 \pm j\kappa a_2 \mp j\kappa a_3 = 0 \tag{8}$$

$$- j\kappa a_1 + \left[\frac{\partial}{\partial z} + j(\beta_e + \beta_q)\right] a_2 = 0 \tag{9}$$

$$- j\kappa a_2 + \left[\frac{\partial}{\partial z} + j(\beta_e - \beta_q)\right] a_3 = 0 \tag{10}$$

where $\kappa = \frac{\beta_e}{2}\sqrt{\frac{K}{W}}$ is the coupling constant between the circuit and fast and slow space charge waves, $\beta_e = \frac{\omega}{u_o}$ is the electronic wave number and $\beta_q = \frac{R\omega_p}{u_o}$ is the reduced plasma wave number. Equations (8) through (10) represent an extension of Pierce's coupling of modes of propagation theory[5] to coupling between <u>three</u> modes together with an explicit expression for the coupling constant. Note that the circuit is coupled equally to the fast and slow space charge waves[7] and that the two space charge waves are not coupled to each other.

It is convenient to extract a phase factor $e^{-j\beta_e z}$ from the definitions of the mode amplitudes by writing

$$\begin{aligned} a_1 &= A_1 e^{-j\beta_e z} \\ a_2 &= A_2 e^{-j\beta_e z} \\ a_3 &= A_3 e^{-j\beta_e z} \end{aligned} \tag{11}$$

and express equations (8), (9) and (10) in terms of the dimensionless traveling wave to be variables, b , d , $\xi = \beta_e Cz$, $\frac{\beta_q}{\beta_e C} = \sqrt{4QC}$, and $\frac{\kappa}{\beta_e C} = k$

$$\left[\frac{\partial}{\partial \xi} + jb \pm d\right] A_1 \pm jkA_2 \mp jkA_3 = 0 \qquad (12)$$

$$- jkA_1 + \left[\frac{\partial}{\partial \xi} + j\sqrt{4QC}\right] A_2 = 0 \qquad (13)$$

$$- jkA_1 + \left[\frac{\partial}{\partial \xi} - j\sqrt{4QC}\right] A_3 = 0 \qquad (14)$$

It is of interest to note that the dimensionless coupling constant k is a function of QC only,

$$k^2 = \frac{1}{2\sqrt{4QC}} \qquad (15)$$

To solve the three simultaneous first order linear equations assume that each independent variable has an exponential dependence on ξ $(e^{\delta\xi})$. For solutions of this type the determinant of the resulting algebraic equations must vanish

$$(\delta + jb \pm d)(\delta^2 + 4QC) \pm j = 0 \qquad (16)$$

This is the familiar traveling wave tube characteristic equation. A general solution may be written as the superposition of the three characteristic waves

$$A_i = \sum_{j=1}^{3} C_{ij}\, e^{\delta_j \xi} \qquad (17)$$

where certain relations exist between the C_{ij} by virtue of equations (12) through (14).

Let us apply these solutions to a length ℓ of traveling wave section shown in Figure 1 to find the mode amplitudes A_1', A_2' and A_3' at the output of the coupler when the input mode amplitudes are $A_1 A_2 A_3$. We know in advance that the outputs are linearly related to the inputs

$$A_1' = M_{11}A_1 + M_{12}A_2 + M_{13}A_3 \tag{18}$$

$$A_2' = M_{21}A_1 + M_{22}A_2 + M_{23}A_3 \tag{19}$$

$$A_3' = M_{31}A_1 + M_{32}A_2 + M_{33}A_3 \tag{20}$$

or simply

$$A' = MA \tag{21}$$

where M is a three by three square matrix and A' and A are three element column and row matrices, respectively. A straightforward application of the solutions (17) to the case of initial conditions $A_1 A_2 A_3$ yields the following expressions for the elements of the M matrix

$$M_{11} = \sum_{i=1}^{3} \frac{\delta_i^{\,2} + 4QC}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{22}$$

$$M_{21} = jk \sum_{i=1}^{3} \frac{\delta_i - j\sqrt{4QC}}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{23}$$

$$M_{31} = jk \sum_{i=1}^{3} \frac{\delta_i + j\sqrt{4QC}}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{24}$$

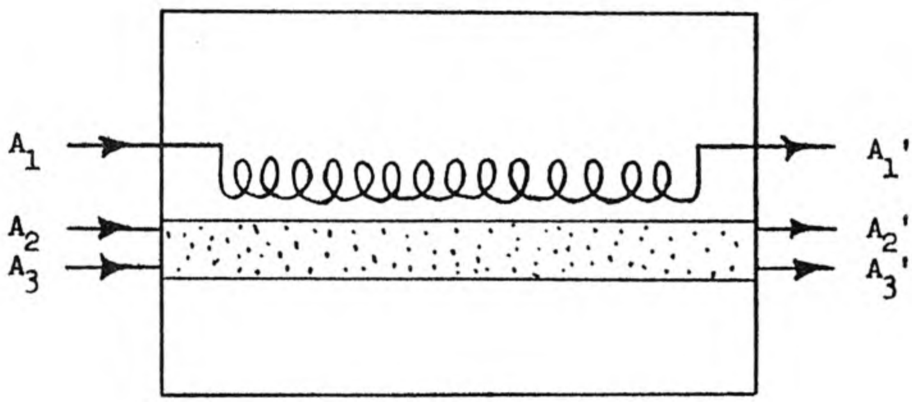

Figure 1.

