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POWER FLOW AND GAP COUPLING TO SLOW WAVE PLASMA MODES

by
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CONTENTS

ABSTRACT

The interaction impedance for the slow wave plasma modes of Trivelpiece and Gould is obtained for a coaxial system of plasma column, surrounding dielectric and conducting surface. The theory presented includes a finite axial magnetic field. Calculations are presented for the special case of no static magnetic field for the so-called surface wave interaction impedance versus βa .

Coupling to the slow wave plasma modes by a known electric field across a gap in the conducting surface at the dielectric surface is analyzed for the coaxial plasma system including a static axial magnetic field. Such an analysis follows that of Sensiper. This results in an impedance as seen by the cavity gap. Calculations are presented for the special case of no static magnetic field.

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Mr. John F. Asmus performed the bulk of the calculations.

1 INTRODUCTION

Consider a plasma column of radius a surrounded by a dielectric of relative dielectric constant κ and radius b. At b, the edge of the dielectric column, there is a conducting boundary. The plasma column is immersed in an axial magnetic field B_o.

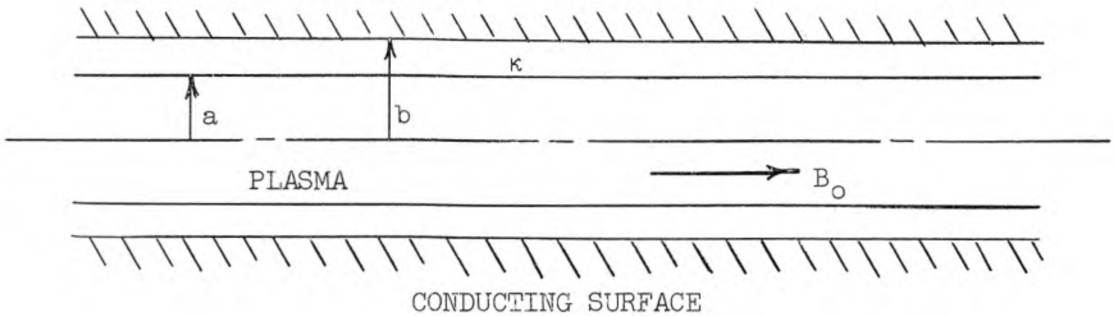

Figure 1.1 Plasma column geometry

Gould and Trivelpiece have shown[1,2,3] that electromechanical modes of propagation exist down the plasma column at frequencies above and below the electron plasma frequency ω_p. Only the electron plasma frequency is considered, as the ions are assumed to be infinitely massive compared to the electrons. Over most of their region of propagation these waves are slow waves, meaning that the phase velocity is small compared to the velocity of light.

It is assumed in this paper that all quantities have an average value plus a small harmonic time-dependent perturbation

$$\underline{F}(r,t) = \underline{F}_o(r) + \underline{F}_1(r)\, e^{i\omega t} \quad . \tag{1.1}$$

The subscripts o and 1 distinguish the d.c. and a.c. terms respectively. The harmonic time-dependent perturbation is assumed to be wave-like in nature

$$\underline{F}_1(x,y,z) = \underline{F}_1(x,y)\, e^{-i\beta z} \tag{1.2}$$

where β is the z directed propagation constant.

If the assumption of slow wave propagation is valid, then it can be shown that these waves can be derived from a single scalar potential. This condition of slow wave propagation means that one can neglect the magnetic field in Maxwell's equation

$$\nabla \times \underline{E}_1 = - \frac{\partial \underline{B}_1}{\partial t} \approx 0 \tag{1.3}$$

which is referred to as the quasi-static approximation. Then $\underline{E}_1$ may be derived as the negative gradient of a scalar ϕ_1 .

If the axial magnetic field, B_o, is finite and different from zero then in Chapter 3 of reference 3 the tensor dielectric constant of the plasma is given by

$$\underline{\underline{\epsilon}} = \epsilon_o \left\| \begin{matrix} \kappa_{rr} & j\kappa_{r\theta} & 0 \\ -j\kappa_{\theta r} & \kappa_{\theta\theta} & 0 \\ 0 & 0 & \kappa_{zz} \end{matrix} \right\| \tag{1.4}$$

where

$$\begin{aligned} \kappa_{rr} &= \kappa_{\theta\theta} = 1 + \frac{\omega_p^2}{\omega_c^2 - \omega^2} \\ \kappa_{r\theta} &= \kappa_{\theta r} = \frac{\omega_c}{\omega} \frac{\omega_p^2}{\omega_c^2 - \omega^2} \\ \kappa_{zz} &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned} \tag{1.5}$$

the electron plasma frequency is given by

$$\omega_p^2 = \frac{-\rho_o e}{m\epsilon_o} = \frac{n_o e^2}{m\epsilon_o} \tag{1.6}$$

where n_o is the number density per unit volume of plasma electrons. The electron cyclotron frequency is given by

$$\omega_c = \frac{eB_o}{m} \quad . \tag{1.7}$$

If one assumes the potential for the nth angular mode to be given by

$$\phi_1 = R_n(r)\ e^{-i(n\Theta + \beta z)} \tag{1.8}$$

then the resulting small signal linear differential equation is given by

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} - \frac{n^2}{r^2} R - \beta^2 \frac{\kappa_{zz}}{\kappa_{rr}} R = 0 \tag{1.9}$$

where

$$\tau^2 = -\beta^2 \frac{\kappa_{zz}}{\kappa_{rr}} \quad , \qquad T^2 = |\tau^2| \quad . \tag{1.10),(1.11}$$

At this point the analysis is restricted to axially symmetric (angularly independent) modes of propagation, thus $n = 0$. The solution to the radial variation of the potential in the region $0 \leq r \leq a$ that is finite at the origin is given by

$$R_{n=0}(r) = \begin{cases} A\ J_o(Tr) & \tau^2 > 0 \\ A\ I_o(Tr) & \tau^2 < 0 \end{cases} \tag{1.12}$$

In the region $a \leq r \leq b$ the radial variation of potential is given by

$$R(r) = C\left[I_o(\beta r)\ K_o(\beta b) - I_o(\beta b)\ K_o(\beta r)\right] \tag{1.13}$$

since the potential must equal zero at $r = b$.

It will be convenient to note that the electric field in the z direction is given by

$$E_{1z} = -\frac{\partial \phi_1}{\partial z} = i\beta\ \phi_1 \tag{1.14}$$

and hence all field quantities can be obtained as derivatives of the z component of electric field. Thus

$$E_{1r} = -\frac{\partial \phi_1}{\partial r} = \frac{i}{\beta}\frac{\partial E_{1z}}{\partial r} \tag{1.15}$$

$$E_{1\theta} \equiv 0 \qquad (n = 0) \quad . \tag{1.16}$$

All solutions for the field strengths will have arbitrary multiplicative constants which will be chosen as the axial field strength at $r = 0$ and denoted by $E_{1z}(0)$. The electric fields of the angularly independent slow wave mode of propagation in the region $0 \leq r \leq a$ are therefore given by

$$E_{1z} = E_{1z}(0) \begin{cases} J_o(Tr) & \tau^2 > 0 \\ I_o(Tr) & \tau^2 < 0 \end{cases} \tag{1.17}$$

$$E_{1r} = E_{1z}(0)\,\frac{i}{\beta}\,T \begin{cases} J_o'(Tr) \\ I_o'(Tr) \end{cases} \tag{1.18}$$

$$E_{1\theta} = 0 \qquad (n = 0) \tag{1.19}$$

Note that at $r = 0$ $J_o(Tr) = I_o(Tr) = 1$.

To satisfy the boundary conditions on the field quantities at $r = a$ one must require that the tangential electric field, E_{1z} (since $E_{1\theta} = 0$), be continuous as well as the normal displacement vector, D_{1r}. Thus from equation 1.13 and 1.17 one obtains in the region $a \leq r \leq b$

$$E_{1z} = E_{1z}(0)\,\frac{I_o(\beta r)\,K_o(\beta b) - I_o(\beta b)\,K_o(\beta r)}{I_o(\beta a)\,K_o(\beta b) - I_o(\beta b)\,K_o(\beta a)} \begin{cases} J_o(Ta) & \tau^2 > 0 \\ I_o(Ta) & \tau^2 < 0 \end{cases} \tag{1.20}$$

The radial electric field for $a \leqq r \leqq b$ from 1.15 is given by

$$E_{1r} = iE_{1z}(0)\,\frac{I_0'(\beta r)K_0(\beta b) - I_0(\beta b)K_0'(\beta r)}{I_0(\beta a)K_0(\beta b) - I_0(\beta b)K_0(\beta a)}\begin{bmatrix} J_0(Ta) & \tau^2 > 0 \\ I_0(Ta) & \tau^2 < 0 \end{bmatrix} \tag{1.21}$$

From 1.18 and 1.21 the remaining boundary condition on the normal displacement vector leads to the determinantal equation for the propagation constant

$$\begin{aligned} \kappa_{rr}(Ta)&\begin{bmatrix} J_0'(Ta)/J_0(Ta) \\ I_0'(Ta)/I_0(Ta) \end{bmatrix} \\ &= \kappa(\beta a)\,\frac{I_0'(\beta a)K_0(\beta b) - I_0(\beta b)K_0'(\beta a)}{I_0(\beta a)K_0(\beta b) - I_0(\beta b)K_0(\beta a)} \quad \begin{matrix} \tau^2 > 0 \\ \tau^2 < 0 \end{matrix} \end{aligned} \tag{1.22}$$

where κ is the relative dielectric constant in the region $a \leqq r \leqq b$. Solutions of the above determine the propagation characteristics (ω-β diagrams) of the plasma column.

