Improving the energy-transfer efficiency of opening switch tokamak Ohmic heating transformers

P. M. Bellan

California Institute of Technology, Pasadena, California 91125

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It is shown that the energy-transfer efficiencies of opening switch and closing switch Ohmic heating transformers differ significantly, and that the efficiency of an opening switch transformer can be optimized by using an iron core with a substantial air gap (gap comparable to the plasma minor radius). A demonstration on the Caltech Encore gapped iron core tokamak is cited.

INTRODUCTION

Tokamak[1,2] Ohmic heating transformers[3] are used to break down gas to form a plasma (breakdown phase), generate a toroidal current in the plasma (ramp-up phase), and sustain the current at a constant value (sustaining phase). Ohmic heating transformers can be divided into two classes, those using opening switches [Fig. 1(a)] and those using closing switches [Fig. 1(b)]. The energy-transfer efficiencies η_E ($\equiv$plasma poloidal field energy/input energy) in the ramp-up phase of these two circuits turn out to be quite different and it will be shown that η_E of the opening switch peaks at a particular set of parameters.

I. OPENING SWITCHES

The sequence of operation of the transformer in Fig. 1(a) starts with current I_1 established in the primary winding (by a crowbarred capacitor discharge circuit, not shown) so that the initial energy in the system is $W_{\text{init}} = \frac{1}{2}L_1 I_1^2$. The primary breaker switch S1 is opened, causing a rapid decrease in I_1, with rate of decrease determined by (large) resistance R_1. This decrease creates an inductive voltage spike ($\propto R_1$) which breaks down the gas in the torus—creating the plasma—and then, as I_1 is depleted, builds up I_2 in the plasma. Ultimately, $I_1 = 0$ and, if the plasma is a perfect conductor, $I_2 =$ constant. Because the plasma is not a perfect conductor, I_2 must be sustained; this will be discussed later. To simplify the analysis, the breakdown process will not be treated here, and instead it will be assumed that a fully ionized, perfectly conducting plasma "springs into life" the instant the breaker switch is opened.

For this idealized plasma the primary and secondary circuit equations are

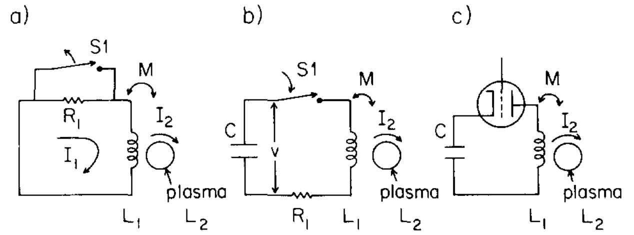

FIG. 1. (a) Opening switch circuit; (b) closing switch circuit; (c) triode switch used in Encore (triode consists of two 8C25 tubes in parallel, $C = 60\ \mu$F charged to 7 kV).

$$0 = L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} + I_1 R_1, \tag{1}$$

$$0 = L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt} + I_2 R_2, \tag{2}$$

where L_1 and L_2 are, respectively, the primary and secondary self-inductances, M is the mutual inductance, and R_2 is the plasma resistance. During the ramp-up phase the inductive voltages in Eq. (2) are much larger than $I_2 R_2$ which, consequently, can be neglected. Substituting Eq. (2) in Eq. (1), multiplying by I_1, and integrating shows the energy dissipated in the primary resistance W_{R_1} is

$$W_{R_1} \equiv \int_0^t I_1^2 R_1\, dt = \left(L_1 - \frac{M^2}{L_2}\right)\frac{I_{1\text{initial}}^2}{2},$$

so that the fraction of initial energy dissipated in R_1 is

$$\frac{W_{R_1}}{W_{\text{init}}} = 1 - \frac{M^2}{L_1 L_2}. \tag{3}$$

Integrating Eq. (2) gives

$$0 = L_2 I_{2\text{final}} - M I_{1\text{initial}}, \tag{4}$$

so that the total energy in the secondary, normalized to W_{init} is

$$\frac{W_{2\text{tot}}}{W_{\text{init}}} = \frac{\frac{1}{2}L_2 I_{2\text{final}}^2}{\frac{1}{2}L_1 I_{1\text{initial}}^2} = \frac{M^2}{L_1 L_2}.$$