Traveling Wave Section

$$M_{22} = jk^2 \sum_{i=1}^{3} \frac{(\delta_i - j\sqrt{4QC})(\delta_j + j\sqrt{4QC})(\delta_k + j\sqrt{4QC})}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{25}$$

$$M_{32} = \pm k^2 \sum_{i=1}^{3} \frac{1}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{26}$$

$$M_{33} = -jk^2 \sum_{i=1}^{3} \frac{(\delta_i + j\sqrt{4QC})(\delta_j - j\sqrt{4QC})(\delta_k - j\sqrt{4QC})}{(\delta_i - \delta_j)(\delta_i - \delta_k)} e^{\delta_i \xi} \tag{27}$$

where i, j, k, are cyclical permutations of the integers 1, 2, 3 and $\xi = \beta_e C \ell$. In writing Equation (26) we have made use of the fact that

$$(\delta_i + j\sqrt{4QC})(\delta_j + j\sqrt{4QC})(\delta_k + \sqrt{4QC}) = \mp j \tag{28}$$

a result which follows from the characteristic equation (16). We have not written the expressions for M_{12}, M_{13}, and M_{23} since it is possible to show, with the aid of equation (28), that

$$M_{12} = \mp M_{21} \tag{29}$$

$$M_{13} = \pm M_{31} \tag{30}$$

$$M_{23} = -M_{32} \tag{31}$$

These expressions indicate certain symmetry properties of the coupler. For example (29) states that a unit amplitude slow wave at the input produces a circuit wave amplitude at the output which is equal to the slow wave amplitude produced by a unit input to the circuit. Similar interpretations may be given to (30) and (31). In addition these matrix elements satisfy certain relations, based on conservation of energy[6],

$$M^{+} P M = P \tag{32}$$

$$M P M^{+} = P \tag{33}$$

where P is the parity matrix

$$P = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{34}$$

and M^{+} is the Hermitian conjugate of M. In terms of the parity matrix the symmetry relations (29) to (31) can be written $(PMP)_{ij} = M_{ji}$ or $\widetilde{PMP} = M$ when the tilde indicates the transpose matrix.

We have verified that these energy conservation relations are indeed satisfied by substitution of some of the numerical results presented in Table 1.

We have computed the matrix elements (22) through (27) for $QC = 1$, $1/4$, and $1/16$, $0 < \xi < 4.75$, and for a number of different b values. Approximately 30 seconds is required for the calculation of a complete set of matrix elements for each combination of QC, ξ, and b. A few qualitative observations are summarized here.

1. There does not appear to be a suitable choice of ξ and b which makes M_{21} equal to zero, although it is small for large QC values. An idea of its magnitude and the magnitude of other matrix elements can be obtained from Table 1.
2. It is possible to remove any residual slow space charge wave with a velocity jump[8] as long as $|M_{31}| > |M_{21}|$.[9] This may be regarded as simply an impedance matching problem.[10]

TABLE I

MATRIX ELEMENTS AT KOMPFNER DIP ($M_{11} = 0$)

(Magnitude and Phase Angle)

QC	b	ξ	M_{21}		M_{23}		M_{22}		M_{31}		M_{33}	
.05	-1.5168	2.0037	1.0936	121.6°	1.6205	-131.3°	2.1959	-13.5°	1.4818	-176.2°	1.1959	-69.1°
.10	-1.5121	2.0367	.8339	108.0°	1.0859	-129.4°	1.6954	-39.3°	1.3021	-162.1°	.6954	-39.6°
.20	-1.5042	2.1129	.5839	248.6°	.6761	-124.6°	1.3409	-78.4°	1.1580	-139.8°	.3409	9.1°
.30	-1.5003	2.2502	.3816	58.2°	.4085	-113.8°	1.1456	-124.7°	1.0703	-111.0°	.1456	77.1°
.40	-1.5046	2.3325	.3078	43.0°	.3221	-105.4°	1.0947	-145.9°	1.0463	-96.5°	.0947	115.1°
.50	-1.5328	2.5070	.2047	8.4°	.2090	-79.9°	1.0419	176.9°	1.0207	-68.4°	.0419	-156.7°
.60	-1.6253	2.7315	.1568	-49.0°	.1587	-26.7°	1.0246	134.4°	1.0122	-30.0°	.0246	- 7.7°
.80	-1.8918	2.9611	.1696	-114.4°	.1721	39.8°	1.0288	69.8°	1.0143	35.6°	.0288	-170.2°
1.00	-2.0718	3.0871	.1339	-156.1°	.1351	78.2°	1.0179	17.7°	1.0089	84.4°	.0179	-41.3°
1.20	-2.2333	3.2678	.0983	139.7°	.0883	139.2°	1.0097	-40.3°	1.0048	139.2°	.0097	138.6°
1.40	-2.4152	3.4206	.1016	79.9°	.1021	-160.7°	1.0103	-95.1°	1.0051	-165.8°	.0103	-46.4°
1.60	-2.5763	3.5131	.0958	40.4°	.0962	-121.8°	1.0092	-141.3°	1.0046	-120.1°	.0092	7.8°
1.80	-2.7164	3.6131	.0786	-3.9°	.0788	-79.1°	1.0062	171.8°	1.0031	62.7°	.0062	-150.0°
2.00	-2.8562	3.7327	.0701	-60.7	.0703	-23.0°	1.0049	121.8°	1.0025	-25.5°	.0049	12.2°

3. It is possible to make M_{11} equal to zero by proper choice of and b (Kompfner dip condition). Under these conditions the magnitude of M_{31} is always larger than the magnitude of M_{21} . This result follows directly from conservation of energy. The matrix elements at Kompfner Dip have been computed and are given in Table 1.
4. It is possible to make M_{33} equal to zero by proper choice of and b . The values of ξ and b which make M_{33} are slightly different from those which make M_{11} equal to zero.