2 POWER FLOW

It is of interest to find the power flow associated with such slow wave plasma modes. The complex power flow is given by the integral of Poynting's vector over the cross-section of the guide

$$\overline{P}_z = \frac{1}{2} \operatorname{Re} \int_{\Sigma} (\underline{E}_1 \times \underline{H}_1^*) \cdot e_z \, d\sigma \quad . \tag{2.1}$$

The real part gives the real power flow and the imaginary part the reactive power flow. Due to the restriction to axially symmetric modes (n = 0), the average complex power flow in the z direction is given by

$$\overline{P}_z = \frac{1}{2} \operatorname{Re} \int_{\Sigma} E_{1r} H_{1\theta}^* \, r dr d\theta = \pi \int_0^b E_{1r} H_{1\theta}^* \, r dr \ . \tag{2.2}$$

Initially the a.c. magnetic field was neglected when making the quasi-static approximation. An approximate value of this magnetic field is obtainable, however, from Maxwell's remaining curl equation.

$$\nabla \times \underline{H}_1 = i\omega \underline{\underline{\epsilon}} \cdot \underline{E}_1 \quad . \tag{2.3}$$

It is readily shown in reference 3 that in the region $0 \leq r \leq a$

$$H_{1\theta} = \frac{\omega}{\beta} \epsilon_o \kappa_{rr} E_{1r} \tag{2.4}$$

and in the region $a \leq r \leq b$

$$H_{1\theta} = \frac{\omega}{\beta} \kappa \epsilon_o E_{1r} \quad . \tag{2.5}$$

Thus equation 2.2 becomes

$$\overline{P}_z = \pi \frac{\omega}{\beta} \epsilon_o \left[\kappa_{rr} \int_0^a E_{1r} E_{1r}^* \, r dr + \kappa \int_a^b E_{1r} E_{1r}^* \, r dr \right] \ . \tag{2.6}$$

To evaluate (2.6) one must substitute relations for E_{1r} in the regions $0 \leqq r \leqq a$ and $a \leqq r \leqq b$ given by 1.18 and 1.21. It will also be convenient to use the following relations:

Reference 4

$$I_o' = I_1 \qquad K_o' = -K_1 \qquad J_o' = -J_1$$

$$I_1' = I_o - \frac{I_1}{x} \qquad -K_1' = K_o + \frac{K_1}{x} \qquad J_1' = J_o - \frac{J_1}{x} \tag{2.7}$$

$$I_2 = I_o - \frac{2I_1}{x} \qquad K_2 = K_o + \frac{2K_1}{x} \qquad -J_2 = J_o - \frac{2}{x} J_1$$

Wronskian $\qquad I_o K_1 + I_1 K_o = \frac{1}{x}$

Reference 5, section 7.14.1, equations 10,13

$$\int x\, J_1^2(x)dx = \frac{x^2}{2}\left[J_1^2 - J_oJ_2\right] = \frac{x^2}{2}\left[J_1^2 - \frac{2J_oJ_1}{x} + J_o^2\right]$$

$$\int x\, I_1^2dx = \frac{x^2}{2}\left[I_1^2 - I_oI_2\right] = \frac{x^2}{2}\left[I_1^2 + \frac{2I_1I_o}{x} - I_o^2\right] \tag{2.8}$$

$$\int x\, K_1^2dx = \frac{x^2}{2}\left[K_1^2 - K_oK_2\right] = \frac{x^2}{2}\left[K_1^2 - \frac{2K_1K_o}{x} - K_o^2\right]$$

Reference 6, page 134, equation 10

$$\int x\, I_1K_1dx = \frac{x^2}{4}\left[2I_1K_1 + I_oK_2 + I_2K_o\right]$$

$$= \frac{x^2}{2}\left[I_1K_1 + I_oK_o + \frac{1}{x}\left(I_oK_1 - I_1K_o\right)\right] \tag{2.9}$$

Also introduce Birdsall's notation[7] for combinations of modified Bessel functions.

$$\mathrm{Becoth}(y,x) = \frac{I_o(y)\ K_1(x) + I_1(x)\ K_o(y)}{I_o(y)\ K_o(x) - I_o(x)\ K_o(y)}$$

$$\mathrm{Betanh}(y,x) = \frac{I_o(y)\ K_o(x) - I_o(x)\ K_o(y)}{I_1(y)\ K_o(x) + I_o(x)\ K_1(y)} \tag{2.10}$$

$$\mathrm{betanh}(y,x) = \frac{I_1(y)\ K_1(x) - I_1(x)\ K_1(y)}{I_o(y)\ K_1(x) + I_1(x)\ K_o(y)} \quad .$$

With these definitions and the relations of 2.7, one can rewrite the determinantal equation of propagation, 1.21, as

$$\frac{\kappa_{rr}}{\kappa}\ (Ta) \begin{bmatrix} -\dfrac{J_1(Ta)}{J_o(Ta)} \\ \dfrac{I_1(Ta)}{I_o(Ta)} \end{bmatrix} +(\beta a)\ \mathrm{Becoth}\ (\beta b,\beta a) = 0 \qquad \begin{matrix} \tau^2 > 0 \\ \\ \tau^2 < 0 \end{matrix} \tag{2.11}$$

The appropriate integrals may now be calculated. From equations 1.18 and 1.21 one obtains

$$E_{1r} = E_{1z}(0)\ i\ \frac{T}{\beta} \begin{bmatrix} -J_1(Tr) & \tau^2 > 0 \\ \\ I_1(Tr) & \tau^2 < 0 \end{bmatrix} \qquad 0 \leq r \leq a \tag{2.12}$$

and in the region $a \leq r \leq b$

$$E_{1r} = i\ E_{1z}(0)\ \frac{I_1(\beta r)\ K_o(\beta b) + I_o(\beta b)\ K_1(\beta r)}{I_o(\beta a)\ K_o(\beta b) - I_o(\beta b)\ K_o(\beta a)} \begin{bmatrix} J_o(Ta) & \tau^2 > 0 \\ \\ I_o(Ta) & \tau^2 < 0 \end{bmatrix} \tag{2.13}$$

Thus from 2.8

$$\int_0^a E_{1r}\ E^*_{1r}\ rdr = \frac{E^2_{1z}(0)}{\beta^2}\ \frac{(Ta)^2}{2}\ \left[J_1^2 - J_oJ_2\right]_{(Ta)} \qquad \tau^2 > 0 \tag{2.14}$$

or

$$\int_0^a E_{1r} E_{1r}^* \, rdr = \frac{E_{1z}^2(0)}{\beta^2} \frac{(Ta)^2}{2} \left[I_1^2 - I_o I_2 \right]_{(Ta)} \qquad \tau^2 < 0 \tag{2.15}$$

From 2.13, eliminating the algebraic steps, one obtains

$$\int_a^b E_{1r} E_{1r}^* \, rdr = \frac{E_{1z}^2(0)}{2\beta^2} \begin{bmatrix} J_o^2(Ta) \\ I_o^2(Ta) \end{bmatrix} \left[\frac{1 - \left\{(\beta a)R + S\right\}^2 + S^2\left\{1+(\beta a)^2\right\}}{S^2} \right] \begin{matrix} \tau^2 > 0 \\ \tau^2 < 0 \end{matrix} \tag{2.16}$$

where one defines

$$R = I_o(\beta b)\, K_1(\beta a) + I_1(\beta a)\, K_o(\beta b) \tag{2.17}$$

$$S = I_o(\beta a)\, K_o(\beta b) - I_o(\beta b)\, K_o(\beta a) \tag{2.18}$$

and notes that

$$\text{Becoth}\,(\beta b, \beta a) = -\frac{R}{S} \quad . \tag{2.19}$$

From equation 2.6 note that

$$\pi \frac{\omega}{\beta} \epsilon_o = \pi \frac{v_{ph}}{c} \sqrt{\frac{\epsilon_o}{\mu_o}} = 2\pi^2 \sqrt{\frac{\epsilon_o}{\mu_o}} \left(\frac{a}{\lambda}\right) \frac{1}{(\beta a)} \tag{2.20}$$

where λ is the free space wavelength corresponding to ω, the velocity of light in free space is given by

$$c = \frac{1}{\sqrt{\epsilon_o \mu_o}} \tag{2.21}$$

and the phase velocity of the slow wave plasma modes of propagation is

$$v_{ph} = \frac{\omega}{\beta} \quad . \tag{2.22}$$

Note that $\sqrt{\mu_o/\epsilon_o} = 377$ ohms and is the characteristic impedance of free space.