As shown in Figs. 2(a) and 2(b) the flux $L_2 I_2$ consists of three

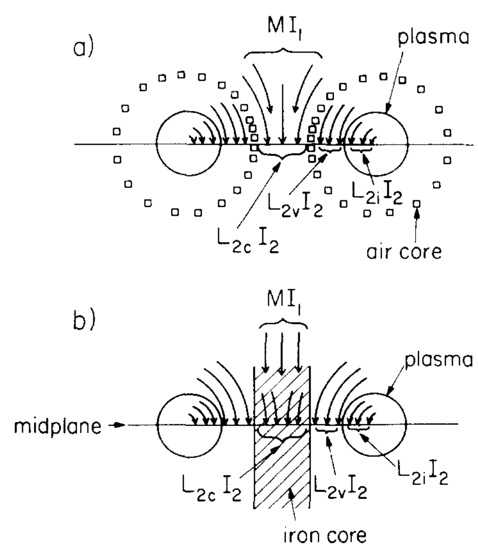

FIG. 2. (a) Air core fluxes; (b) iron core fluxes.

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distinct components: $L_{2c}I_2$, $L_{2v}I_2$, and $L_{2i}I_2$, where $L_{2c}I_2$ is the flux common to both the primary and secondary circuits, $L_{2v}I_2$ is the vacuum flux that lies between the plasma and the transformer, and $L_{2i}I_2$ is the flux internal to the plasma.

It is easy to see that the inductance of a one-turn coil generating the common flux is just M/n, where n is the turns ratio, so

$$L_{2c} = M/n.$$

Substituting for L_{2c} in Eq. (4) gives the current transfer efficiency η_I as

$$\eta_I \equiv \frac{I_{2\text{final}}}{nI_{1\text{initial}}} = \frac{M/n}{M/n + (L_{2i} + L_{2v})}, \tag{5}$$

which is plotted in Fig. 3(a). Equation (5) shows that maximum plasma current is obtained when $M/n \gg L_{2i} + L_{2v}$.

Now, consider the efficiency of creating poloidal field energy (i.e., $\frac{1}{2}L_{2i}I_2^2$) in the plasma; this is the quantity one wants to maximize since it is the poloidal field that provides confinement. Defining the poloidal field energy-transfer efficiency as η_E, one finds

$$\eta_E \equiv \frac{\text{poloidal energy}}{W_{\text{init}}} = \frac{\frac{1}{2}L_{2i}I_2^2}{\frac{1}{2}L_1 I_1^2} = \left(\frac{L_{2i}}{L_1/n^2}\right)\left(\frac{I_2}{nI_1}\right)^2.$$

On using Eq. (5) and noting that $L_1 = n^2 L_{2c} = n^2 M/n$, this becomes

$$\eta_E = \left(\frac{L_{2i}}{L_{2i} + L_{2v}}\right)\left(\frac{(L_{2i} + L_{2v})M/n}{[M/n + (L_{2i} + L_{2v})]^2}\right). \tag{6}$$

The factor $L_{2i}/(L_{2i} + L_{2v})$ is determined by the space between the plasma and the transformer (cf. Fig. 2). The other factor in Eq. (6) maximizes at the value 1/4 when

$$M/n = L_{2i} + L_{2v}. \tag{7}$$

In this case $L_2 = 2M/n$ so that the normalized total secondary energy becomes $W_{2\text{tot}}/W_{\text{init}} = 1/2$ with the other half of the initial energy dissipated in R_1. Furthermore, when η_E is maximized, half of $W_{2\text{tot}}$ consists of common flux energy ($\frac{1}{2}L_{2c}I_2^2$) and half consists of the sum of the internal and vacuum flux energies $[\frac{1}{2}(L_{2i} + L_{2v})I_2^2]$. Thus, the initial energy W_{init} is divided as follows: half into R_1, one-quarter into common flux, and one-quarter into the sum of internal plus vacuum flux. The fraction of the sum of internal plus vacuum flux energy that goes into internal flux energy, i.e., $L_{2i}/(L_{2i} + L_{2v})$, depends only on the machine geometry. It is appropriate to factor this fraction out of Eq. (6) by defining the geometry-independent figure of merit $\eta_E^* \equiv \eta_E (L_{2i} + L_{2v})/L_{2i}$ which then can be used to compare machines having different geometry. Figure 3(b) shows a plot of η_E^*, as determined by Eq. (6). Now, consider where conventional iron and air core transformers lie on Figs. 3(a) and (b).

