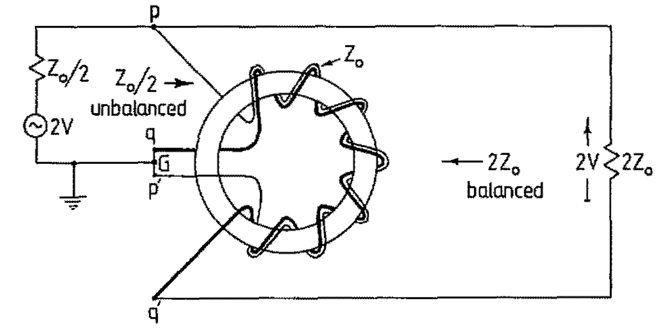

Fig. 5. Wide-band balun with 4:1 impedance ratio. Derived from Fig. 4 by wrapping the line around a magnetic core.

load with impedance $Z_0/2$ at pq. Since points p and q' are at equal but opposite potentials with respect to G, the device also functions as a balun (balanced-to-unbalanced) transformer when G is grounded.

Since signals are transmitted in the transmission-line mode and since the transmission lines are correctly terminated, the bandwidth is not restricted by leakage reactance and stray capacitance as in the conventional transformer. At high frequencies the phase shift in the transmission line winding can no longer be neglected. Thus, the response is down 1 dB when the line length is $\lambda/4$ and the response is zero at $\lambda/2$ (1).

At low frequencies the longitudinal-mode currents in $p - p'$ and $q - q'$ become appreciable. The existence of these currents means that less power is available for the desired transmission-line mode, and this is responsible for the fact that the response drops off at low frequency. To improve the low frequency performance the longitudinal-mode inductive reactance must be increased. The preferred method of doing this is to use high permeability core material rather than to increase the number of turns, since the latter increases the transmission-line phase shift and thus degrades the high frequency response (2).

ACKNOWLEDGMENT

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Far-Field Radiation Patterns of Y and T Shaped Arrays with Chebyshev Loading

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***Abstract*—The far-field radiation patterns for Chebyshev loaded Y and T shaped array antennas are presented for a number of cases. These include full and half-length loaded array arms.**

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I. INTRODUCTION

An N-element antenna array, with excitation coefficients equal to the Chebyshev polynomial T_n will provide the narrowest main lobe beamwidth while keeping the sidelobes below a predetermined level [1]. Inversely, for a specified beamwidth, the sidelobes level is at a minimum. In order to obtain the narrowest beam in two dimensions, an $N \times N$ elements array with Chebyshev loading is required. In this communication we present the far-field radiation patterns for a Chebyshev loaded antenna array with only three array booms in the form of a Y or T. The purpose is to determine the effect of different Chebyshev loading arrangements on this type of arrays.

II. FORMULATION

The far-field pattern of a linear array of half-wave dipoles lying along an arm in the $X - Z$ plane at an angle α from the X axis (see Fig. 1) is given by [2]:

$$F_\alpha(\theta, \phi) = \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \cdot \sum_{n=0}^{N-1} A_n e^{-inkL(\cos\alpha\sin\theta\cos\phi + \sin\alpha\cos\theta)} \right| \qquad (1)$$

where L is the spacing between the dipoles, N is the total number of dipoles on the arm, and the dipoles are assumed to be always oriented with their long axis parallel to the Z axis.

For several booms at different angles α_j, the above expression is summed for different values of j before calculating the argument.

III. RESULTS

The radiation patterns were calculated for a variety of $Y(\alpha_1 = 0°, \alpha_2 = 120°, \alpha_3 = 240°)$ and $T(\alpha_1 = 0°, \alpha_2 = 90°, \alpha_3 = 270°)$ shaped antennas with either a complete Chebyshev loading on each arm or half Chebyshev loading per arm (i.e., the arm corresponds to only half the length of a Chebyshev loaded array). Typical examples of the results are shown in Fig. 2. The radiation pattern is presented in the $\theta - \phi$ plane

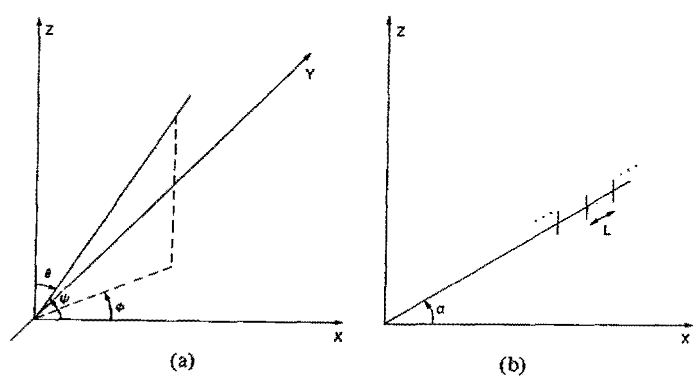

Fig. 1. (a) Geometry of the coordinates system used. (b) Geometry of the antenna elements; L is the separation between dipoles and α is the angle between the arm and X-axis.

