$$E_z = A\left[\frac{\exp\left(-i\int_{x_0}^{x} k_{x_1}\,dx'\right)}{k_{x_1}^{1/2}} - \frac{\exp\left(-i\int_{x_0}^{x_c} k_{x_1}\,dx'\right)}{k_{x_2}^{1/2}}\right.$$
$$\left.\times \exp\left(-i\int_{x_c}^{x} k_{x_2}\,dx'\right)\right]. \qquad (5)$$

The damping factor $\exp(-D)$ of the ion thermal wave is obtained by changing the elements of the dielectric tensor due to collisions and Landau damping: $K_{\parallel} \to K_{\parallel} + i\delta K_{\parallel}$, $K_{\perp} \to K_{\perp} + i\delta K_{\perp}$. $D$ is given by the following expression:

$$D = \int_{x_0}^{x_c} |k_z| \left(\frac{-K_{\parallel}(x)}{K_{\perp}(x)}\right)^{1/2}$$
$$\times \sin\left[\frac{1}{2}\ \mathrm{arctg}\left(\frac{\delta K_{\parallel}(x)}{-K_{\parallel}(x)} + \frac{\delta K_{\perp}(x)}{K_{\perp}(x)}\right)\right] dx. \qquad (6)$$

The difference between our result and Eq. (44) of Ref. 1, is that the integration is extended only to the conversion point and not to the lower-hybrid resonance point. Notice that $K_{\perp}(x)$ in the denominator never vanishes in the region of integration. Since $n_z$ is above accessibility, there is never a contribution from the region very close to the lower hybrid resonance and $\delta K_{\perp}(x) \ll K_{\perp}(x)$ for all $x \in (x_0, x_c)$. The damping is proportional to $\delta K_{\perp}$, and not to $(\delta K_{\perp})^{1/2}$ as in Eq. (45), of Ref. 1. This reduces the collisional effect by at least one order of magnitude. A rough estimate of the integral in (6) is

$$D \simeq |k_z|\, L\, \frac{[-K_{\parallel}(x_0)]^{1/2}}{[K_{\perp}(x_c)]^{3/2}}\, \delta K_{\perp}(x_c), \qquad (7)$$

where $L = x_c - x_0$. A more precise evaluation requires numerical integration of (6) with reasonable assumptions about the density and temperature profiles.

There might be a strong electron Landau damping for sources, which excite waves with large parallel refractive indices.

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[1]P. M. Bellan and M. Porkolab, Phys. Fluids 17, 1592 (1974).
[2]A. Bers, in *Proceedings of the Third Symposium on Plasma Heating in Toroidal Devices*, Varenna-Como, Italy (Editrice Compositori, Bologna, 1976), p. 99.
[3]C. L. Grabbe, Ph.D. thesis, California Institute of Technology (1977).

# Reply to Comments by Krapchev and Bers

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Krapchev and Bers are correct in criticizing the calculation presented in Ref. 1 on the effect of damping on linear mode conversion, but their revised calculation is oversimplified and not quite correct in itself. In this reply the oversimplifications by Krapchev and Bers will be discussed, and then a more detailed correction to the calculation of Ref. 1 will be reviewed.[2]

Krapchev and Bers have, by the joining in their Eq. (4) of the contour integration solution to the WKB solution, implied that their

$$I_{1,2} = \frac{1}{k_{x_{1,2}}^{1/2}} \exp\left(-i\int_{x_c}^{x} k_{x_{1,2}}\, dx\right).$$

From Eq. (35) of Ref. 1, the saddle point solutions are seen to be expansions about $x_{th}$ and not $x_c$, so that the lower limit in the integral of the previous expression should be $x_{th}$. Furthermore, their Eq. (3), combined with the previous expression, suggests that the amplitude coefficients of $I_{1,2}$ are both unity. In fact, these coefficients differ by a factor of $i$; the right angle between the directions of the saddle points marked 1 and 2 in Fig. 7(b) of Ref. 1 is the reason for this phase factor. [An equivalent explanation is that the second derivative of the exponential argument of Eq. (38) of Ref. 1 has opposite signs when evaluated at the respective hot and cold saddle points.] This factor of $i$ is also found in Refs. 3 and 4.

Let us now take this opportunity to present a corrected derivation[2] which shows that, as suggested by Krapchev and Bers, there is no enhancement of damping at the mode conversion layer. With damping, Eq. (37) (the equation numbers in the following refer to Ref. 1) becomes

$$\frac{d^4E_z}{d\xi^4} + b\,\frac{d^3E_z}{d\xi^3} + (\xi + i\bar{\epsilon})\,\frac{d^2E_z}{d\xi^2} + \frac{dE_z}{d\xi} + \mu E_z = 0,$$

where $\bar{\epsilon} = \alpha^{-1/3}(\mathrm{Re}K'_{xx})^{1/3}\,\mathrm{Im}(K_{xx})/\mathrm{Re}(K'_{xx})$. Equation (41) now assumes the same form as for the nondamping case, except $x$ is everywhere replaced by $x + i\epsilon$, where $\epsilon = \mathrm{Im}K_{xx}/\mathrm{Re}K'_{xx}$; i.e., Eq. (41) becomes

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$$E_z(x,\ z) = i|k_z|^{1/2}\left(\frac{\alpha}{[(x+i\epsilon)\mathrm{Re}K'_{xx}]^3}\right)^{1/4}$$

$$\times \exp\left[\frac{2}{3}\, i\left(\frac{\mathrm{Re}K'_{xx}}{\alpha}\right)^{1/2}(x+i\epsilon)^{3/2}\right]$$

$$+[-(x+i\epsilon)\,\mathrm{Re}(K'_{xx})K_{zz}]^{-1/4}$$

$$\times \exp\left[2i\left(\frac{-k_z^2(x+i\epsilon)K_{zz}}{\mathrm{Re}K'_{xx}}\right)^{1/2}\right]\ .$$