First consider air core transformers. From Fig. 2(a), it is seen that M/n is the inductance of a one-turn surface-current toroid having the same minor radius r_{OH} as the Ohmic heating transformer (the major radius R of the plasma and the Ohmic heating transformer are the same), i.e.,

$$M/n = \mu_0 R\,[\ln(8R/r_{\text{OH}}) - 2] \tag{8}$$

(this form of the inductance must be modified for very small aspect ratios, $R/r_{\text{OH}} < 3$). Also from Fig. 2(a), it is seen that the vacuum flux is the difference between the fluxes of two surface current toroids having minor radii of r_{OH} and r_{pl}, respectively, so that

$$L_{2v} = \mu_0 R \ln(r_{\text{OH}}/r_{\text{pl}}). \tag{9}$$

Making the reasonable assumption of a parabolic plasma current-density profile gives

$$L_{2i} = 3\mu_0 R/4. \tag{10}$$

Combining these last three equations shows that, for an air core,

$$\frac{M/n}{L_{2i} + L_{2v}} = \frac{\ln(8R/r_{\text{OH}}) - 2}{3/4 + \ln(r_{\text{OH}}/r_{\text{pl}})}. \tag{11}$$

Figure 4 shows a plot of Eq. (11) for the typical aspect ratio of

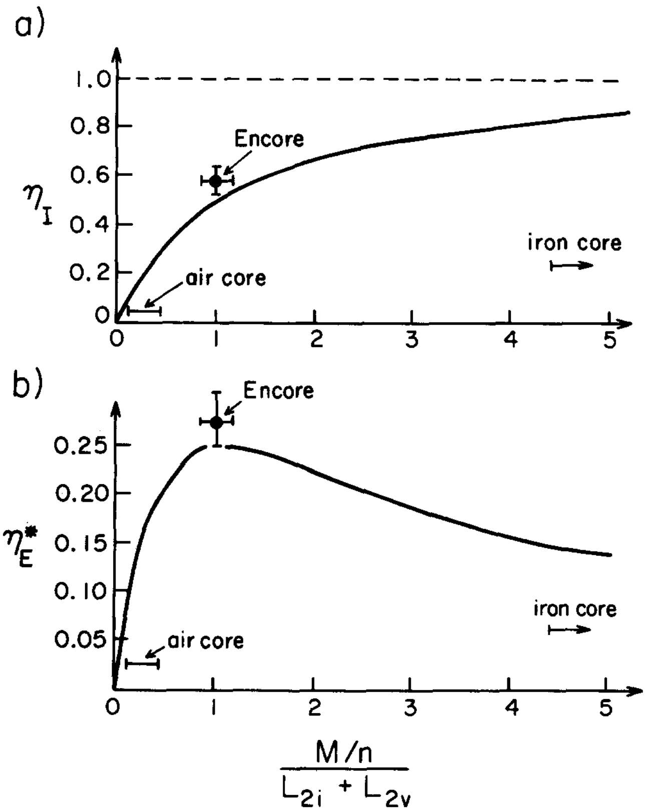

FIG. 3. (a) Current transfer efficiency η_I from Eq. (5); (b) poloidal energy-transfer figure of merit $\eta_E^* = \eta_E(L_{2i} + L_{2v})/L_{2i}$ for opening switch, from Eq. (6).

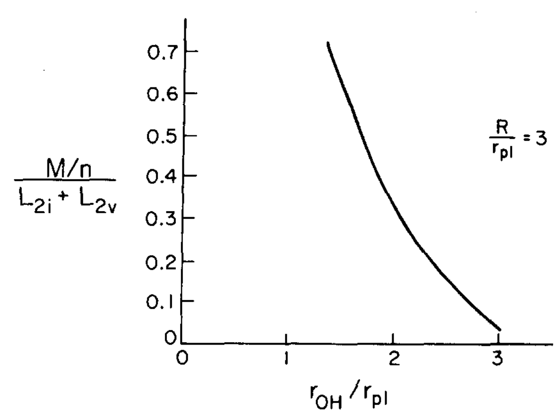

FIG. 4. $(M/n)/(L_{2i} + L_{2v})$ vs transformer size r_{OH}/r_{pl} for an air core transformer from Eq. (11), $R/r_{pl} = 3$.