III VELOCITY JUMPS, DRIFT SPACES, AND COMPOSITE SECTIONS

Since traveling wave couplers will be used in conjunction with drift spaces and velocity jumps the matrices describing the latter are also presented here. In a drifting beam the phases of the fast and slow space charge waves are delayed by $(\beta_e - \beta_q)$ and $(\beta_e + \beta_q)$ respectively if ℓ is the drift distance. The amplitudes are unchanged. If we neglect the common phase delay $\beta_e \ell$ and define $\theta = \beta_q \ell$ the drift space equations are

$$A_2' = A_2 e^{-j\theta} \tag{35}$$

$$A_3' = A_3 e^{+j\theta} \tag{36}$$

Although the circuit amplitude is not involved here it is convenient for matrix multiplication to use a three by three matrix and introduce the additional relation for the circuit amplitude $A_1' = A_1$. The matrix appropriate to a drift region is then

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-j\theta} & 0 \\ 0 & 0 & e^{j\theta} \end{pmatrix} \qquad \theta = \beta_q \ell \tag{37}$$

The equations describing a velocity jump are obtained by noting that in a velocity jump the kinetic voltage V_1 and a.c. beam current I_1 are invariant in an abrupt jump[10]. Using relations (6) and (7) the invariant principle is expressed

$$(A_2' + A_3')\sqrt{W'} = (A_2 + A_3)\sqrt{W} \tag{38}$$

$$(-A_2' + A_3')/\sqrt{W'} = (-A_2 + A_3)/\sqrt{W} \tag{39}$$

where the primed symbols refer to quantities after the jump and unprimed symbols refer to quantities before the jump. Solving for the matrix elements

$$M_{22} = M_{33} = \frac{1}{2}\left(\sqrt{\frac{W}{W'}} + \sqrt{\frac{W'}{W}}\right) \tag{40}$$

$$M_{23} = M_{32} = \frac{1}{2}\left(\sqrt{\frac{W}{W'}} - \sqrt{\frac{W'}{W}}\right) \tag{41}$$

To complete the matrix we assume that $A_1' = A_1$, hence $M_{11} = 1$ and $M_{12} = M_{21} = M_{13} = M_{31} = 0$. Thus the matrix for a velocity jump is

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\alpha^2+1}{2\alpha} & \frac{\alpha^2-1}{2\alpha} \\ 0 & \frac{\alpha^2-1}{2\alpha} & \frac{\alpha^2+1}{2\alpha} \end{pmatrix} \qquad \alpha = \sqrt{\frac{W}{W'}} \tag{42}$$

It is of interest to note that the matrix for a jump from impedance W to impedance W' is the same as the matrix for a jump from W' to W except for a change in sign of the off-diagonal elements.

The matrix which describes a composite section consisting of cascaded individual sections of these types can be written as the product of the matrices describing the individual sections. For example, a traveling wave section (matrix M) followed by a drift region (matrix M') followed by a velocity jump (matrix M'') has the properties given by the resultant matrix

$$M''M'M \tag{43}$$

IV FAST SPACE CHARGE WAVE COUPLERS

We now apply the preceding results to synthesize fast wave couplers for longitudinal beam type parametric amplifiers. First, consider the input coupler. The input coupler should perform two functions (a) remove the noise from the fast space charge wave, (b) place the input signal on the beam in the form of a fast space charge wave as effectively as possible. If we describe the coupler by the matrix M (in general it will consist of a number of cascaded elementary sections), the first requirement can be stated

$$M_{32} = M_{33} = 0 \tag{44}$$

i.e. there should be no noise output on the fast space charge wave due to noise inputs on <u>either</u> the fast or slow space charge waves. Assume that such a coupler can be constructed and consider the restrictions imposed by the assumption that the coupler is lossless (Equations (32) and (33)). The 33 component of Equation (33)

$$M_{31}M_{31}^{*} - M_{32}M_{32}^{*} + M_{33}M_{33}^{*} = 1 \tag{45}$$

together with Equation (44) shows that

$$\left| M_{31} \right| = 1 \tag{46}$$

The 32 component of Equation (33)

$$M_{31}M_{21}^{*} - M_{32}M_{22}^{*} + M_{33}M_{23}^{*} = 0 \tag{47}$$

together with Equations (44) and (46) show that

$$M_{21} = 0 \tag{48}$$

The 31 component of Equation (33)

$$M_{31}M_{11}^{*} - M_{32}M_{12}^{*} - M_{33}M_{13}^{*} = 0 \tag{49}$$

together with Equations (44) and (46) show that

$$M_{11} = 0 \tag{50}$$

We conclude then that an input on the circuit must produce no output on the circuit and no output on the slow space charge wave. The input signal is transferred completely to the fast space charge wave. Thus the second requirement of the coupler is automatically satisfied. The remainder of the restrictions imposed by Equations (32) and (33) are

$$|M_{22}|^2 - |M_{23}|^2 = 1 \tag{51}$$

$$-|M_{12}|^2 + |M_{13}|^2 = 1 \tag{52}$$

$$|M_{13}|^2 - |M_{23}|^2 = 1 \tag{53}$$

$$-|M_{12}|^2 + |M_{22}|^2 = 1 \tag{54}$$

$$M_{22}M_{12}^{*} = M_{23}M_{13}^{*} \tag{55}$$

$$M_{12}M_{13}^{*} = M_{23}^{*}M_{22} \tag{56}$$

From (55) and (56) or from (51) and (53) or from (52) and (54) it is seen that

$$|M_{13}|^2 = |M_{22}|^2 \tag{57}$$

From (57) we may write

$$M_{13} = \gamma e^{-j\theta_3} \qquad\qquad M_{22} = \gamma e^{-j\theta_2} \tag{58}$$

and the remaining relations (51) through (56) will be satisfied if

$$M_{12} = \sqrt{\gamma^2 - 1}\, e^{-j\theta_2} \qquad\qquad M_{23} = \sqrt{\gamma^2 - 1}\, e^{-j\theta_3} \tag{59}$$

provided that $\gamma^2 \geqslant 1$. The resultant matrix is

$$M = \begin{pmatrix} 0 & \sqrt{\gamma^2-1}\, e^{-j\theta_2} & \gamma e^{-j\theta_3} \\ 0 & \gamma e^{-j\theta_2} & \sqrt{\gamma^2-1}\, e^{-j\theta_3} \\ e^{-j\theta_1} & 0 & 0 \end{pmatrix} \tag{60}$$

and there are four remaining variables γ , θ_1 , θ_2 and θ_3 .