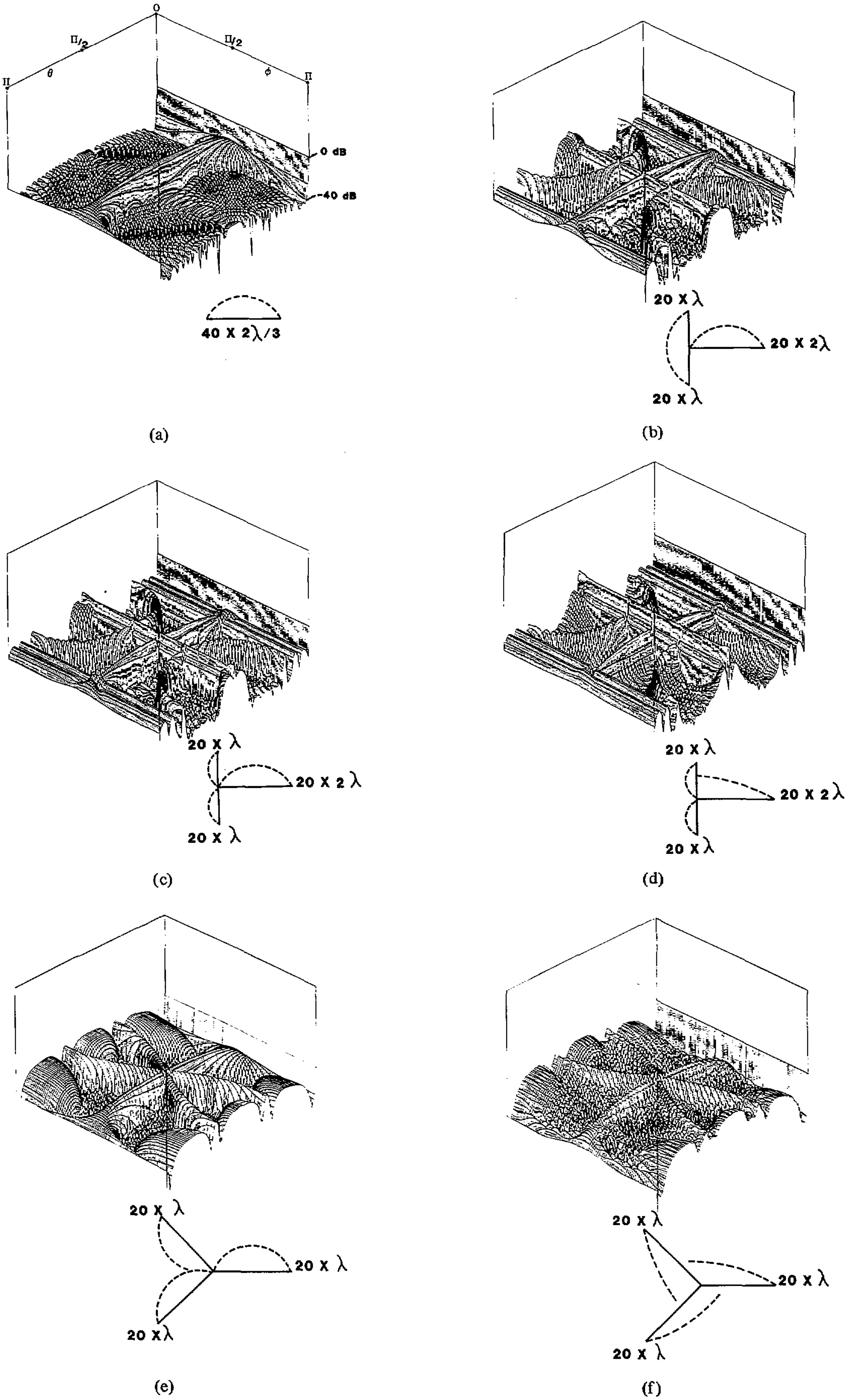

Fig. 2. Radiation patterns of the different antenna configurations which were investigated (illustrated in the small inserts). The dashed lines in the inserts are illustrative sketches of the loading distribution.

format. The coefficients of the Chebyshev polynomial were selected such that the sidelobes for each arm by itself are −40 dB below the main lobe.

Fig. 2(a) shows, as a reference, the pattern for a 40-element linear array ($\alpha = 0°$) with $L = 2\lambda/3$ and a −40 dB sidelobes level. Fig. 2(b) corresponds to a T shape array where the X-axis arm has 20 elements with $L = 2\lambda$ and a complete Chebyshev loading. The two arms on the Z axis consists each of 20 elements with $L = \lambda$. Both arms together form the elements of a Chebyshev array. In Fig. 2(c) each arm on the Z axis by itself has a complete Chebyshev loading. The main change from Fig. 2(b) is the additional high sidelobe at $\theta = \pm\pi/2$. In Fig. 2(d) the antenna is similar to the one in Fig. 2(c) except the X-axis arm has a half Chebyshev loading. This results in the gradual increase of the sidelobes near the main lobe (i.e., along $\theta = 0$ and $\theta = \pi$). This is due to the fact that the X-axis arm is not optimally loaded.