Pierce defines the interaction impedance Z_O for a slow wave propagating circuit in terms of the average z directed power flow.

$$\overline{P}_z = \frac{\phi_1^2(0)}{2Z_o} = \frac{E_{1z}^2(0)}{2\beta^2 Z_o} \quad . \tag{2.23}$$

Thus the interaction impedance on the axis of a slow wave circuit is defined as

$$\frac{1}{Z_o} = \frac{\overline{P}_z}{\frac{E_{1z}^2(0)}{2\beta^2}} \quad . \tag{2.24}$$

Combining equations 2.6, 2.11, 2.14, 2.15, 2.16, 2.19, 2.20 and 2.24 results in the interaction impedance being given by

$$\frac{1}{Z_o} \cdot \frac{1}{\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}} = (\beta a)(Ta) \begin{bmatrix} \frac{J_o}{J_1}(J_1^2 - J_oJ_2) \\ -\frac{I_o}{I_1}(I_1^2 - I_oI_2) \end{bmatrix}_{(Ta)} \mathrm{Becoth}(\beta b, \beta a)$$

$$+ \begin{bmatrix} J_o^2 \\ I_o^2 \end{bmatrix}_{(Ta)} \left\{ \frac{1}{S^2} + 1 + (\beta a)^2 - \left[1 - (\beta a)\mathrm{Becoth}(\beta b, \beta a)\right]^2 \right\} \quad \begin{matrix} \tau^2 > 0 \\ \tau^2 < 0 \end{matrix} \tag{2.25}$$

where S is given by 2.18.

The following limiting cases can be obtained for equation 2.25. As $\beta a \to 0$ regardless of Ta

$$Z_o \left[\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}\right] \to \frac{1}{2} \ell n \frac{b}{a} \quad . \tag{2.26}$$

For $\tau^2 < 0$ and $\beta(b-a) \gg \ell n(\beta a, \beta b)$ then as $\beta a \to \infty$ one obtains

$$Z_o \left[\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}\right] \to 2\pi \frac{Ta}{(\beta a)^2} e^{-2Ta} \quad . \tag{2.27}$$

The interaction impedance of equation 2.25 is for a plasma column of radius a surrounded by a medium of relative dielectric constant κ out to a radius b. At radius b is a conducting surface.

In the limit of $b \to \infty$ the situation is that of a plasma column of radius a surrounded by a dielectric medium κ extending to infinity. In this limit equation 2.25 becomes (with the aid of equation 2.7)

$$\frac{1}{Z_o} \frac{1}{\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}} =$$

$$(\beta a)(Ta)\begin{bmatrix} \frac{J_o}{J_1}(J_1^2 - J_oJ_2) \\ \\ -\frac{I_o}{I_1}(I_1^2 - I_oI_2) \end{bmatrix}_{(Ta)} \cdot \frac{K_1(\beta a)}{K_o(\beta a)} + (\beta a)^2 \begin{bmatrix} J_o^2 \\ \\ I_o^2 \end{bmatrix}_{(Ta)} \left\{\frac{K_2}{K_o} - \frac{K_1^2}{K_o^2}\right\}_{(\beta a)} \begin{matrix} \tau^2 > 0 \\ \\ \tau^2 < 0 \end{matrix} \tag{2.28}$$

The determinantal equation of propagation in this limiting case of $b \to \infty$ is obtainable from equation 2.11 as

$$\frac{\kappa_{rr}}{\kappa}(Ta)\begin{bmatrix} -\frac{J_1}{J_o} \\ \\ \frac{I_1}{I_o} \end{bmatrix}_{(Ta)} + (\beta a)\frac{K_1(\beta a)}{K_o(\beta a)} = 0 \tag{2.29}$$

An electron beam of finite size will interact with the electric fields of the axis as well as at $r = 0$. In the region $0 \leq r \leq a$ the axial electric field is given by equation 1.17. For $\tau^2 < 0$ the axial electric field increases with radius and thus by equation 2.24 the interaction impedance increases. In general for a finite size electron beam of radius d $(d \leq a)$ and of uniform current density one may define an average interaction impedance as

$$\overline{Z}_o = \frac{1}{2\beta^2 \overline{P}_z} \frac{1}{\pi d^2} \int_0^{2\pi}\int_0^{d} E_{1z}^2(r)\, r\, dr\, d\Theta$$

$$= Z_o \frac{2}{d^2} \int_0^{d} \begin{bmatrix} J_o^2(Tr) \\ \\ I_o^2(Tr) \end{bmatrix} r\, dr \qquad \begin{matrix} \tau^2 > 0 \\ \\ \tau^2 < 0 \end{matrix} \tag{2.30}$$

which becomes

$$\frac{\overline{Z}_o}{Z_o} = \begin{bmatrix} J_o^2 + J_1^2 \\ \\ I_o^2 - I_1^2 \end{bmatrix}_{(Td)} \qquad \begin{matrix} \tau^2 > 0 \\ \\ \tau^2 < 0 \end{matrix} \quad . \tag{2.31}$$

2.1 Surface Waves

If there is no axial magnetic field $(B_o = 0)$ the slow wave plasma modes are called surface waves by Trivelpiece and Gould. With no magnetic field $\omega_c = 0$ and thus $\tau^2 < 0$ and $T = \beta$.

In this special case the determinantal propagation equation is obtainable from equations 2.11 and 1.5 as

$$\frac{\omega_p^2}{\omega^2} = 1 + \kappa \frac{I_o(\beta a)}{I_1(\beta a)} \text{Becoth}\ (\beta b, \beta a) \quad . \tag{2.1.1}$$

The interaction impedance for the surface wave mode of propagation becomes

$$\frac{1}{Z_o} \cdot \frac{1}{\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}} = -(\beta a)^2 \frac{I_o}{I_1} \left\{ I_1^2 - I_o I_2 \right\} \text{Becoth}(\beta b, \beta a)$$

$$+ I_o^2 \left\{ \frac{1}{S^2} + 1 + (\beta a)^2 - \left[1 - (\beta a)\text{Becoth}(\beta b, \beta a) \right]^2 \right\} \tag{2.1.2}$$

The Bessel functions are of argument βa and S is given by equation 2.18. In the limit of the conducting surface at infinity, i.e., the radius $b \to \infty$, the interaction impedance becomes

$$\frac{1}{Z_o} \cdot \frac{1}{\kappa \frac{v_{ph}}{c} \pi \sqrt{\frac{\epsilon_o}{\mu_o}}} = (\beta a)^2 I_o^2 \left\{ \left(\frac{K_2}{K_o} - \frac{K_1^2}{K_o^2}\right) + \frac{I_o K_1}{I_1 K_o} \left(\frac{I_2}{I_o} - \frac{I_1^2}{I_o^2}\right) \right\} \tag{2.1.3}$$

where the Bessel functions are of argument βa . Equation 2.1.3 can be shown to be equivalent to equation V.16 of reference (3) except that it should read

$$K = \frac{1}{\omega_p \epsilon_o \pi a} \left\{ \left(\frac{\omega}{\omega_p}\right) (\beta a) I_o^2 \left[\left(1 - \frac{\omega_p^2}{\omega^2}\right) \left(\frac{I_1^2}{I_o^2} - \frac{I_2}{I_o}\right) - \kappa \left(\frac{K_1^2}{K_o^2} - \frac{K_2}{K_o}\right) \right] \right\}^{-1}$$

with $\kappa = 1$ in the special case considered there.

The determinantal propagation equation for no magnetic field and $b \to \infty$ is obtainable from equation 2.29 as

$$\frac{\omega_p^2}{\omega^2} = 1 + \kappa \left[\frac{I_o K_1}{I_1 K_o} \right]_{(\beta a)} \quad . \tag{2.1.4}$$

Computations have been performed to obtain the normalized interaction impedance, on the axis $(r = 0)$, for the special case of no magnetic field for the cases of $b/a = 1.5$ and $b/a \to \infty$. The former corresponds to the ratio of outside to inside radii of the glass tube used for the plasma column of the cavity modulation tube of reference (8). However, in reference (8) the glass column was normally in free space instead of having a conducting coating at radius b as assumed in this paper.

In Figure 2.1 is presented the ω-β diagrams for the plasma column of Figure 1.1 for two values of the relative dielectric constant $(\kappa = 1$ and $\kappa = 4.6$, the latter value corresponding to the glass used in reference 8) surrounding the plasma of radius a . In the limit of $\beta a \to \infty$,

$\frac{\omega}{\omega_p} \rightarrow \frac{1}{\sqrt{1+\kappa}}$. For purposes of comparison the three region geometrical situation of plasma $(0 \leq r \leq a)$ dielectric κ $(a \leq r \leq b)$ and free space $(b \leq r \leq \infty)$ to infinity is presented also in Figure 2.1 and is labeled "Column in Free Space". The determinantal equation of propagation for this case is given by equation 3.0.4 of reference 8 and is plotted in Figure 3.1 of that reference.

In Figure 2.2 the interaction impedance, Z_o , times the dielectric constant, κ , of the surrounding medium times the ratio of the slow wave plasma mode phase velocity $(v_{ph} = \omega/\beta)$ to the velocity of light, c , is plotted versus βa as obtained from equations 2.1.2 and 2.1.3 for $b/a = 1.5$ and $b/a = \infty$ respectively. This figure presents the surface wave interaction impedance on the axis versus βa for a constant phase velocity of the slow surface wave mode of propagation.

Also shown in Figure 2.2 is the normalized interaction impedance on the axis of a sheath helix in free space $(\kappa = 1)$ versus βa . This was obtained from equation 3.9 and Figure 3.4 of Pierce[9].

In Figure 2.3 is presented the surface wave interaction impedance versus βa normalized in terms of a fixed frequency of propagation ω . This is obtained from Figure 2.2 with the aid of equation 2.20 reduced to

$$\frac{v_{ph}}{c} = \frac{2\pi}{\beta a} \left(\frac{a}{\lambda}\right) \quad . \tag{2.1.5}$$

In Figure 2.3 the phase velocity varies inversely as βa for a fixed frequency ω (free space wavelength λ). The maximum interaction impedance occurs near βa equal to unity.