$R/r_{pl} = 3$. For typical values of $r_{\rm OH}/r_{pl} \sim 1.5$–$2.5$, it is seen that $M/n(L_{2i} + L_{2v})$ ranges from 0.15 to 0.5. Thus, typical air core transformers lie in the region marked "air core" on Figs. 3(a) and 3(b) and so are characterized by low energy-transfer efficiency and also by low I_2/nI_1.

Now consider iron cores. The magnetic field in a toroidal core due to a one turn winding (e.g., plasma) with current I_2 is $B = \mu_{\rm Fe} I_2/l$, where $\mu_{\rm Fe}$ is the permeability of iron and l is the core circumference. The flux ϕ_{21} that I_2 generates in the n-turn primary winding is $\phi_{21} = n\mu_{\rm Fe} I_2 A/l$, where A is the core cross-section area. Thus, the mutual inductance $M \equiv d\phi_{21}/dI_2 = n\mu_{\rm Fe} A/l$ and $M/n = \mu_{\rm Fe} A/l$ (or, in magnetic circuit terminology, the inverse reluctance). For an iron core, L_{2v} is again given by Eq. (9) with $r_{\rm OH}$ taken to be the effective minor radius of the iron; e.g., a square cross-section iron core with sides of length $2a$, would have an effective circular radius of $a\sqrt{4/\pi}$, so that $r_{\rm OH} \approx R - a\sqrt{4/\pi}$ gives the outer limit of the vacuum flux region. As Fig. 5(a) shows, a typical iron core has $A \sim \pi r_{pl}^2$ and $l \sim 8R$ so that, for an iron core,

$$\frac{M/n}{L_{2i}+L_{2v}} \approx \frac{\mu_{\rm Fe}\pi r_{pl}^2/8R}{\mu_0 R\left\{3/4+\ln\left[(R-a\sqrt{4/\pi})/r_{pl}\right]\right\}} \approx \frac{\pi}{8}\frac{\mu_{\rm Fe}}{\mu_0}\left(\frac{r_{pl}}{R}\right)^2 \gg 1, \tag{12}$$

where the inequality holds because $\mu_{\rm Fe}/\mu_0 \approx 10^3$–$10^4$. Thus, typical iron core transformers lie in the region marked "iron core" in Figs. 3(a) and 3(b) and are characterized by low energy-transfer efficiency and by high I_2/nI_1.

It should now be apparent that for an opening switch circuit, conventional iron core transformers are analogous to a transmission line driving a short circuit, while air core transformers are analogous to a transmission line driving an open circuit. Clearly, the energy coupling efficiency could be improved by satisfying Eq. (6) so as to operate at the peak of Fig. 3(b). Figure 5(b) shows a way this desirable outcome may be achieved: an air gap is inserted in the iron core so as to make M/n adjustable. From elementary magnetic circuit considerations[4]

$$\left(\frac{M}{n}\right)^{-1} = \frac{l}{\mu_{\rm Fe}A} + \frac{d}{\mu_0 A}, \tag{13}$$

where d is the air gap width. Since $\mu_{\rm Fe} \gg \mu_0$,

$$M/n \approx \mu_0 A/d. \tag{14}$$

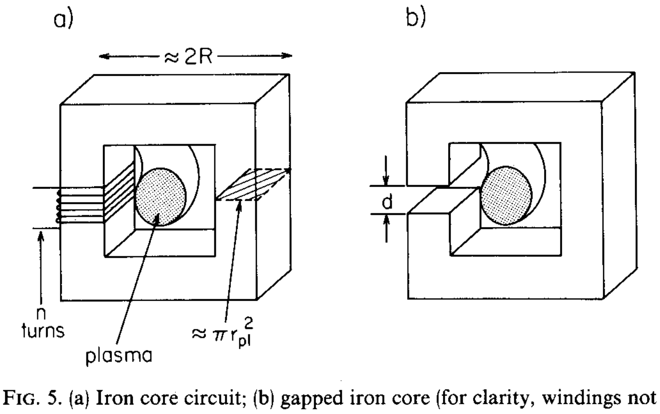

FIG. 5. (a) Iron core circuit; (b) gapped iron core (for clarity, windings not shown).

