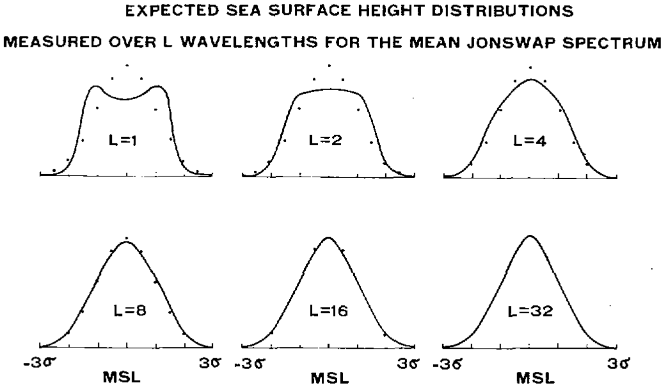

Fig. 8. Histograms of expected surface height distribution for mean JONSWAP spectrum. Ordinate is number of observations, and departures indicated on abscissa are in terms of standard deviation observed in *each* segment used for observation. Dots corresponding to Guassian curve are included for comparison.

tracking with an accelerometer to increase the time interval and the number of wavelengths considered so the MLP has a Gaussian surface height distribution to work with. However, this approach could not be used with beam-limited, target-referenced radar systems such as the two-frequency radar interferometer system proposed by Weissman [9] or the Fairchild Industries SCOR system being investigated at WFC. They determine SWH by developing a measure of the range extent of the sea surface within their beamwidths [4]. Since beam-limited, target-referenced systems only consider the range extent of the target and not the range to the target they can only work on what is within their beamwidth at any instant and the effect indicated in Fig. 8 may considerably complicate the analysis of the data and instrument performance for such systems.

VI. Conclusions

Care must be taken in trying to use aircraft to experimentally verify measurement accuracies for radio oceanographic instruments. The random variation in the sea surface parameters of interest may far exceed the instrument's ability to measure them when only a small number of wavelengths are examined. Also, some of the basic assumptions used in the analysis of such systems, such as assuming a Gaussian surface height distribution, break down when only a few wavelengths are considered. Random oscillatory motion introduced by the aircraft autopilot in maintaining altitude can degrade system performance and require increased system complexity over satellite applications.

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Effects of Random Phase Changes on the Formation of Synthetic Aperture Radar Imagery

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Abstract—**The effects of Gaussian random and linear phase change on the response of the matched azimuth processor of a synthetic aperture imaging radar is analyzed numerically.**

I. Introduction

One of the key questions in developing a theory for synthetic aperture radar imaging of the ocean surface [1]–[6] is what is the effect of the random temporal changes of the surface on the image formation. The basic information that is used in generating the high resolution imagery is the phase history of the echo from each element of the surface. In the case of the ocean surface, a random component is added to the phase history which tends to disturb the image formation process. In this succinct paper, some insight is achieved by considering the effects of random perturbation on the radar image formation by numerically analyzing the effect of a random phase component on the matched processor response for a point target or resolution element. In particular, we consider Gaussian and linear temporal phase history perturbations. The parameters selected correspond to airborne and spaceborne radar systems.

For a thorough understanding of the principle of imaging radars the reader is referred to the literature [7]–[9]. Here, we will only briefly discuss the aspects which are relevant to the analysis in this paper.