As a typical example assume $\kappa = 1$, $\omega/2\pi = 490$ Mc $(\lambda = 61.2$ cm$)$, $a = .275$ cm. This corresponds to the cavity modulation tube used in reference 8. The axial interaction impedance at $\beta a = 1$ for $b/a = \infty$ is

$Z_o = 1743$ ohms which is in agreement with Figure 3.2 of reference 8.

For an electron beam of radius d $(d \leq a)$ the average interaction impedance $\overline{Z}_o$ is related to the interaction impedance on the axis, Z_o, by equation 2.31. In the case of no static magnetic field this reduces to

$$\frac{\overline{Z}_o}{Z_o} = I_o^2(\beta d) - I_1^2(\beta d) \quad . \tag{2.1.6}$$

This is plotted in Figure 2.4.

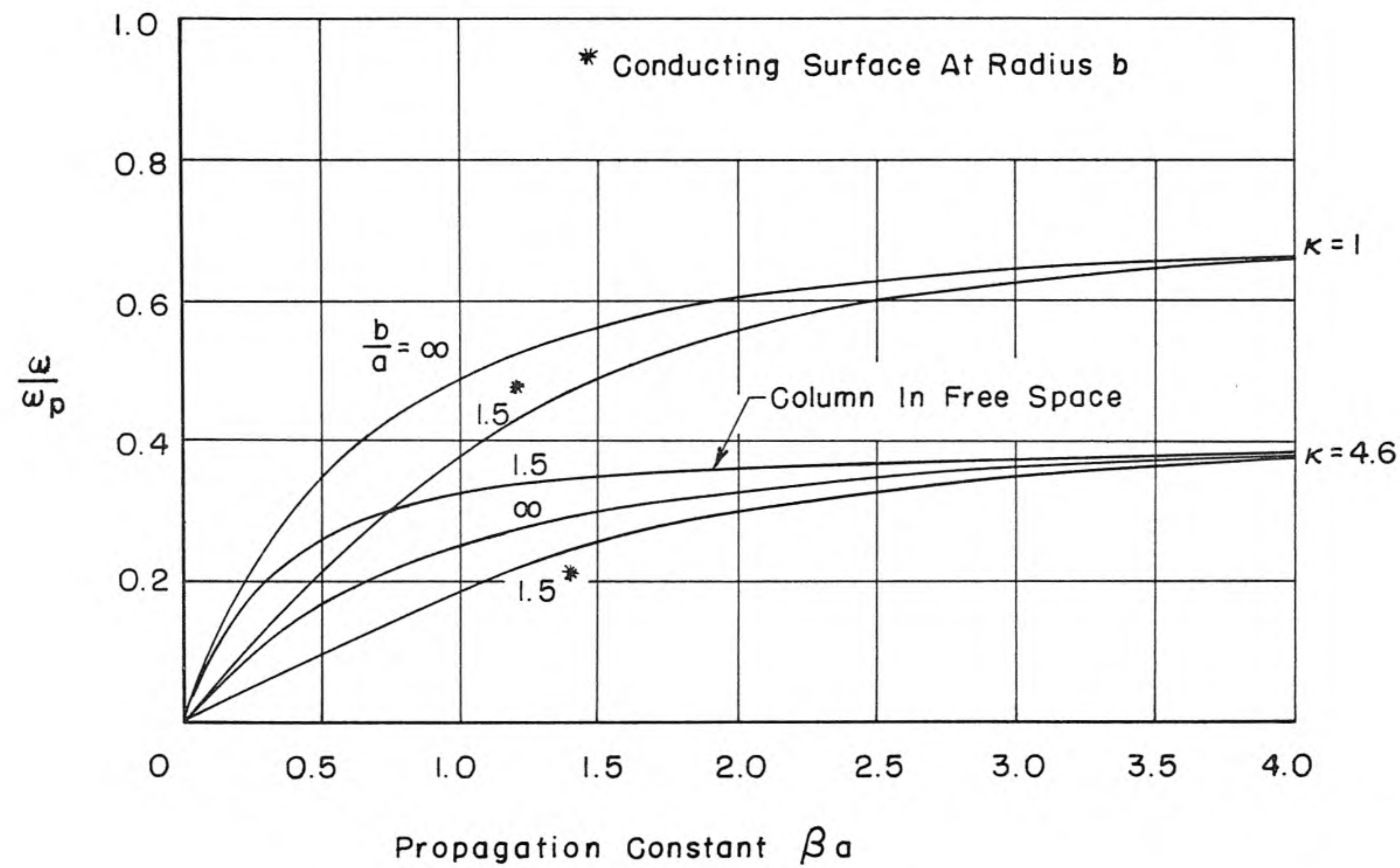

Figure 2.1 Frequency versus propagation constant for the plasma column. $B_o = 0$

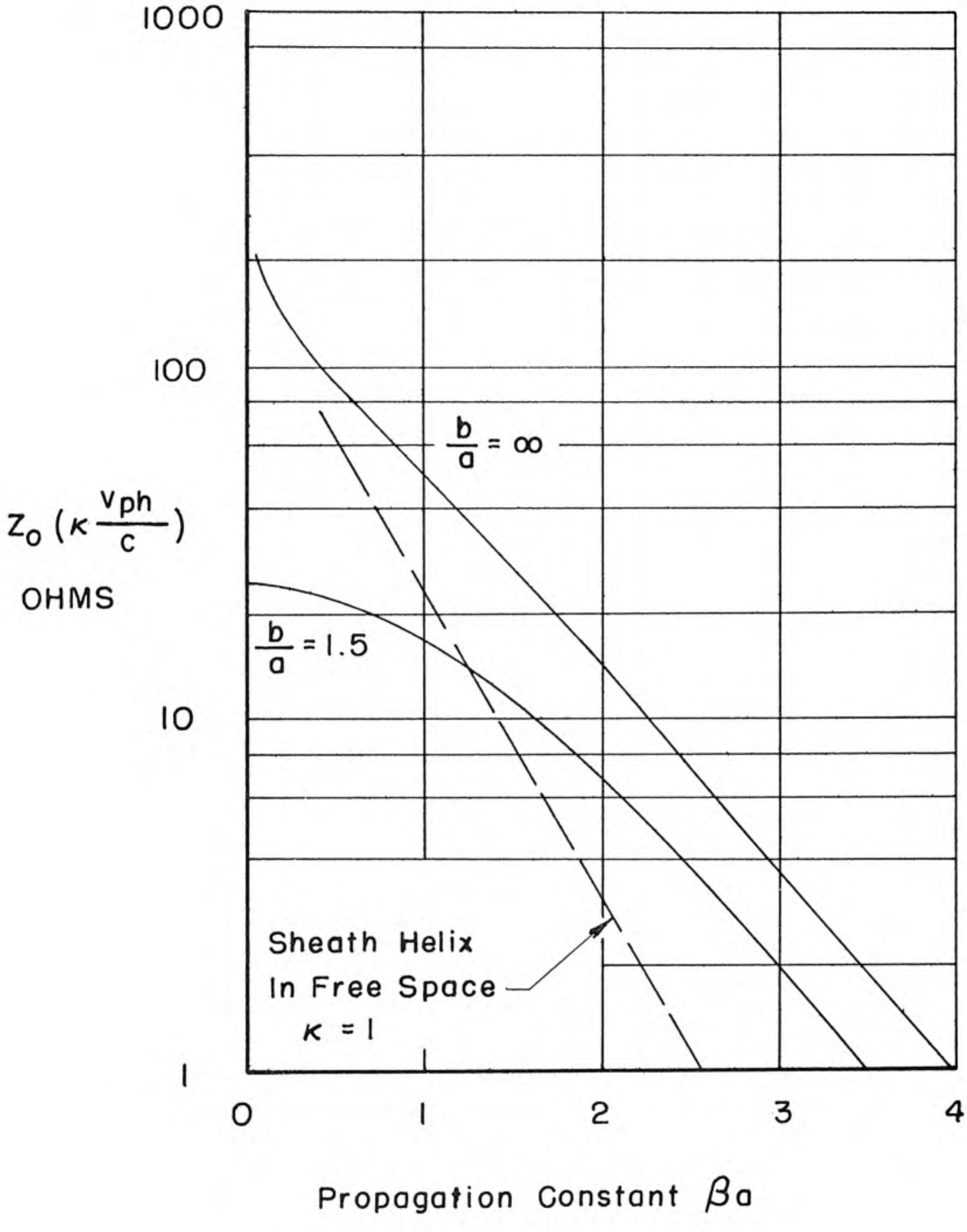

Figure 2.2 Axial interaction impedance at a fixed phase velocity versus the propagation constant. $B_o = 0$.