Thus, optimum energy transfer efficiency η_E is obtained by setting

$$d = \frac{\mu_0 A}{L_{2i}+L_{2v}}. \tag{15}$$

By using the very rough approximations $A \sim \pi r_{pl}^2$ and $R \sim \pi r_{pl}$, an approximate scaling for d may be obtained:

$$d \sim \frac{\mu_0 \pi r_{pl}^2}{\mu_0 R\left\{3/4+\ln\left[(R-a\sqrt{4/\pi})/r_{pl}\right]\right\}} \sim \frac{\pi r_{pl}^2}{R} \sim r_{pl}. \tag{16}$$

Hence, an iron core with air gap width comparable to the plasma minor radius will give optimum energy transfer. (Alternately, if core saturation is acceptable during the slow bank phase, an ungapped core with a small cross section could be used instead.)

A test of this idea has been made on the Caltech Encore high repetition rate (15 shots/s) tokamak which has $R = 0.38$ m, $r_{pl} = 0.12$ m, an iron core with a square cross section with side $2a = 0.2$ m, and a primary winding with $n = 430$ turns. Since it is technologically difficult to obtain breaker switches that will recycle quickly, to attain high repetition rates Encore uses the triode circuit shown in Fig. 1(c); here biasing the triode to conduct establishes I_1, while subsequently biasing the tube to cutoff is equivalent to opening switch S1 in Fig. 1(a). An adjustable air gap is located on the midplane inside the primary winding of the iron core; this location is chosen so as to minimize field errors due to stray flux leaking from the gap. The gap is set to 0.06 m, so that from Eq. (14) $M/n = 0.8\,\mu$H. From Eqs. (9) and (10) the internal and vacuum inductances are calculated to be $L_{2i} = 0.4\ \mu$H and $L_{2v} = 0.4\ \mu$H so that $(M/n)/(L_{2i} + L_{2v}) = 1$, satisfying Eq. (7). The primary inductance is $L_1 = n^2(M/n) = 0.15$ H. Figure 6 shows a typical experimental measurement of I_1 and I_2 (gas fill of 5×10^{-5} H, 1-kG toroidal field, and in order to compare with this theory, no slow bank was used to sustain I_2). Here, the primary winding is energized with $I_1 = 41$ A, so that the initial energy is $\frac{1}{2}L_1 I_1^2 = 130$ J. The plasma current is $I_2 = 10$ kA giving $\eta_I = 0.57$. Also, the internal poloidal field energy is $\frac{1}{2}L_{2i} I_2^2 = 18$ J, so that $\eta_E = 0.14$ and $\eta_E^* = 0.28$. These measured values of η_I and η_E^* are plotted in Figs. 3(a) and 3(b) and are seen to be in excellent agreement with the theory.

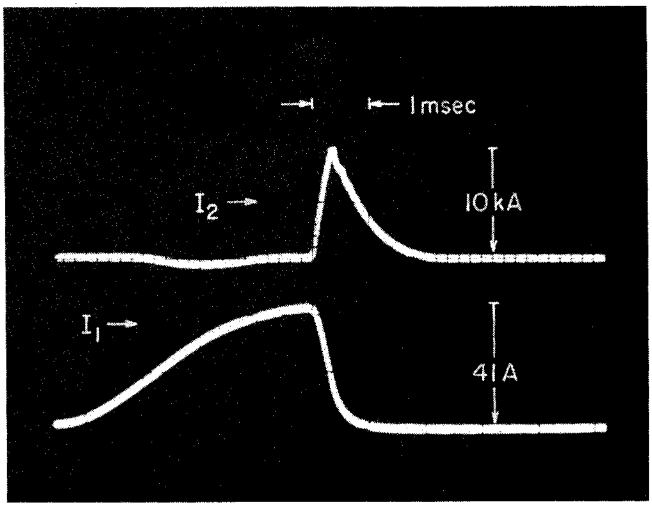

FIG. 6. Plasma current I_2, primary current I_1 measured on Encore.