It has been shown that $M_{11} = M_{21} = 0$ is a necessary condition for the complete removal of beam noise from the fast space charge wave. In a similar way it can be shown that if $M_{11} = M_{21} = 0$, then M_{32} and M_{33} are also zero. Thus the condition $M_{11} = M_{21} = 0$ is also sufficient.[11]

A coupler which has these properties can be constructed using a traveling wave section in conjunction with velocity jumps and drift regions.. Since M_{11}

is to be zero the traveling wave section must be operated at Kompfner Dip. Under this condition the fast wave modulation of the beam is greater than the slow wave modulation, hence it is possible to completely remove the slow wave with an appropriate velocity jump following the traveling wave section. The matrix element connecting the circuit input to the slow wave output of such a composite coupler is, using the results of the previous section

$$M_{21} = \frac{\alpha^2 + 1}{2\alpha} e^{-j\theta} M'_{21} + \frac{\alpha^2 - 1}{2\alpha} e^{j\theta} M'_{31} \tag{61}$$

where $\alpha^2 = \frac{W}{W'}$ is the ratio of the beam impedance W before the velocity jump to the beam impedance W' after the jump, and M'_{21} and M'_{31} are matrix elements of the traveling wave section alone. This may be made zero with either of two choices

$$\theta = \frac{1}{2} \arg \frac{M'_{21}}{M'_{31}} + n\pi , \qquad \frac{W'}{W} = \frac{1 - \left|\frac{M'_{21}}{M'_{31}}\right|}{1 + \left|\frac{M'_{21}}{M'_{31}}\right|} \tag{62}$$

$$\theta = \frac{1}{2} \arg \frac{M'_{21}}{M'_{31}} + (n + \frac{1}{2})\pi , \qquad \frac{W'}{W} = \frac{1 + \left|\frac{M'_{21}}{M'_{31}}\right|}{1 - \left|\frac{M'_{21}}{M'_{31}}\right|} \tag{63}$$

where θ is the length of the drift region and $\frac{W'}{W}$ is the ratio of the beam impedance after the jump to the beam impedance before the jump. The first choice corresponds to a jump to lower velocity and the second to a jump to higher velocity. The velocity jump locations for the two cases differ by a quarter space charge wavelength. The magnitude and location of a jump to a lower velocity which makes $M_{21} = 0$ is shown in Figure 2. The physical