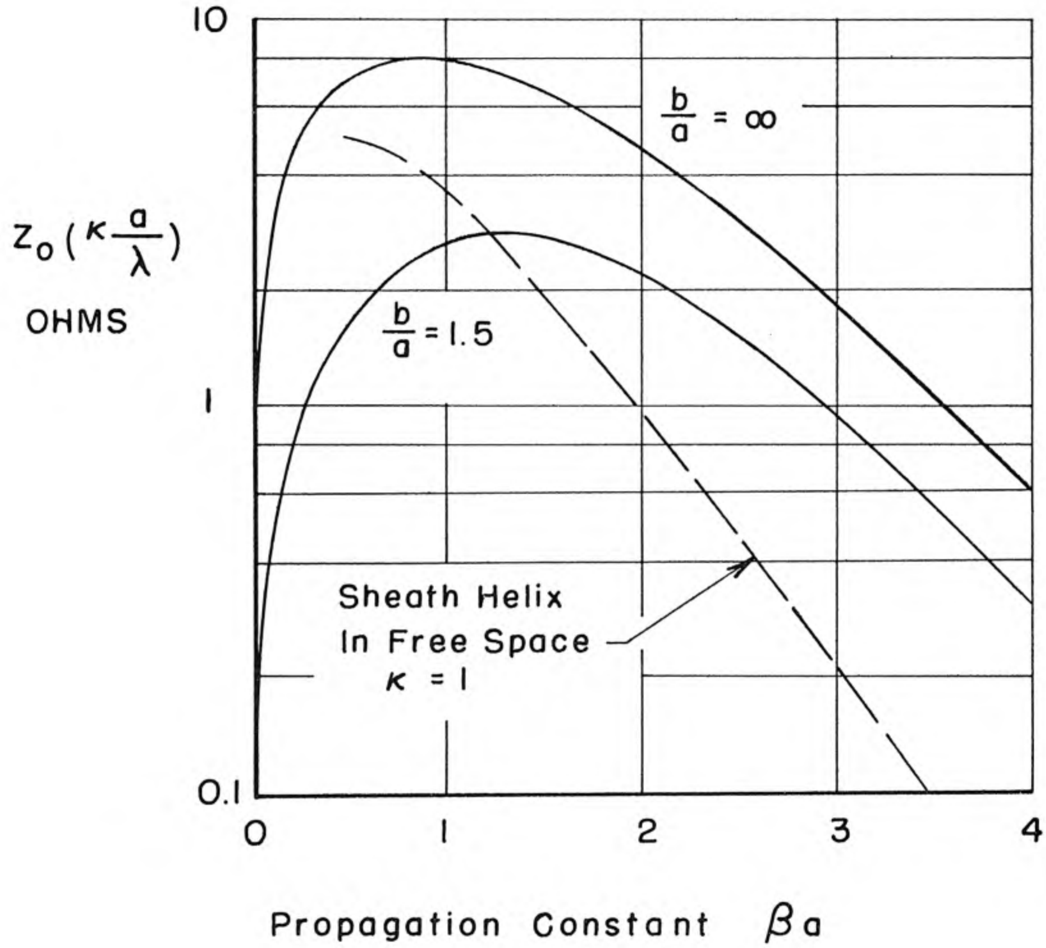

Figure 2.3 Axial interaction impedance at a fixed frequency versus the propagation constant. $B_o = 0$

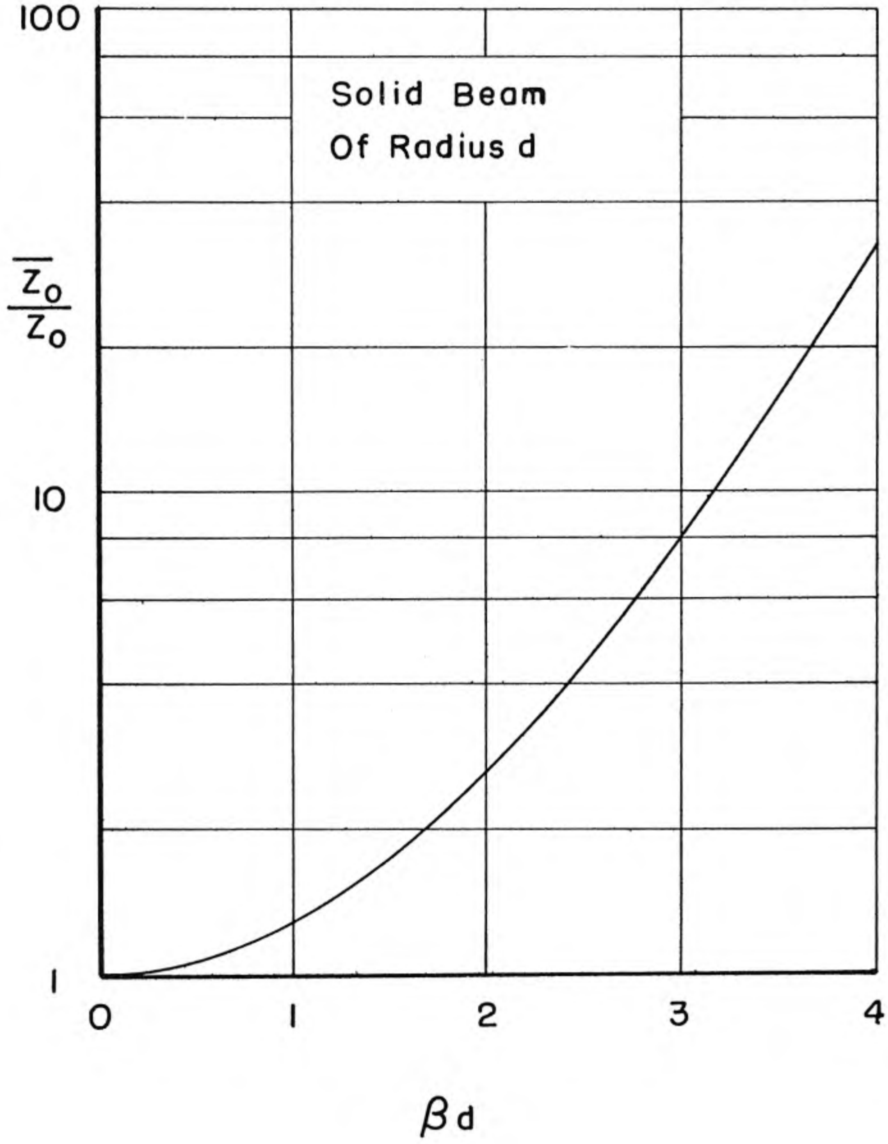

Figure 2.4 Average interaction impedance for an electron beam of finite radius. $B_o = 0$

3 COUPLING TO SLOW WAVE PLASMA MODES

The propagating system of Figure 1.1, Section 1 was considered to be of infinite length. It is useful to consider the coupling of energy to such plasma modes by a cavity. As an idealized case that is amenable to solution, consider that there is a gap in the conducting surface at $r = b$ of length h for values of z between $-h/2$ and $+h/2$. As a further assumption, assume that the gap field at $r = b$ is known and is a uniform alternating electric field of strength E_{1zb}. Naturally, for $|z| > h/2$ the electric field in the z direction at $r = b$ is zero since there is a conducting surface at this radius for all z except inside the gap. As a further assumption, assume that all energy coupled from the gap to the slow wave plasma modes travels outward from $z = 0$ and that none returns. This is equivalent to assuming that the plasma column is perfectly terminated far from the gap so that no energy is reflected. Only coupling to angularly independent modes of propagation will be considered. The approach used is that of Sensiper[10].

All fields have time and z dependence given by $e^{i(\omega t - \beta z)}$. The gap field at $r = b$ is shown in Figure 3.1.

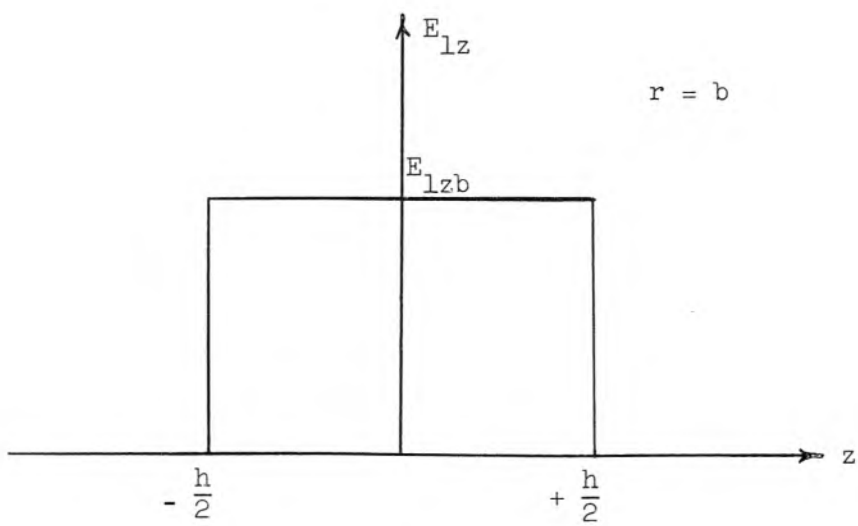

Figure 3.1 Gap field distribution

By the methods of Fourier integral analysis the field distribution across the gap can be expanded as

$$E_{1z}(r=b) = \begin{cases} E_{1zb} & |z| \leq \frac{h}{2} \\ 0 & |z| \geq \frac{h}{2} \end{cases}$$

$$= \frac{1}{2\pi} \int_C A(\beta)\, e^{-i\beta z}\, d\beta \tag{3.1}$$

$$A(\beta) = \int_C E_{1z}(r=b)\, e^{i\beta z}\, dz = 2E_{1zb} \frac{\sin \beta \frac{h}{2}}{\beta} \tag{3.2}$$

where the contour C is simply the entire real axis of the complex β plane from $-\infty$ to $+\infty$. Here β is considered as a general complex variable and should not be confused with the axial propagation constants of the slow wave plasma modes, although these latter values are, of course, definite points on the real axis of the complex β plane for lossless systems.