II. Formulation

In Fig. 1 we present the geometry in the plane of the flightline. For simplicity, we only consider the radar azimuth plane. The locus of the phase of successive echoes from a fixed point target or resolution element P is given by

$$\phi(t) = \frac{4\pi}{\lambda} D(t) = \frac{4\pi}{\lambda} [h^2 + v^2 t^2]^{1/2} \tag{1}$$

which can be approximated by

$$\phi(t) \simeq \phi_0 + at^2$$

where we assumed that $h \gg vt$ during the formation of the synthetic aperture, h and v are the height and velocity of the radar platform, λ is the radar wavelength, t is time referenced such that at $t = 0$ the point P is viewed at 90° relative to the line of flight, and $a = 2\pi v^2/(\lambda h)$. The resulting echo $U(t)$ can be represented as

$$U(t) \sim Ae^{i\phi_0 + iat^2}, \qquad \text{for } |t| \le T \tag{2}$$

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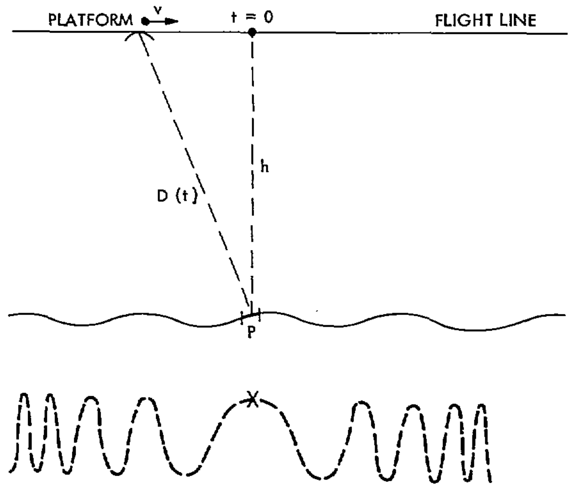

Fig. 1. Geometry. For simplicity we assumed that target is in vertical plane of flight. At bottom we show history of real part of echo, i.e., $Re(U)$.

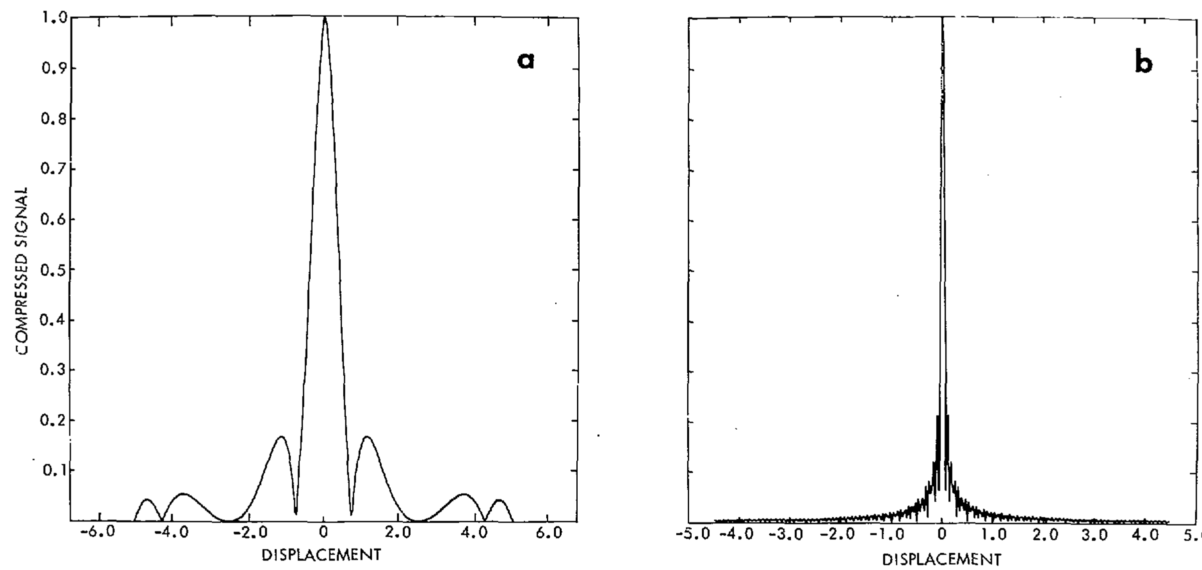

Fig. 2. Compressed signal (i.e., correlation function) in case of fixed nonfluctuating target. (a) Airborne platform. (b) Spaceborne platform. Displacement is in normalized time $a^{1/2}t$.