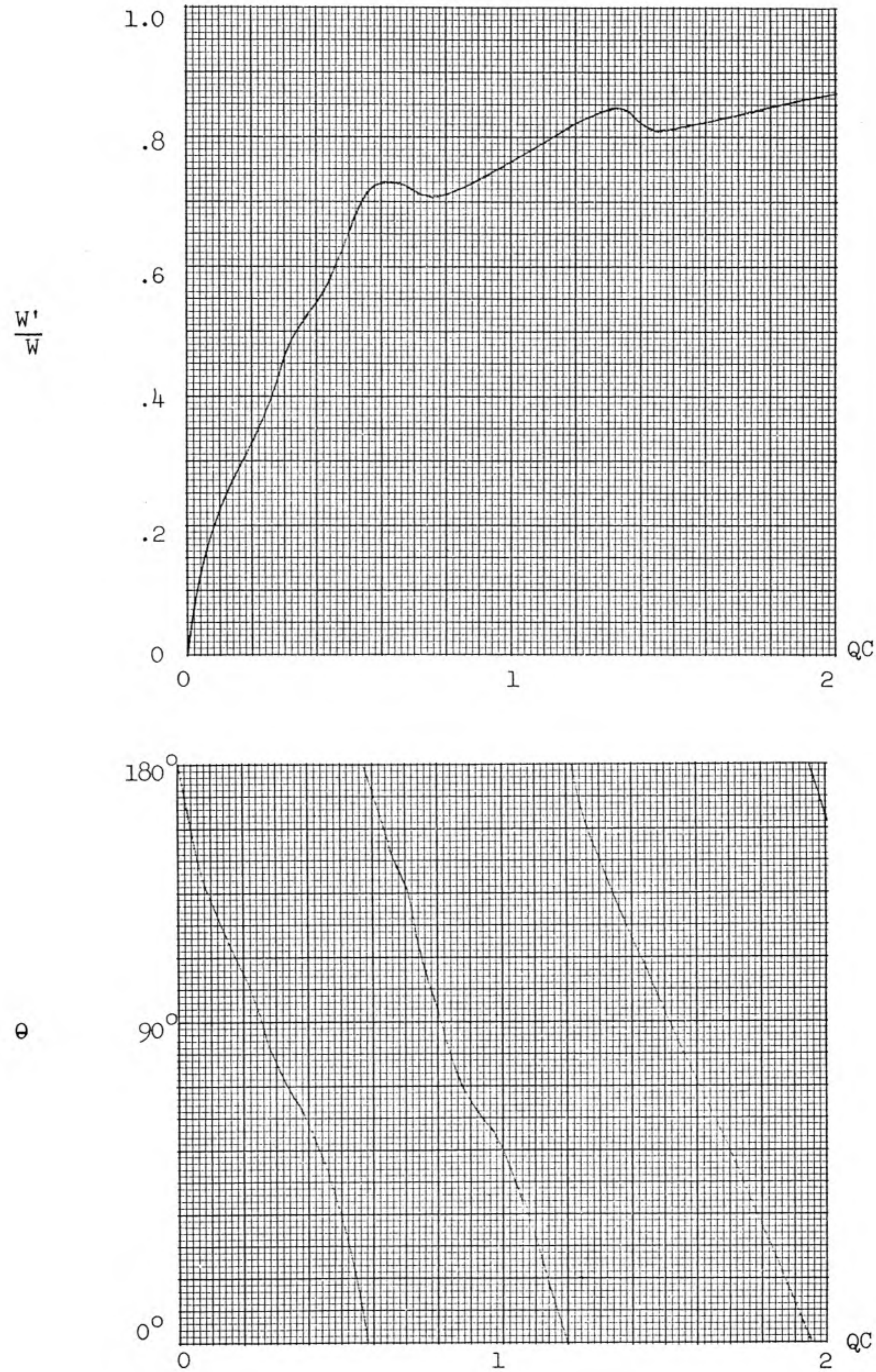

Figure 2. Magnitude of Impedance Change and Position of Velocity Jump for no Coupling to the Slow Space Charge Wave ($M_{21} = 0$).

configuration of the coupler is shown in Figure 3. This coupler transfers the entire input signal to the fast space charge wave and removes all beam noise from the fast space charge wave. Its matrix has the same form as Equation (60) where

$$\gamma = |M'_{13}| = |M'_{31}|$$

$$\theta_3 = \arg M'_{13} \tag{64}$$

$$\theta_2 = \arg M'_{12}$$

and M'_{13} and M'_{12} are elements of the traveling wave matrix given in Table 1. Furthermore by preceding the traveling wave section by drift regions and velocity jumps it is possible to obtain other values of γ, θ_2, and θ_3 without affecting the fundamental properties of the coupler expressed by Equations (44), (48), and (50).

The requirements on the output coupler of a parametric amplifier are different from those of the input coupler. First, the output coupler should not be sensitive to a slow wave input since the slow wave may be noisy (although the slow wave noise will not be amplified if the pump is in the form of a pure fast wave), or

$$M_{12} = 0 \tag{65}$$

Furthermore, the coupling to the fast wave (M_{13}) should be maximized. The 11 component of Equation (33) can be written

$$|M_{13}|^2 = 1 - |M_{11}|^2 + |M_{12}|^2 \tag{66}$$

From this relation it is seen that the coupling to the fast wave is maximized

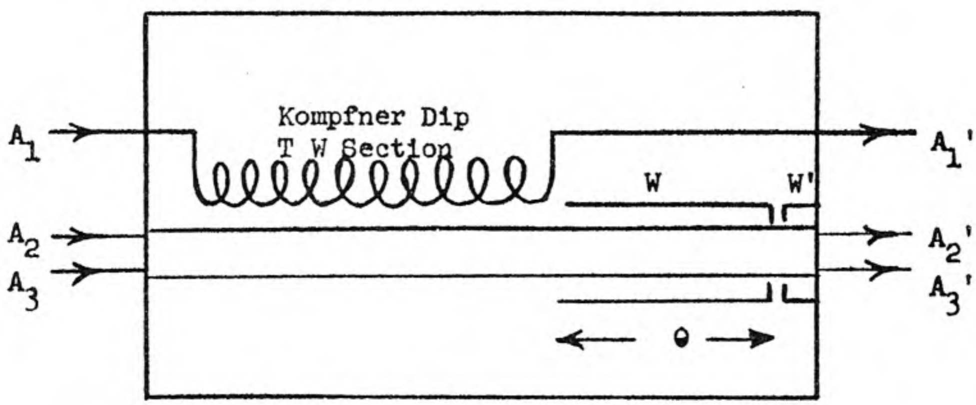

Figure 3.

Input Coupler consisting of Kompfner Dip Traveling Wave Section, Drift Region, and Velocity Jump.

when

$$M_{11} = 0 \tag{67}$$

or when the traveling wave section is operated at Kompfner Dip. When (65) and (67) are satisfied the energy conservation relations can be used to show that M_{23} and M_{33} are zero. It is possible to satisfy (65) by preceding the traveling wave section by a velocity jump and drift region. The 12 matrix element of the composite coupler is

$$M_{12} = M'_{12}\, e^{-j\theta} \frac{\alpha^2 + 1}{2\alpha} + M'_{13}\, e^{j\theta} \frac{\alpha^2 - 1}{2\alpha} \tag{68}$$

where $\alpha^2 = \frac{W'}{W}$ is the ratio of the beam impedance before the velocity jump to the beam impedance after the velocity jump and M'_{12} and M'_{13} are matrix elements of the traveling wave section alone. This relation is similar to Equation (61) which applies to the input coupler and it is possible to make M'_{12} equal to zero in either of two ways

$$\theta = \frac{1}{2} \arg \frac{M'_{12}}{M'_{13}} + n\pi \,, \qquad \frac{W}{W'} = \frac{1 - \left|\frac{M'_{12}}{M'_{13}}\right|}{1 + \left|\frac{M'_{12}}{M'_{13}}\right|} \tag{69}$$

$$\theta = \frac{1}{2} \arg \frac{M'_{12}}{M'_{13}} + (n + \tfrac{1}{2})\pi \,, \qquad \frac{W}{W'} = \frac{1 + \left|\frac{M'_{12}}{M'_{13}}\right|}{1 - \left|\frac{M'_{12}}{M'_{13}}\right|} \tag{70}$$

By virtue of Equations (29) and (30)

$$\frac{M'_{21}}{M'_{31}} = - \frac{M'_{12}}{M'_{13}} \tag{71}$$

so that Equations (69) and (70) become identical with Equations (63) and (62). Thus the results shown in Figure 1 are also applicable to the output coupler. Stated in words, the drift length which is required between a velocity jump in which the beam impedance is increased and traveling wave section in order to make $M_{12} = 0$, is the same as that required between the traveling wave section and the velocity jump in which the beam impedance is decreased in order to make $M_{21} = 0$. The resulting coupler is depicted in Figure 4.