Inside a uniform plasma column the fields are given by equations 1.17 and 2.12. These functions represent the solution of the appropriate differential equation. Outside the plasma column the fields are given by 1.20 and 2.13. When a gap is placed in the conducting surface at $r = b$ these solutions can exist only at values of $|z| \gg h$. Thus in the neighborhood of the gap it is necessary to represent the fields by a Fourier integral and choose the contour in the manner of Sensiper[10] so as to allow only outgoing waves from the gap $-\frac{h}{2} \leq z \leq +\frac{h}{2}$. For convenience in this section only the $\tau^2 < 0$ solutions will be considered. The solutions for $\tau^2 > 0$ can easily be obtained by observing that

$$I_n(x) = i^{-n} J_n(ix) \quad . \tag{3.3}$$

In the region $0 \leq r \leq a$, from 1.17, represent the electric field as

$$E_{1z}(r) = \frac{1}{2\pi} \int_C B(\beta) \frac{I_o(Tr)}{I_o(Ta)} e^{-i\beta z} d\beta \tag{3.4}$$

From 1.20 in the range $a \leq r \leq b$

$$E_{1z}(r) = \frac{1}{2\pi} \int_C B(\beta) \frac{C(\beta)\, I_o(\beta r) + K_o(\beta r)}{C(\beta)\, I_o(\beta a) + K_o(\beta a)} e^{-i\beta z} d\beta \quad . \tag{3.5}$$

The continuity of tangential electric field at $r = a$ has been satisfied automatically in 3.4 and 3.5. The continuity of the normal displacement vector D_{1r} results in

$$\frac{\kappa_{rr}}{\kappa} \left(\frac{T}{\beta}\right) \frac{I_1(Ta)}{I_o(Ta)} = \frac{C(\beta)\, I_1(\beta a) - K_1(\beta a)}{C(\beta)\, I_o(\beta a) + K_o(\beta a)} \quad . \tag{3.6}$$

Solving for $C(\beta)$ by the bilinear transformation, one obtains

$$C(\beta) = \frac{-\left\{\frac{\kappa_{rr}}{\kappa} \left(\frac{T}{\beta}\right) \frac{I_1(Ta)}{I_o(Ta)}\right\} K_o(\beta a) - K_1(\beta a)}{\left\{\frac{\kappa_{rr}}{\kappa} \left(\frac{T}{\beta}\right) \frac{I_1(Ta)}{I_o(Ta)}\right\} I_o(\beta a) - I_1(\beta a)} \quad . \tag{3.7}$$

From 3.1 and 3.2 we obtain the Fourier integral expansion of the fields at $r = b$

$$E_{1z}(r = b) = \frac{E_{1zb}}{\pi} \int_C \frac{\sin \beta \frac{h}{2}}{\beta} e^{-i\beta z} d\beta \quad . \tag{3.8}$$

Equating integrands of equations 3.8 and 3.5 at $r = b$ we obtain

$$B(\beta) = 2E_{1zb} \frac{\sin \beta \frac{h}{2}}{\beta} \frac{C(\beta)\, I_o(\beta a) + K_o(\beta a)}{C(\beta)\, I_o(\beta b) + K_o(\beta b)} \tag{3.9}$$

where $C(\beta)$ is given by equation 3.7.

To find the coupling between the gap fields and the slow wave plasma modes it will be convenient to find the ratio between the field strength on the axis, $E_{1z}(0)$, and that in the gap. Thus from 3.4 and 3.9

$$\frac{E_{1z}(0)}{E_{1zb}} = \frac{1}{\pi} \int_C \frac{\sin \beta \frac{h}{2}}{\beta\, I_o(Ta)} \; \frac{C(\beta)\, I_o(\beta a) + K_o(\beta a)}{C(\beta)\, I_o(\beta b) + K_o(\beta b)} \; e^{-i\beta z}\, d\beta \quad . \tag{3.10}$$

This can be simplified from 3.7 to obtain

$$\frac{E_{1z}(0)}{E_{1zb}} = -\frac{1}{\pi} \int_C \frac{\sin \beta \frac{h}{2}}{\beta\, D(\beta)}\, e^{-i\beta z}\, d\beta \tag{3.11}$$

where

$$D(\beta) = \frac{\kappa_{rr}}{\kappa} (Ta)\, I_1(Ta)\, S - (\beta a)\, I_o(Ta)\, R \tag{3.12}$$

and R and S are defined in 2.17 and 2.18. Note that from 2.11 and 2.19 the quantity $D(\beta)$ in the denominator is the determinantal equation for the propagation constants of the plasma waves under consideration. Thus there are poles in the integrand of 3.11 at values of β on the real axis corresponding to the propagation constants of the ideal infinite plasma column without the gap. Label the various roots of the propagation equation

$$D(\beta) = 0 \tag{3.13}$$

by the subscript m as β_m .

To determine the appropriate contour of integration, restrict to positive values of z. Consider two representative roots β_1 and β_2 of the propagation equation 3.13. Inspection of 3.12 will show that $-\beta_1$ and $-\beta_2$ are also solutions. Assume that β_1 and β_2 are positive real numbers. The fact that they are real implies that they are propagating modes and not cutoff modes of propagation. Figure 3.2 represents their position in the complex β plane.

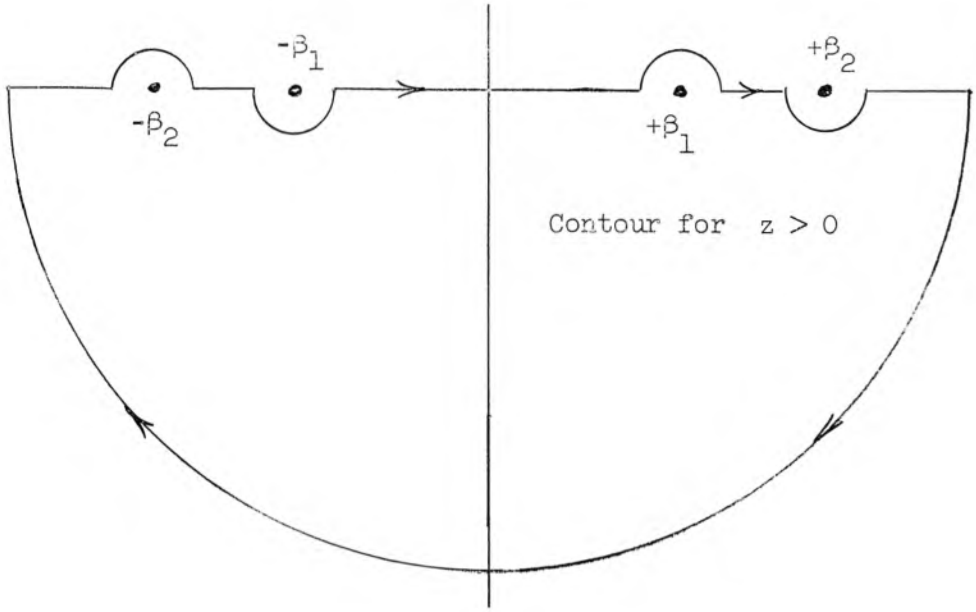

Figure 3.2 Contour of integration in the complex β plane

If with the assumption of loss the roots become $\underset{\sim}{\beta}_m = \beta_m - i\alpha_m$ then waves vary as

$$e^{-i(\underset{\sim}{\beta}_m z)} = e^{-i\beta_m z - \alpha_m z} \quad . \tag{3.14}$$

For forward waves of positive β_m and positive z note that α_m must be positive for waves to travel outward in the positive z direction and decay as the insertion of loss must require. For the case of a pole at $-\beta_m$ and positive z the phase velocity is inward, but if it is a backward wave the group velocity is outward. Since energy propagates in the direction of the group velocity, this mode with its power flow outward is one which physically can be excited by the gap. The above physical arguments show that for positive z an outward traveling forward wave will have a propagation constant in the fourth quadrant, and an outward traveling (group velocity) backward wave will have a propagation constant in the third quadrant. Thus in Figure 3.2 is drawn the appropriate contour for positive z, β_1 being a forward wave and β_2 being a backward wave.

From the theory of residues of complex variables it is well known that

$$\int_C f(z)\, dz = 2\pi i \sum \text{Residues } (r_m) \tag{3.15}$$

if

$$f(z) = \frac{p(z)}{q(z)} \tag{3.16}$$

then the residue at z_m is given by

$$r_m = \frac{p(z_m)}{q'(z_m)} \quad , \tag{3.17}$$

if the pole is of order 1, i.e., a simple pole. Since the coupling to one particular mode at a time is desired, it is not necessary to calculate the residues of all the poles of the integrand of 3.11, but only the residue of the propagating mode under consideration. Call the propagation constant of this mode β_m. Thus in the present case

$$\begin{aligned} p(\beta) &= \sin \beta \frac{h}{2} \cdot e^{-i\beta z} \\ q(\beta) &= \beta\, D(\beta) \end{aligned} \tag{3.18}$$

and

$$\begin{aligned} \left.\frac{dq}{d\beta}\right|_{\beta=\beta_m} = (\beta a)^2 \Bigg[& \frac{\kappa_{zz}}{\kappa} I_o(Ta)S + \frac{\sqrt{\kappa_{rr}\kappa_{zz}}}{\kappa} I_o(Ta) \left\{R - \frac{b}{a} M\right\} \\ & - \sqrt{\frac{\kappa_{zz}}{\kappa_{rr}}}\, I_1(Ta)R - I_o(Ta) \left\{S + \frac{b}{a} N\right\} \Bigg] \end{aligned} \tag{3.19}$$

where

$$M = I_o(\beta a)\, K_1(\beta b) + I_1(\beta b)\, K_o(\beta a) \tag{3.20}$$

$$N = I_1(\beta b)\, K_1(\beta a) - K_1(\beta a)\, K_1(\beta b) \tag{3.21}$$

and for $\tau^2 < 0$

$$T = \beta \sqrt{\frac{\kappa_{zz}}{\kappa_{rr}}}\ . \tag{3.22}$$

Then from 3.11 and 3.15

$$\frac{E_{1z}(0)}{E_{1zb}} = -i\ 2\ \frac{p(\beta_m)}{\left.\frac{dq}{d\beta}\right|_{\beta=\beta_m}}\ . \tag{3.23}$$

Thus we have evaluated the coupling to the plasma mode whose propagation constant is β_m .