The quantities $(x+i\epsilon)^{1/2}$, $(x+i\epsilon)^{3/2}$ in the phases of the above expression can be written as integrals:

$$\frac{2}{3}(x+i\epsilon)^{3/2} = \int_{-i\epsilon}^{x}(x+i\epsilon)^{1/2}\,dx,$$

$$2(x+i\epsilon)^{1/2} = \int_{-i\epsilon}^{x}(x+i\epsilon)^{-1/2}\,dx.$$

Note that the lower limit of the integrals is at $x=-i\epsilon$. This was missed in Ref. 1 because the damping modification was inappropriately introduced *after* matching the boundary layer solutions to the WKB solutions rather than before, hence, the lower limit was improperly taken to be at $x=0$. This incorrect choice of lower limit led to the erroneous result of Ref. 1. It should be emphasized that the change suggested by Krapchev and Bers, namely changing the integration limit from $x_{1h}$ to $x_c$, is not required.

Equation (42) now becomes

$$E_z(x,\ z) = i|k_z|^{1/2}\left(\frac{\alpha}{K_{xx}^3}\right)^{1/4}\exp\left[i\int_{-i\epsilon}^{x}\left(\frac{K_{xx}}{\alpha}\right)^{1/2}dx\right]$$

$$+(-K_{xx}K_{zz})^{-1/4}\exp\left[i|k_z|\int_{-i\epsilon}^{x}\left(-\frac{K_{zz}}{K_{xx}}\right)^{1/2}dx\right]$$

where $K_{xx}=\mathrm{Re}K_{xx}+i\,\mathrm{Im}K_{xx}$. This expression is undetermined with respect to an overall constant, but this may be found by requiring that the second (i.e., cold) term correspond to the cold plasma wave launched from the antenna located at $x_0$. This correspondence may be obtained (in a manner similar to Ref. 1) by multiplying the entire expression by the constant,

$$\exp\left[i|k_z|\int_{x_0}^{-i\epsilon}\left(\frac{-K_{zz}}{K_{xx}}\right)^{1/2}dx\right]$$

giving

$$E_z(x,\ k_z) = i|k_z|^{1/2}\left(\frac{\alpha}{K_{xx}^3}\right)^{1/4}$$

$$\times\exp\left[i\int_{-i\epsilon}^{x}\left(\frac{K_{xx}}{\alpha}\right)^{1/2}dx + i|k_z|\int_{x_0}^{-i\epsilon}\left(\frac{-K_{zz}}{K_{xx}}\right)^{1/2}dx\right]$$

$$+(-K_{xx}K_{zz})^{-1/4}\exp\left[i|k_z|\int_{x_0}^{x}\left(\frac{-K_{zz}}{K_{xx}}\right)^{1/2}dx\right].$$

This expression parallels the nondamping Eq. (43), except here $K_{xx}$ is complex, and (unlike Ref. 1) the integration limits at $x=0$ are replaced by limits at $x=-i\epsilon$.

The attenuating effect of the mode conversion layer may be found by examining the real part of the hot wave phase at a point just beyond the mode conversion layer. At such a point explicit damping of the hot plasma wave is not yet important, so the hot plasma wave attenuation can come only from (i) damping at the mode conversion layer (if any), or (ii) damping of the cold plasma wave before it reaches the mode conversion layer. Noting that $\mathrm{Re}K'_{xx}=L^{-1}$, where $L$ is the scale length near the mode conversion layer, we may split the second integral of the hot wave phase into two regions $x<L$ and $x>L$ (by assumption the first integral has $x<L$). Then $D$, the real part of the hot wave phase is

$$D = -\left(\frac{\mathrm{Re}K'_{xx}}{\alpha}\right)^{1/2}\epsilon x^{1/2} + |k_z|\left(\frac{-K_{zz}}{\mathrm{Re}K'_{xx}}\right)^{1/2}\frac{\epsilon}{L^{1/2}}$$

$$+\mathrm{Re}\left[i\int_{x_0}^{L}|k_z|\left(\frac{-K_{zz}}{K_{xx}}\right)^{1/2}dx\right].$$

The second term has the wrong sign for damping; however, comparing the magnitudes of the first and second terms and making use of $x<L$ together with the criterion for the existence of mode conversion [Eq. (40)] shows that the first term always dominates the second. Hence,

$$D \simeq -\left(\frac{\mathrm{Re}K'_{xx}}{\alpha}\right)^{1/2}\epsilon x^{1/2} - \mathrm{Im}\int_{x_0}^{L}|k_z|\left(\frac{-K_{zz}}{K_{xx}}\right)^{1/2}dx.$$

The two terms in this expression are just the ordinary WKB damping experienced by the hot and cold waves outside the conversion layer. Similar results have been found by Schuss *et al.*[3]

[1]P. M. Bellan and M. Porkolab, Phys. Fluids **17**, 1592 (1974).
[2]P. M. Bellan, in *Proceedings of the Third Topical Conference on Plasma Heating* (California Institute of Technology, Pasadena, Calif., 1977), paper E1.
[3]J. Schuss, M. Porkolab, and R. R. Parker, in *Proceedings of the Third Topical Conference on Plasma Heating* (California Institute of Technology, Pasadena, Calif., 1977), paper E2.
[4]T. H. Stix, Phys. Rev. Lett. **15**, 878 (1965).