Similar arguments can be applied to synthesize a pump coupler, although the bandwidth afforded by a traveling wave coupler is not required. The requirements for a pump coupler are (a) produce no slow wave modulation ($M_{21} = 0$) and (b) to maximize the fast wave modulation (maximize M_{31}). The latter condition is achieved by taking $M_{11} = 0$. Thus the pump coupler is electrically identical with the input coupler (although it operates at a different frequency).

Finally we consider the symmetric coupler shown in Figure 5. The traveling wave section is preceded by a velocity jump from impedance W' to impedance W and a drift region of length θ. It is followed by a drift region of length θ and a velocity jump back to the original impedance W'. It is readily verified that this composite coupler has the following symmetry

$$M_{12} = \mp M_{21}$$

$$M_{13} = \pm M_{31}$$

$$M_{23} = - M_{32}$$

which is the symmetry of the traveling wave coupler alone. Furthermore by choosing the location and magnitude of the velocity jump in the manner already described (Figure 2), it is possible to construct a kind of ideal coupler,

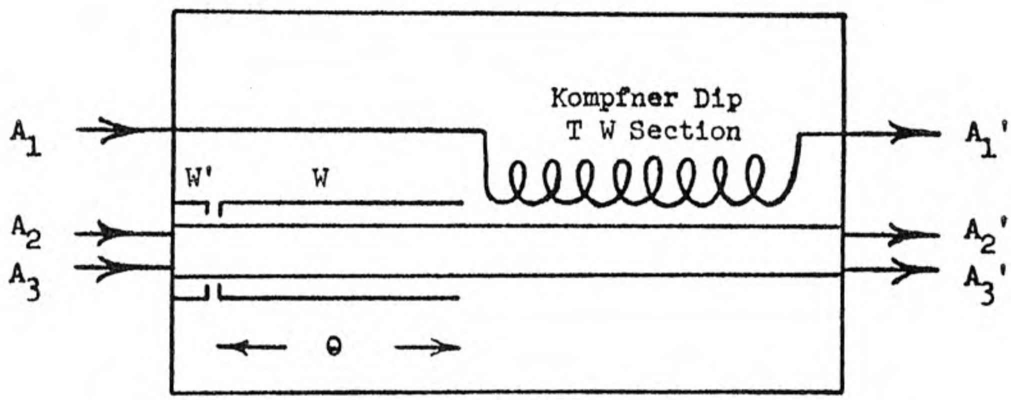

Figure 4.

Output Coupler consisting of Velocity Jump, Drift Region, and Kompfner Dip Traveling Wave Section.

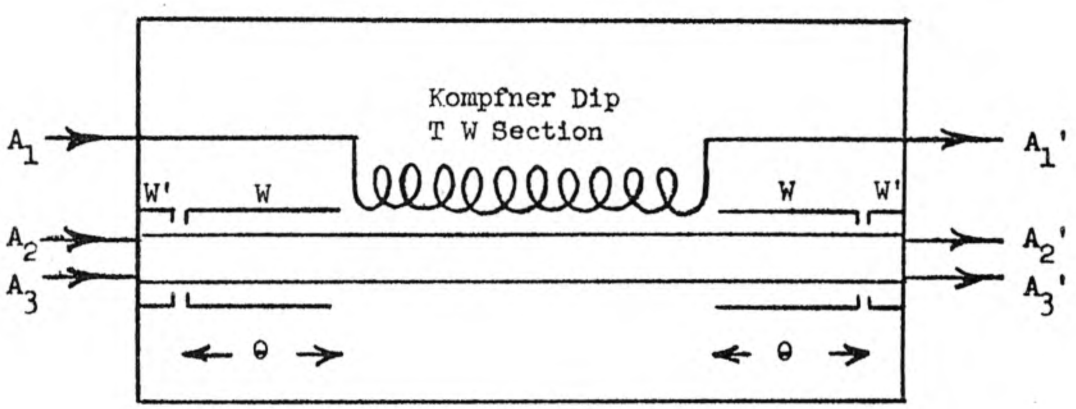

Figure 5.

Ideal Coupler consisting of Velocity Jump, Drift Region, Kompfner Dip Traveling Wave Section, drift region, and Velocity Jump.

whose matrix is

$$M = \begin{pmatrix} 0 & 0 & e^{-j\theta_3} \\ 0 & e^{-j\theta_2} & 0 \\ e^{-j\theta_1} & 0 & 0 \end{pmatrix}$$

The slow space charge wave passes through the coupler with only a shift in phase and there is a complete transfer of energy from the circuit wave to the slow space charge wave and vice versa.

V DISCUSSION

The theory of longitudinal beam traveling wave couplers has been developed and applied to the design of couplers for parametric amplifiers. A coupler can be constructed for any QC value which couples only to the fast space charge wave and furthermore this same coupler also removes the beam noise from the fast space charge wave. This coupler consists of a traveling wave section, drift region, and velocity jump. For certain QC values the velocity jump can be placed very close to the traveling wave section, making a very compact coupler.

Results for the traveling wave section by itself (Table 1) indicate that the coupling to the slow space charge wave is down 20 db or more at Kompfner Dip for $QC > 1.2$ ($|M_{21}| < .10$) and that beam noise is reduced by a similar amount ($|M_{32}| < .1$, $|M_{33}| < .01$) under these same conditions. Where used with a low noise electron gun this less perfect but inherently simpler type of coupler also appears very attractive.

Finally, it should be pointed out that the presence of the pump signal on the beam may modify these results slightly. Locating the input coupler before the pump coupler will eliminate any possible effect in the input coupler where noise is eliminated from the fast wave.