Previously in equation 2.24 an interaction impedance Z_o for plasma waves was defined. Define an impedance as seen at the gap by

$$Z = \Gamma\ Z_o \tag{3.24}$$

where

$$\overline{P}_z = \frac{\phi_{1b}^2}{2Z} = \frac{h^2\ E_{1zb}^2}{2Z}\ . \tag{3.25}$$

Equating 2.23 and 3.25 and substituting 3.23, yields

$$\Gamma = (\beta h)^2\ \frac{E_{1zb}^2}{E_{1z}^2(0)} = (\beta\ \frac{h}{2})^2\ \left|\frac{\left.\frac{dq}{d\beta}\right|_{\beta=\beta_m}}{p(\beta_m)}\right|^2\ . \tag{3.26}$$

Thus from 3.26, 3.18 and 3.19 obtain for the case of $\tau^2 < 0$

$$\Gamma = \frac{(\beta a)^4 (\beta\ \frac{h}{2})^2}{\sin^2(\beta\ \frac{h}{2})}\ S^2 \left[\frac{\kappa_{zz}}{\kappa}\ I_o(Ta) + \frac{\sqrt{\kappa_{rr}\kappa_{zz}}}{\kappa}\ I_1(Ta) \left\{ \frac{R}{S} - \frac{b}{a}\frac{M}{S} \right\} - \sqrt{\frac{\kappa_{zz}}{\kappa_{rr}}}\ I_1(Ta)\ \frac{R}{S} - I_o(Ta) \left\{ 1 + \frac{b}{a}\frac{N}{S} \right\} \right]^2 \tag{3.27}$$

where

$$\frac{R}{S} = -\ \mathrm{Becoth}\ (\beta b, \beta a) \tag{3.28}$$

$$\frac{M}{S} = \frac{-1}{\mathrm{Betanh}\ (\beta b, \beta a)} \tag{3.29}$$

$$\frac{N}{S} = \frac{N}{R} \cdot \frac{R}{S} = -\ \mathrm{betanh}\ (\beta b, \beta a) \cdot \mathrm{Becoth}(\beta b, \beta a)\ . \tag{3.30}$$

In equation 3.27 note the multiplying constant

$$\frac{(\beta \frac{h}{2})^2}{\sin^2(\beta \frac{h}{2})}\ . \tag{3.31}$$

This is the reciprocal of the well known gap factor squared. Such is plotted in Figure 3.3 for several values of h/a. h is the width of the cavity gap and a is the radius of the plasma column.

3.1 Surface Waves

As a special case find the coupling coefficient to surface waves, $\omega_c = 0$. In this case $\tau^2 < 0$ and $T = \beta$. Then from 1.5

$$\kappa_{rr} = \kappa_{zz} = 1 - \frac{\omega_p^2}{\omega^2} \tag{3.1.1}$$

and from 2.11

$$\begin{aligned} \frac{\kappa_{rr}}{\kappa} &= -\frac{I_0(\beta a)}{I_1(\beta a)}\ \mathrm{Becoth}\ (\beta b, \beta a) \\ &= +\frac{I_0(\beta a)}{I_1(\beta a)}\ \frac{R}{S}\ . \end{aligned} \tag{3.1.2}$$

From equation 3.27 obtain

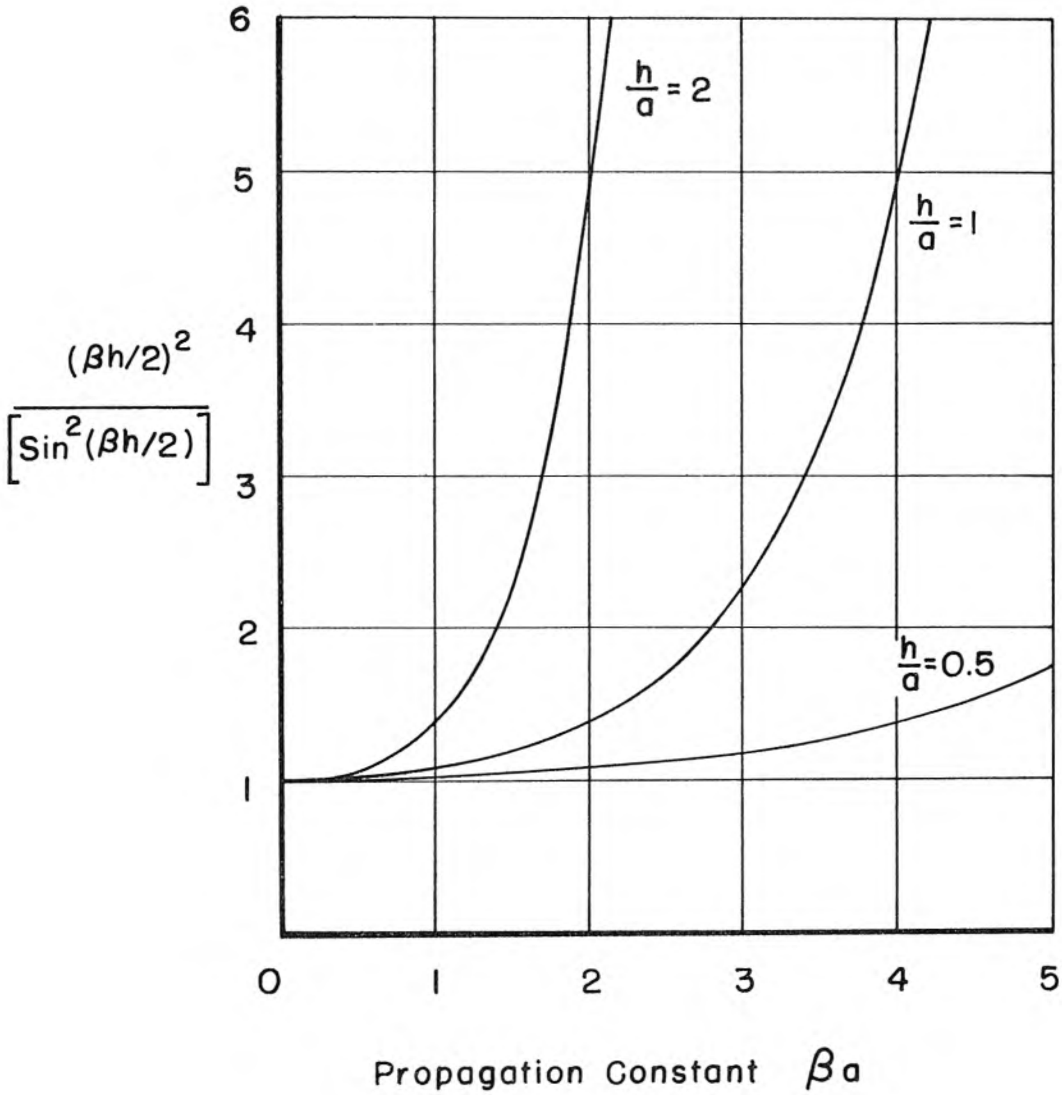

Figure 3.3 Reciprocal of the gap factor squared for several values of the gap length divided by the plasma column radius

$$\Gamma = \frac{(\beta\,\frac{h}{2})^2}{\sin^2(\beta\,\frac{h}{2})}\,(\beta a)^4 S^2 \Bigg\{ \mathrm{Becoth}(\beta b,\beta a)\left[I_o\,\mathrm{Becoth}(\beta b,\beta a) + I_1 - \frac{I_o^2}{I_o} \right]$$

$$- \frac{b}{a}\, I_o \left[\frac{\mathrm{Becoth}(\beta b,\beta a)}{\mathrm{Betanh}(\beta b,\beta a)} - \mathrm{betanh}(\beta b,\beta a)\cdot \mathrm{Becoth}(\beta b,\beta a) \right] - I_o \Bigg\}^2 \qquad (3.1.3)$$

where the Bessel functions are of argument βa .

The impedance factor $\Gamma / \left[\frac{(\beta h/2)^2}{\sin^2(\beta h/2)} \right]$ is plotted in Figure 3.4 for a specific value of $b/a = 1.5$ versus βa . The normalized impedance as seen at the gap is plotted in Figure 3.5. This figure is obtained from a combination of Figures 2.2 and 3.4 as specified by equation 3.24.

It is well to point out that the impedance given in Figure 3.5 represents coupling only to the wave traveling in one direction from the gap since only the positive propagation constant β_m was used in the computation of the residue in equation 3.23. In practice for gap coupling to a plasma column extending to infinity in both directions from the gap, one would couple to both the $+\beta_m$ and $-\beta_m$ outward propagating modes and thus the gap impedance would be half that given by Figure 3.5.