II. CLOSING SWITCHES

Here, Eq. (1) is replaced by the primary circuit equation

$$V = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt} + I_1R_1, \tag{17}$$

where V is the voltage on the capacitor, and R_1 is now the (small) resistance of the primary winding. The operating sequence is very simple: switch S1 is closed discharging capacitor C into L_1; the rising current I_1 generates a voltage spike which first breaks down the gas and then establishes I_2. Unlike the opening switch circuit, the voltage spike is not adjustable and is just V/n. Substitution of Eq. (2) into Eq. (17) gives

$$V = \left(\frac{(L_1L_2 - M^2)}{L_2}\right)\left(\frac{dI_1}{dt}\right),$$

where I_1R_1 and I_2R_2 have been dropped since they are relatively small during the ramp-up phase. On substitution for L_1,L_2

$$V = \left(\frac{nM(L_{2i} + L_{2v})}{M/n + L_{2i} + L_{2v}}\right)\left(\frac{dI_1}{dt}\right).$$

Multiplication by I_1 and integration gives the total input energy to the circuit

$$W_{\text{input}} = \frac{nM(L_{2i} + L_{2v})}{M/n + L_{2i} + L_{2v}}\,\frac{I^2_{1\text{final}}}{2}. \tag{18}$$

Integration of Eq. (2) gives

$$I_{2\text{final}} = -(M/L_2)I_{1\text{final}}, \tag{19}$$

so that η_I is the same [Eq. (5) and Fig. 3(a)] as it was for the opening switch (except for a change of sign). However, calculation of η_E gives a substantially different result from the opening circuit; substitution of Eq. (19) in Eq. (18) gives

$$\eta_E = \frac{\frac{1}{2}L_{2i}I^2_{2\text{final}}}{W_{\text{input}}} = \left(\frac{L_{2i}}{L_{2i} + L_{2v}}\right)\left(\frac{M/n}{M/n + L_{2i} + L_{2v}}\right). \tag{20}$$

Hence, $\eta_E^* \equiv \eta_E(L_{2i} + L_{2v})/L_{2i}$ has the same functional form as η_I; i.e., that of Fig. 3(a).

Thus, for a closing switch it is best to have $M/n \gg L_{2i} + L_{2v}$ in which case an ungapped iron core would be the best method. The maximum η_E^* of a closing switch is unity, i.e., four times greater than the maximum possible η_E^* of an opening switch circuit. The reason for this difference is that an ideal closing switch dissipates no energy in R_1, and, since the net common flux is zero, has no energy stored in common flux. Thus, a closing switch can transfer all the initial energy to the sum of the internal plus vacuum fluxes. In contrast, an ideal opening switch (with inductances set for maximum energy transfer) will split the initial energy into half for dissipation in R_1, one-quarter for common flux, and one-quarter for the sum of internal plus vacuum fluxes.

III. OPENING VS CLOSING SWITCHES

Closing switches are clearly more efficient for establishing I_2; however, they are less efficient when sustaining I_2 is taken into account because the large initial I_1 must be continuously maintained in the primary causing substantial dissipation in R_1. In iron core closing switch systems, saturation of the iron places an upper limit on I_2 and its duration.

Air core transformers are typically used when one desires fluxes greater than unsaturated iron can provide; such large fluxes allow the sustaining phase of the plasma to last longer. In these applications opening switches are used so that a large constant I_1 does not have to be maintained in the primary circuit during the sustaining phase. As shown here, the efficiency of these opening switch air core transformers could be significantly improved by adding a gapped iron core (or equivalently, a small cross-section ungapped core with $\mu_{\text{Fe}}A/l = L_{2i} + L_{2v}$). Saturation of the iron would not be a concern during the ramp-up phase, because B in the iron would be small ($\sim$the poloidal field); however, the iron would eventually saturate during the operation of the slow bank.

It might be desirable to use a distributed air gap, i.e., a large number of smaller air gaps evenly spaced over the transformer circumference, so that stray fields at the air gap are minimized, and also so that disassembly of the iron for maintenance would be facilitated.

ACKNOWLEDGMENT

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