VI ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Mr. D. C. Forster for several illuminating discussions of this problem and to Mr. K. Hebert for writing the program for machine computation of the matrix elements. Support for research on microwave electronics by the Office of Naval Research and for the investigation reported here by the Sperry Rand Corporation is gratefully acknowledged.

REFERENCES

1. W. H. Louisell and C. F. Quate, "Parametric Amplification of Space Charge Waves," Proc. IRE, vol. 46, pp. 707-716; April 1958.

2. R. Kompfner, "On the Operation of the Traveling Wave Tube at Low Level," British Jour. IRE, vol. 10, pp. 283-289; August-September 1955. See also H. R. Johnson, "Kompfner Dip Conditions," Proc. IRE, vol. 43, p. 874; July 1955.

3. R. W. Gould, "A Coupled Mode Description of the Backward Wave Oscillator and the Kompfner Dip Condition," IRE Trans. P.G.E.D., vol. ED-2, pp. 37-42; October 1955.

4. For a given beam $4QC^3 = \left(\frac{\omega_q}{\omega}\right)^2$ is approximately constant. Therefore $4QC \sim 1/C^2$.

5. J. R. Pierce, "Coupling of Modes of Propagation," Jour. Appl. Phys., vol. 25, pp. 179-183; February 1954. "The Wave Picture of Microwave Tubes," Bell Syst. Tech. J., vol. 33, pp. 1343-1372; November 1954.

6. H. A. Haus and F. N. H. Robinson, "The Minimum Noise Figure of Microwave Beam Amplifiers," Proc. IRE, vol. 43, pp. 981-991; August 1955.

7. The factor j preceding κ in Equations (8) through (10) does not appear in reference 3. This difference is due to the choice of the phases of a_1, a_2, a_3.

8. The use of a velocity jump following the helix for this purpose was first described by A. Ashkin at the Sixteenth Annual Conference on Electron Tube Research, June 25-27, 1958, Quebec, Canada.

9. $|M_{31}|$ will be larger than $|M_{21}|$ if $|M_{11}| < 1$, or if the traveling wave section has no gain. This follows from the 1-1 component of Equation (32).

10. S. Bloom and R. Peter, "A Transmission Line Analog of a Modulated Electron Beam", RCA Review, Vol.15, pp. 95-112; March 1954.

11. It is of interest to note that these same general arguments also apply to lossless cavity couplers if one lets A_1 and A_1' refer to the incident and reflected waves, respectively, on the transmission line leading to the cavity system. For a cavity system to be considered as lossless, the energy dissipated in the cavity must be small compared to the power transferred to the beam.

DISTRIBUTION OF TECHNICAL REPORTS

Chief of Naval Research 2
Navy Department - CODE 427
Washington 25, D. C.

Director, Naval Research Lab.
Washington 25, D. C.
Attn: Code 5240 1
Code 7130 1
Code 2000 6
Code 5430 1

Commanding Officer 1
Office of Naval Res.Branch Office
1000 Geary Street
San Francisco, California

Scientific Liaison Officer 25
Office of Naval Research, London
c/o Navy 100, Box 39, F.P.O.
New York, New York

Commanding Officer 2
Office of Naval Res.Branch Office
1030 E. Green Street
Pasadena, California

Commanding Officer 1
Office of Naval Research Branch Of.
The John Crerar Library Building
86 East Randolph Street
Chicago 1, Illinois

Commanding Officer 1
Office of Naval Res.Branch Office
346 Broadway
New York 13, New York

Officer-in-Charge 3
Office of Naval Research
Navy No. 100
Fleet Post Office
New York, New York

Chief, Bureau of Aeronautics EL 4 1
Navy Department EL 43 1
Washington 25, D. C. EL 45 1

Chief, Bureau of Ordnance
Navy Department Re 4 1
Washington 25, D.C. Re 9 1

Periodicals Librarian
General Library
California Inst. of Technology 1

Lincoln Laboratory
Massachusetts Inst. of Technology 1
Cambridge 39, Massachusetts

Signal Corps Resident Engineer
Electronic Defense Laboratory 1
Post Office Box 205
Mountain View, California

Cornell Aeronautical Laboratory 1
Cornell Research Foundation
Buffalo 21, New York

Director, Electronics Defense Gr. 1
Engineering Research Inst.
University of Michigan
Ann Arbor, Michigan

Georgia Inst. of Technology 1
Atlanta, Georgia
Attn: Librarian

Fred D. Wilimek 1
Varian Associates
611 Hansen Way
Palo Alto, California

John Dyer 1
Airborne Instrument Laboratory
Mineola, L.I., New York

Bell Telephone Laboratories
Murray Hill, New Jersey
Attn: Librarian 1
J. R. Pierce 1

Hughes Aircraft Company
Culver City, California 1
Attn: Mr. Milek, Tech.Librarian

RCA Laboratories
Princeton, New Jersey 1
Attn: Mr.Herold and H.Johnson

Federal Tele. Laboratories
500 Washington Avenue
Nutley, New Jersey
Attn: W. Derrick 1
K. Wing 1

Technical Library
G. E. Microwave Laboratory
601 California Avenue 1
Palo Alto, California

Chief of Naval Operations Op 20X 1
Navy Department Op 421 1
Washington 25, D. C. Op 55 1

Director, Naval Ordnance Laboratory 1
White Oak, Maryland

Director, Naval Electronics Lab. 1
San Diego 52, California

Dept. of Electronics and Physics 1
U.S. Naval Post Graduate School
Monterey, California

Commander Code 366 1
Naval Air Missile Test Center
Point Mugu, California

U. S. Naval Proving Ground 1
Attention: W. H. Benson
Dahlgren, Virginia

Commander 1
U.S. Naval Air Development Center
Johnsville, Pennsylvania

Committee on Electronics 1
Research and Development Board
Department of Defense
Washington 25, D. C.