where we assumed that the amplitude of the echo is constant during the time $2T$ when the synthetic aperture is formed. Thus the normalized transfer function of a matched processor should be

$$f(\xi) = e^{ib\xi^2}, \qquad \text{for } |\xi| \leq 1 \tag{3}$$

where

$$b = aT^2.$$

The value of T is related to the image resolution desired in the following way. To achieve a resolution r on the surface we require a synthetic aperture beamwidth $\theta_s = r/h$ which implies a synthetic aperture length $L = \lambda/\theta_s = \lambda h/r$. If the platform is moving with a velocity v, the time required to move a distance L is $2T = L/v = \lambda h/rv$ which is the time of formation of the aperture. Thus the expression of b is

$$b = \lambda\pi h/(2r^2). \tag{4}$$

The response of a matched processor is

$$F = f * \bar{f}$$

where the asterisk (*) is the correlation operator, and $\bar{f}$ is the complex conjugate of f.

In the case of the ocean, the surface structure in a resolution element changes in a somewhat random way as a function of time, thus leading to a random change in the phase ϕ of the echo from the resolution element P. An evaluation of the phase as a function of the surface profile using an exact solution of the wave scattering problem is very complicated, however it is quite reasonable to approximate the phase ϕ by

$$\phi(\xi) = \phi_0 + b\xi^2 + n(\xi) \tag{5}$$

where $n(\xi)$ is a random function. The resulting response of the processor would then be

$$F' = f * \bar{f}' \tag{6}$$

where

$$f' = e^{ib\xi^2 + in(\xi)}.$$

We also considered the case where a linear component is added to the phase term as a result of the average vertical component