As an example, assume $h/a = 1$ and $\beta a = 2$. From Figure 3.3 and 3.5 one obtains

$$Z\kappa \frac{v_{ph}}{c} = 91.87 \times 1.410 = 129.5 \text{ ohms} .$$

If the phase velocity is a tenth the speed of light and $\kappa = 4.6$, then the impedance as seen by the gap is 282 ohms.

In Figure 3.5 it is apparent that the impedance as seen by the gap is fairly constant over the range of βa shown. Of course as $\beta a \to \infty$ the impedance Z must also approach infinity. For practical ranges of βa, however, one may obtain a fair estimate of the impedance as seen by the gap by computing the impedance Z in the limit of $\beta a \to 0$.

From equation 2.1.2 in the limit of $\beta a \to 0$

$$Z_o(\kappa \frac{v_{ph}}{c}) = \frac{\sqrt{\mu_o/\epsilon_o}}{2\pi} \ \ell n \ \frac{b}{a} \tag{3.1.4}$$

and from equation 3.1.3 as $\beta a \to 0$

$$\frac{\Gamma}{\frac{(\beta h/2)^2}{\sin^2(\beta h/2)}} = 4 \quad . \tag{3.1.5}$$

Therefore as $\beta a \to 0$

$$\frac{Z\ (\kappa \ \frac{v_{ph}}{c})}{\frac{(\beta h/2)^2}{\sin^2(\beta h/2)}} = \frac{2}{\pi}\sqrt{\frac{\mu_o}{\epsilon_o}} \ \ell n \ \frac{b}{a} \quad . \tag{3.1.6}$$

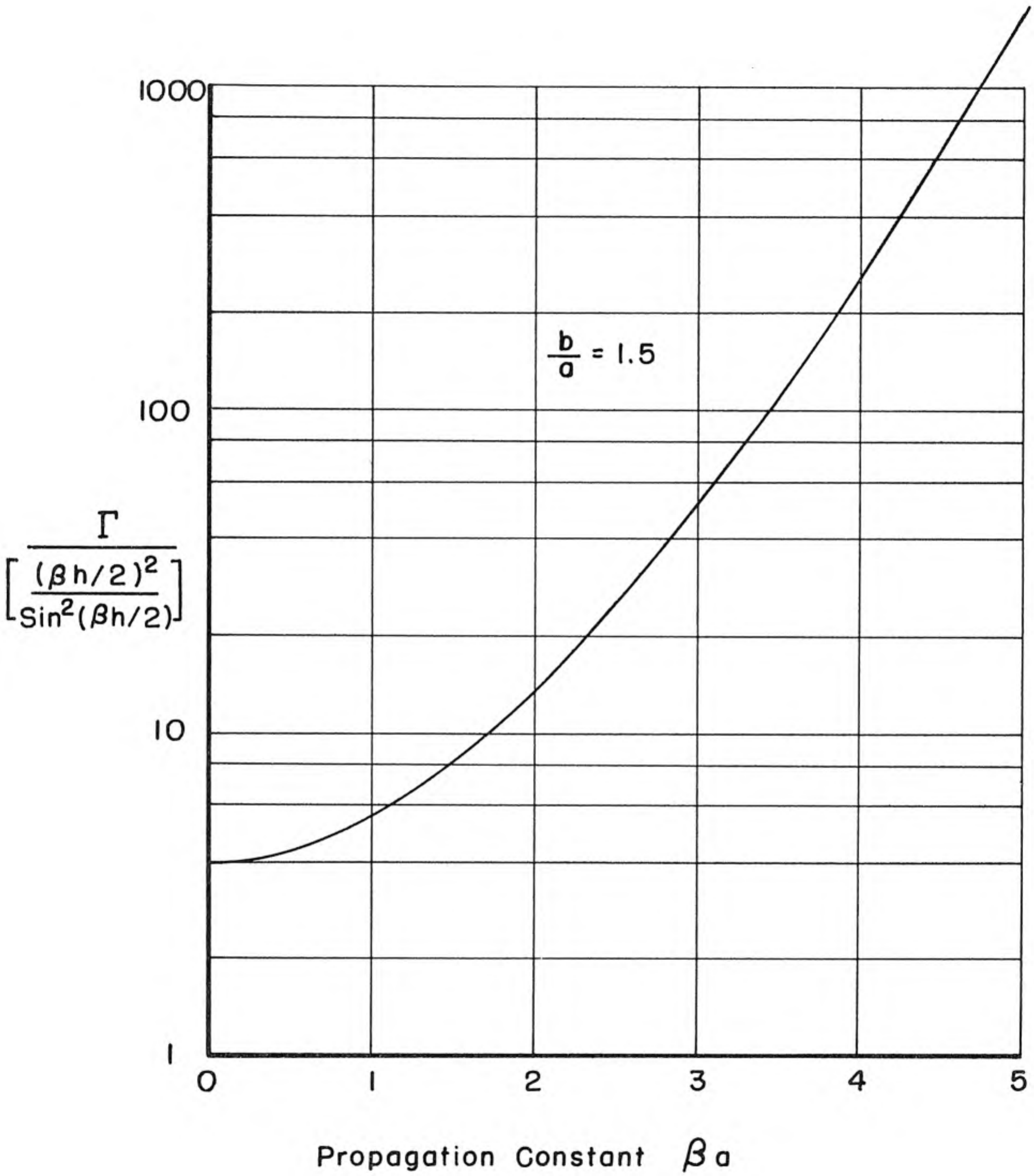

Figure 3.4 Ratio Γ of the impedance as seen by the gap to the axial interaction impedance. Normalized in terms of the gap factor squared. $B_o = 0$

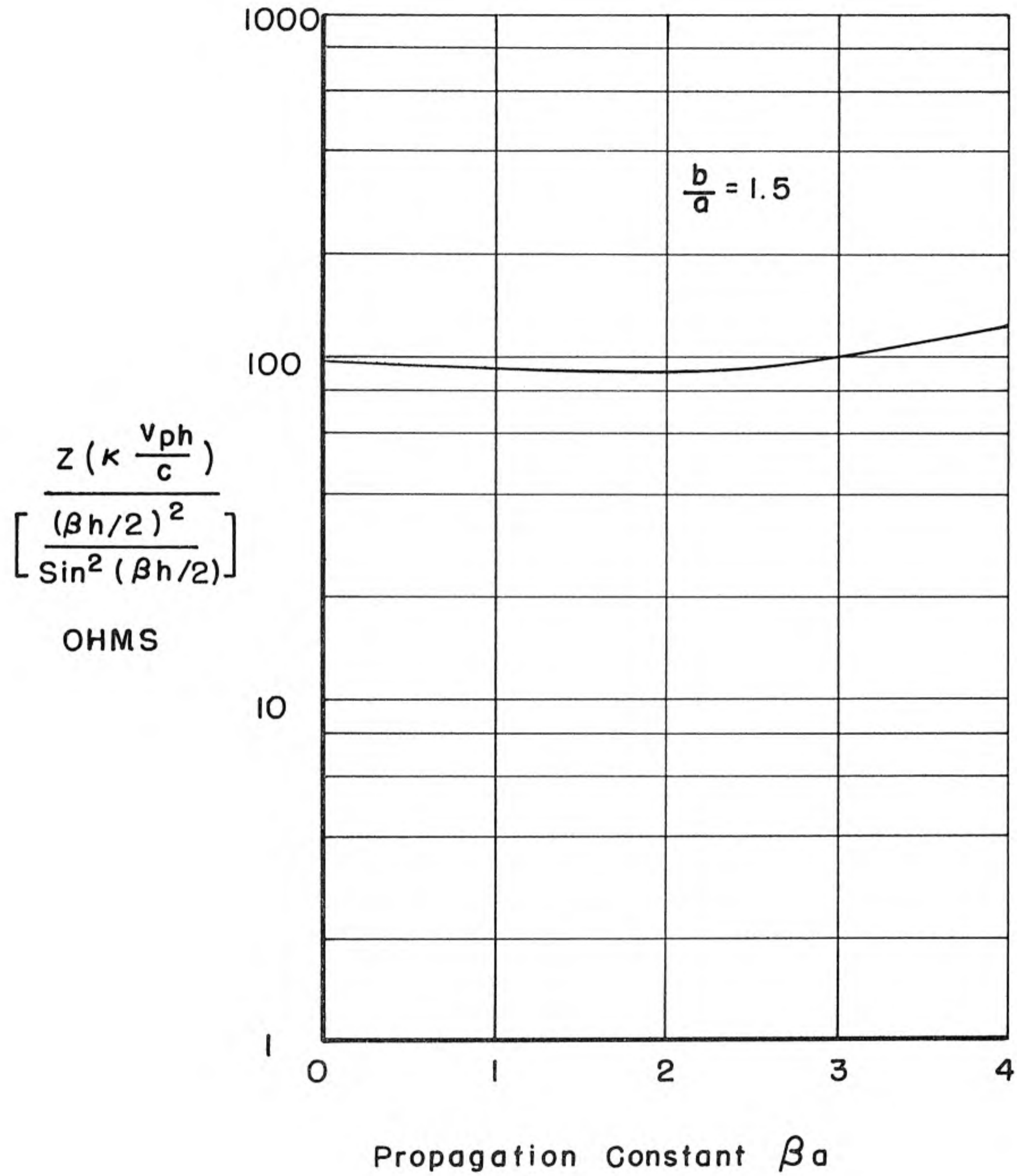

Figure 3.5 Normalized gap impedance for coupling to a fixed phase velocity slow wave plasma mode of propagation as a function of the propagation constant. $B_o = 0$.

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