Director, National Bureau of Stds. 1
Washington 25, D. C.
Attn: Div. 14.0 CRPL, Librarian

Commanding Officer 1
Engineering Res. and Dev. Lab.
Fort Belvoir, Virginia

Ballistics Research Laboratories 2
Aberdeen Proving Ground, Maryland
Attn: D. W. H. Delsasso

Chief, Ordnance Development Div. 2
National Bureau of Standards
Connecticut Ave and VanNess St, NW
Washington 25, D. C.

Commanding Officer 1
Frankford Arsenal
Bridesburg, Philadelphia, Pa.

Columbia Radiation Laboratory 1
538 W. 120th Street
New York 27, New York

Countermeasures Laboratory 1
Gilfillan Brothers, Inc.
1815 Venice Boulevard
Los Angeles, California

The Rand Corporation 1
1700 Main Street
Santa Monica, California
Attn: Librarian

Technical Library 1
Research and Development Board
Pentagon Building
Washington 25, D. C.

The Motorola Riverside Research Lab. 1
8330 Indiana Avenue
Riverside, California
Attn: Mr. John Byrne

Chief, Bureau of Ships CODE 816 1
Department of the Navy 820 1
Washington, D. C. 840 1

Panel on Electron Tubes 1
346 Broadway (8th Floor)
New York 13, New York

Supervisor of Research Laboratory 1
Electrical Engineering Building
Purdue University
Lafayette, Indiana

W. E. Lear 1
University of Florida
Department of Electrical Eng.
Gainesville, Florida

Director, Microwave Research Inst. 1
Polytechnic Institute of Brooklyn
55 Johnson Street
Brooklyn 1, New York

Material Laboratory Library,CODE 912B
New York Naval Shipyard 1
Brooklyn 1, New York

University of Washington
Department of Electrical Engineering
Seattle, Washington
Attn: E. A. Harrison 1
A. V. Eastman 1

Thermionics Branch 5
Signal Corps Eng. Laboratories
Evans Signal Lab, Bldg. 42
Belmar, New Jersey

Commanding General
Air Research and Dev.Command
Post Office Box 1395
Baltimore 3, Maryland
Attn: RDRR 1
RDDDE-3 1
RDDDE-5 1

Commanding General WCLC 1
Wright Air Devel.Center WCLRC 1
Wright-Patterson Air Force Base
Ohio

Commanding General CRRE 1
Air Force Cambridge Research Cen.
230 Albany Street
Cambridge 39, Massachusetts

Commanding General RCRW 1
Rome Air Development Center
Griffiss Air Force Base
Rome, New York

Armed Services Tech.Information 5
Document Service Center DSC-SA
Knott Building
Dayton 2, Ohio

Director CR4582 1
Air University Library
Maxwell Air Force Base, Alabama

Chief, Western Division 1
Air Research and Devel. Command
Office of Scientific Research
P.O. Box 2035, Pasadena, Calif.

Microwave Laboratory 1
Stanford University
Stanford, California
Attn: F.V.L. Pindar

University of Michigan 1
Electron Tube Laboratory
Ann Arbor, Michigan
Attn: E. Rowe

Mr. John S. McCullough 1
Eitel-McCullough, Inc.
San Bruno, California

University of Colorado 1
Department of Electrical Engineering
Boulder, Colorado

Ramo-Wooldridge Corporation 1
Control Systems Division
P.O. Box 900B
Hawthorne, California
Attn: Librarian

Electrical Engineering Dept. 1
Princeton University
Princeton, New Jersey

National Union Radio Company 1
350 Scotland Road
Orange, New Jersey
Attn: Dr. A. M. Skellet

Dr. J. E. Shepherd 1
Sperry Gyroscope Company
Great Neck, L.I., New York

W. L. Maxson Corporation 1
460 West 34th Street
New York 1, New York
Attn: M. Simpson

Mr. H. R. Argento
Raytheon Corporation 1
Waltham, Massachusetts

Dr. E. D. McArthur
Electron Tube Laboratory
General Electric Company 1
The Knolls
Schenectady, New York

General Electric Company 1
Electronic Components Division
Power Tube Department
Microwave Laboratory at Stanford
Palo Alto, California

Johns Hopkins University 1
Radiation Laboratory
1315 St. Paul Street
Baltimore 2, Maryland
Attn: Margaret Poole, Librarian

Raytheon Corporation 1
Waltham, Massachusetts
Attn: Librarian

Cascade Research
53 Victory Lane 1
Los Gatos, California

Engineering Library 1
Stanford University
Stanford, California

Research Laboratory of Electronics 1
Massachusetts Inst. of Technology
Cambridge 39, Massachusetts

Sloane Physics Laboratory 1
Yale University
New Haven, Connecticut
Attn: R. Beringer

Mr. H J Reich
Department of Electrical Eng. 1
Yale University
New Haven, Connecticut

Electron Tube Section 1
Electrical Engineering Dept.
University of Illinois
Champaign, Illinois

Chairman, Div. of Electrical Eng. 1
University of California
Berkeley 4, California

Technical Report Collection 1
303A, Pierce Hall
Harvard University
Cambridge 38, Massachusetts

Laboratory for Insulation Research
Massachusetts Inst. of Technology 1
Cambridge 39, Massachusetts
Attn: A. von Hippel

Chief, West Coast Office 1
Signal Corps Eng. Laboratories
75 So. Grand Avenue
Pasadena, 2, California

Office of Technical Services 1
Department of Commerce
Washington 25, D. C.

Professor W. P. Dyke 1
Linfield College
McMinnville, Oregon

Additional copies for Staff and
Future Requirements 25

Stanford Electronics Laboratories 1
Stanford University
Stanford, California
Attn: Applied Electronics Laboratory
Document Library

Mr. E. C. Okress, Technical Director
Westinghouse Microwave Center
310 N. Aurora Street 1
Ithaca, New York