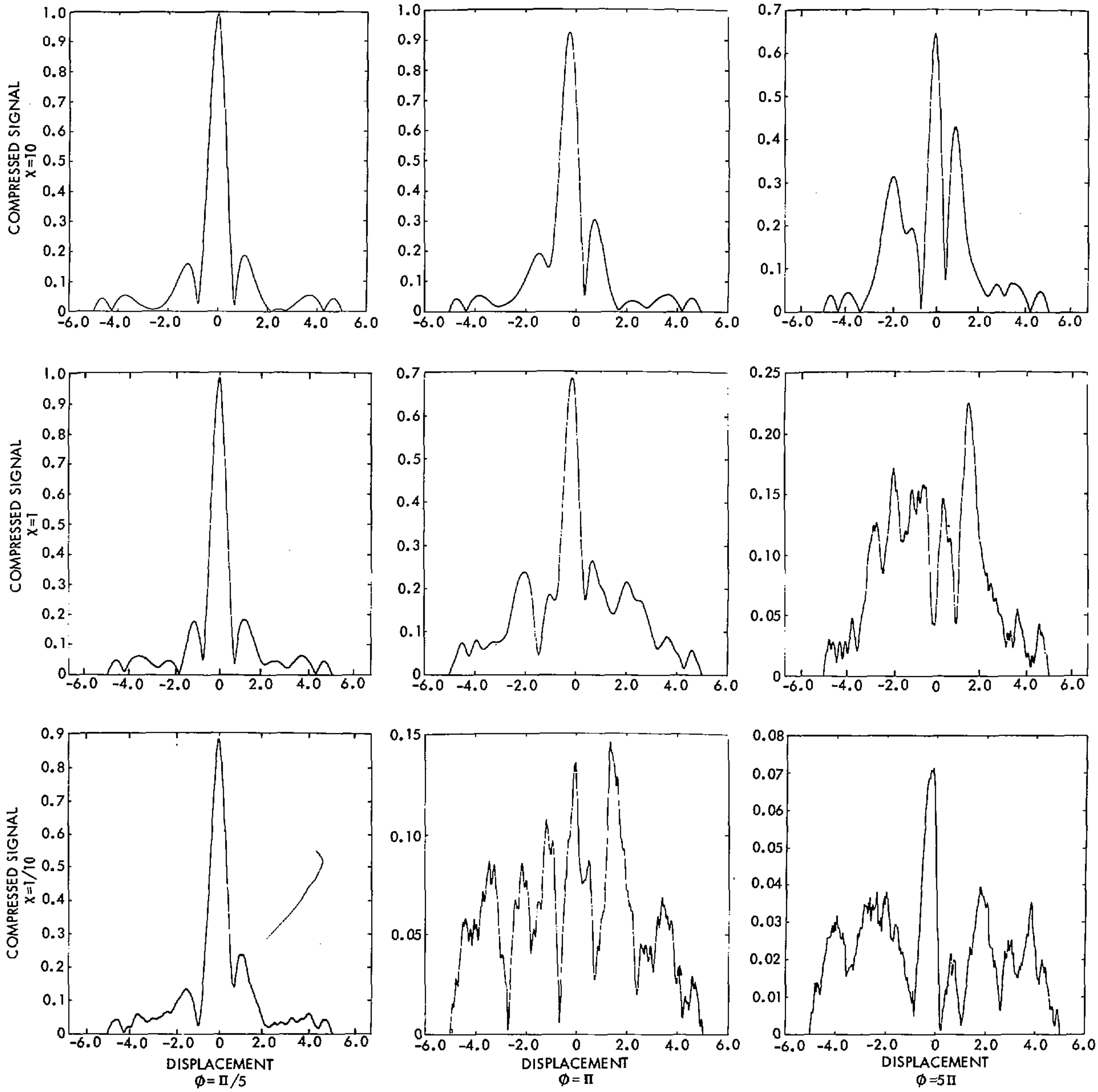

Fig. 3. Compressed signal (i.e., correlation function) for airborne platform in case of randomly Gaussian fluctuating target. Φ is rms change and χ is correlation time of random phase of echo. Note change in scale. Displacement is in normalized time $a^{1/2}t$.

of motion of the scatterers. The resulting response of the processor would then be

$$F'' = f * \bar{f}'' \tag{7}$$

where

$$f'' = e^{ib\xi^2 + in(\xi) + ic\xi}.$$

III. Numerical Analysis

To illustrate the effect of the random phase change, we computed the magnitude of F' for the case of a Gaussian $n(\xi)$ with a standard deviation Φ and a normalized correlation time $\chi = \tau/T$, where τ is the coherence time of the fluctuation and T is the synthetic aperture formation time. The value of Φ expresses the magnitude of the phase change, while χ is inversely proportional to how fast the change occurs. The parameters used were the following.

	Airborne	Spaceborne
r(m)	25	25
v(m/s)	250	8000
h(km)	10	800
λ(cm)	25	25

These give $b = 2\pi$ and $T = 0.2$ s for an airborne system and $b = 160\pi$ and $T = 0.5$ s for a spaceborne system. The random function $n(\xi)$ was numerically generated.

Fig. 2 shows the correlation function F when no temporal variations are present. In Figs. 3 and 4 we present one Monte Carlo case illustrating the effects of random phase perturbation for the cases of airborne and spaceborne platforms. We observe that for small values of Φ, the peak of the correlation function decreases slightly while the sidelobes are slightly perturbed even for $\chi = 1/10$ which corresponds to fast temporal change. For $\Phi = \pi$, and χ large, the main effect is a displacement of the