Fig. 2(e) and 2(f) correspond to a Y shaped array with 20 elements on each arm and $L = \lambda$. In Fig. 2(e) each arm has a complete Chebyshev loading. In Fig. 2(f) each arm has only a half-Chebyshev loading. This results in an increase of the sidelobes levels because of the nonoptimum loading. In Fig. 2(e) except for the main lobes, all the small sidelobes are at a level of −40 dB.

In summary, in both the Y and T shaped arrays the main lobes are defined by the shape of the array, and the level of the sidelobes is mainly dependent on the Chebyshev loading (i.e., complete or partial).

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On the Concept of Reaction Field for Electromagnetic Fields in Dispersive Media

MASAHIRO AGU

Abstract—**The concept of "reaction field" is introduced for electromagnetic fields in dispersive media as a measure of the amount of the reactions of a radiation source on spatially distributed field detectors. It is found that the equations of the "reaction field" in the medium take the adjoint forms to those of the electromagnetic field. A reciprocity theorem having a clear-cut physical meaning is derived with the use of the concept of the reaction field. Some variational expressions are derived with the use of the reaction field.**

I. INTRODUCTION

The concept of "reaction" in vacuum electromagnetic theory has been introduced as a physical observable representing the amount of interaction between a field source and some other field source [1]. The concept has been used to simplify the boundary value problems or to obtain variational expressions in the scattering problems [1]-[5]. The reaction in an anisotropic dispersive medium has been introduced into the harmonic field by using the "complementary medium" whose permittivity tensor is defined by transposing the original one [2], [3]. In this communication, the reaction of a radiation source on field detectors is treated as a "reaction field," which is expressed as a function of the time and the position of the radiation source. In other words, the "reaction field" is introduced as a measure of the amount of the reactions of a point radiation source on spatially distributed field detectors. It is found that the "reaction field" in a medium is subject to the equations adjoint to those of electromagnetic field. Here the "adjoint" means not only the transposition of original permittivity tensor but also the space–time inversiona ($\mathbf{r} \to -\mathbf{r}$, $t \to -t$) of original field equations. A reciprocity theorem having a clear-cut physical meaning is derived without using the complementary medium. The reciprocity theorem gives us a systematic way to derive the Huygens's principle in an anisotropic dissipative medium. Some variational formulations of the field equations are obtained with the use of the adjoint field.

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II. REACTION FIELD IN DISPERSIVE MEDIA

Let us consider electromagnetic field in a homogeneous despersive medium. The Maxwell equations of the field are assumed to be

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t}\mu_0 \mathbf{H}(\mathbf{r}, t), \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t}\mathbf{D}(\mathbf{r}, t) + \mathbf{J}(\mathbf{r}, t). \tag{2}$$

where the electric displacement $\mathbf{D}(\mathbf{r}, t)$ is defined by

$$D_\alpha(r, t) = \int d\mathbf{r}' \int_{-\infty}^{t} dt' \cdot \epsilon_{\alpha\beta}(t - t', \mathbf{r} - \mathbf{r}')E_\beta(\mathbf{r}', t'). \tag{3}$$

Here $\epsilon_{\alpha\beta}(t - t', \mathbf{r} - \mathbf{r}')$ is a permittivity tensor of temporally and spatially dispersive medium, and the subscripts α, β run over x, y, z and the repeated indices are assumed to be summed. The electric field $\mathbf{E}(\mathbf{r}, t)$ at point $\mathbf{r}$ at time t which is produced by the current source $\mathbf{J}(\mathbf{r}', t')$ at point $\mathbf{r}'$ at time t' is represented with the use of the dyadic Green's function $G_{\alpha\beta}$;

$$E_\alpha(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^{t} dt' G_{\alpha\beta}(r - r', t - t')J_\beta(\mathbf{r}', t'). \tag{4}$$

Taking into account the fact that the "reaction" has been introduced into the vacuum electromagnetic theory as a measure of the interactions between a field source and some other field source [1], we define reaction field $E_\alpha^{+}(\mathbf{r}, t)$ in the dispersive medium by the equation

$$E_\alpha^{+}(\mathbf{r}, t) = \int d\mathbf{r}'' \int_{t}^{\infty} dt'' S_\beta(\mathbf{r}'', t'')G_{\beta\alpha}(\mathbf{r}'' - \mathbf{r}, t'' - t). \tag{5}$$

Here $S_\beta(\mathbf{r}'', t'')$ means the amount of the interactions at time t'' of the detector source located at $\mathbf{r}$ with the β-component of the electric field on that point. The reaction $E_\alpha^{+}(\mathbf{r}, t)$ defined