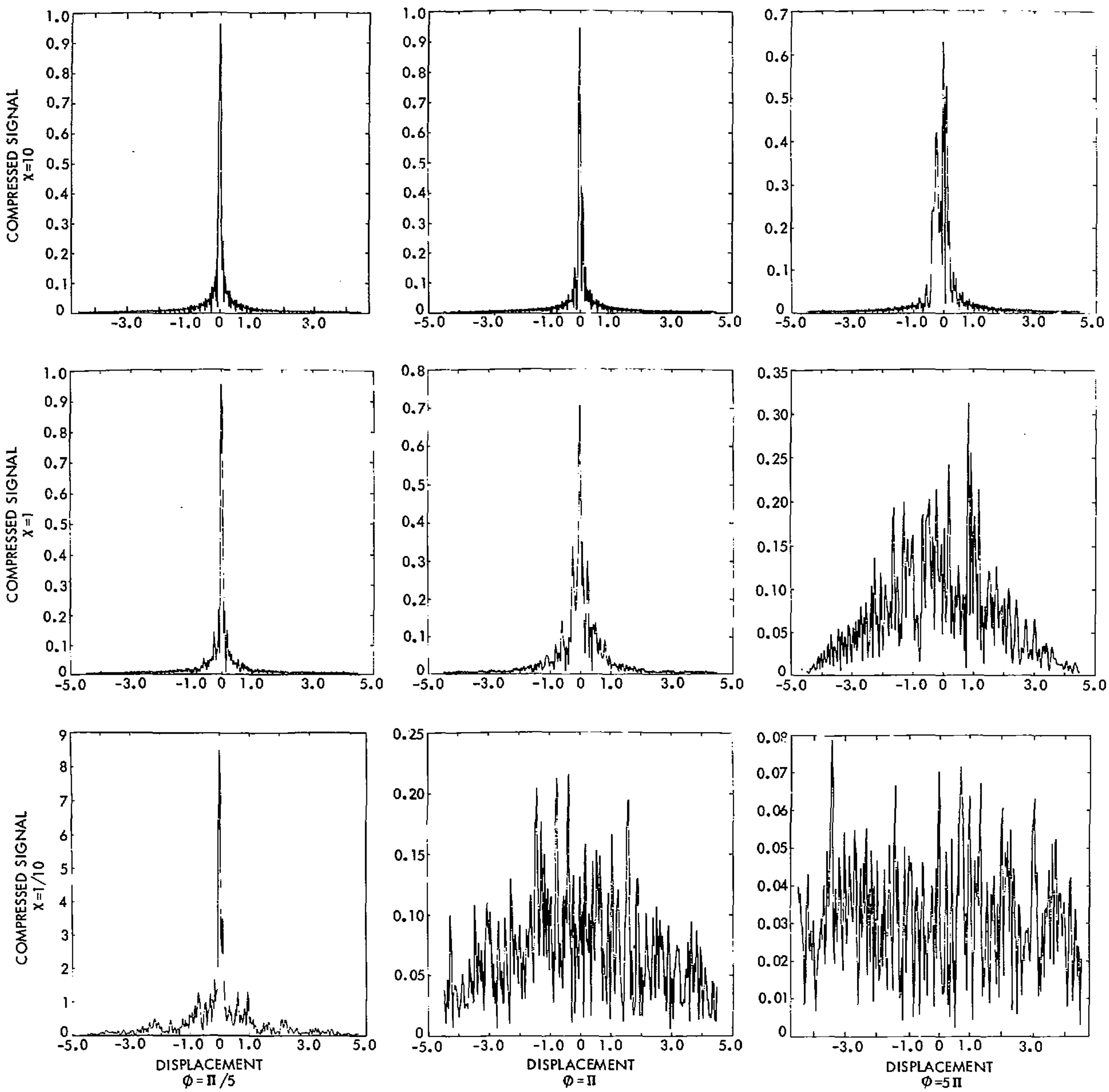

Fig. 4. Same as Fig. 3 but for spaceborne platform.

position and slight decrease in the value of the correlation peak. As χ decreases, the correlation function gets more and more distorted until it becomes completely different from the original case (Fig. 2) when χ is equal to or larger than 10. In that case, the energy is mainly in the sidelobes.

For $\Phi = 5\pi$, the correlation function is badly distorted for $\chi = 1$ or larger. In summary we can say that the random phase perturbation will drastically distort the correlation function if Φ is relatively large while χ is relatively small. The limits of acceptability are hard to define exactly and are somewhat subjective.

To illustrate the effect of a linear change in the phase, we show in Fig. 5 the resulting correlation function derived by considering no Gaussian noise ($n(\xi) = 0$) but a linear change in phase ($c = \gamma b^{1/2}$ where $\gamma = 8.8$ for the aircraft case and 30 for the spacecraft case). As expected, the central peak is shifted leading to a fictitious displacement in the image.

IV. Summary and Conclusions

We analyzed numerically the effects of Gaussian random fluctuation and linear change of the echo phase on the response of the azimuth matched processor of a synthetic aperture radar. Our conclusion is that the random fluctuation would not alter appreciably the processor response if its standard deviation Φ is less than π while its normalized correlation time χ is larger than 1. If these two bounding conditions are not satisfied, then the sidelobes level gets relatively large and overshadows the central peak. In the case of linear displacement, we found that the main effect is a spatial displacement of the point scatterer in the image plane.

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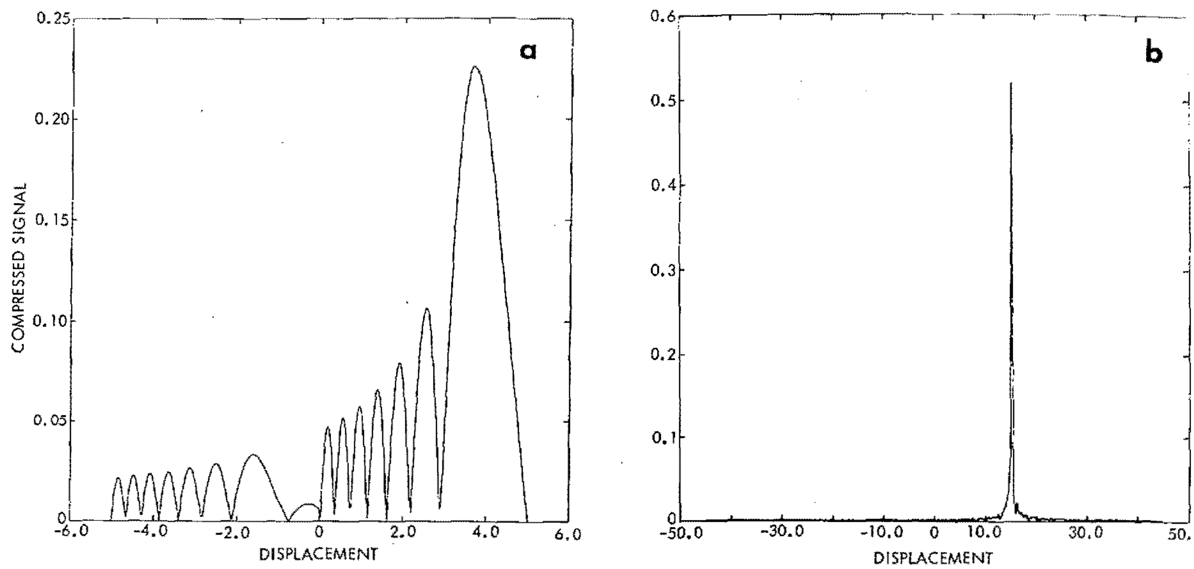

Fig. 5. Compressed signal (i.e., correlation function) in case of linearly moving target. (a) Airborne platform. (b) Spaceborne platform. In spacecraft case horizontal scale has been expanded to encompass entire compressed signal.

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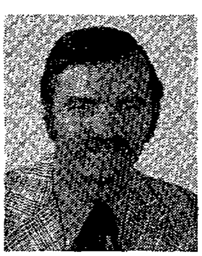
Contributors



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