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Microstrip Reactive Circuit Elements

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Abstract —**Quantitative design information is given for some planar distributed microwave circuit elements. A microstripline section is calculated as a parallel tuning element, and the radial-line stub and open- and short-circuited coupled microstrip stubs are treated. Typical applications showing measurements on circuits utilizing planar tuning elements are also presented.**

I. INTRODUCTION

In the design of microstrip circuits for impedance matching, tuning, and bias-line functions, a relatively limited number of planar circuit configurations is available to fulfill circuit design requirements for inductive, capacitive, and resonant elements. Distributed circuit elements, having dimensions comparable with a wavelength in size, typically require significant amounts of area on the microstrip surface but are simpler to realize in thin film technology than lumped capacitors or inductors.

II. THE RADIAL-LINE STUB

The quarter-wavelength open-ended shunt stub of microstripline is conventionally employed to establish a point of low impedance relative to the ground plane in mixers and switching circuits and in bias-line circuits. For a stub with low characteristic impedance Z_0, the point of low impedance at the stub input is indeterminate by an amount equal to the width of the microstripline. A radial-line stub has been proposed to overcome this indeterminancy [1], [2], but a simple calculation for the radii of the resonant radial-line sector (Fig. 1) is not widely available. Based on an assumed radial-wave solution in the substrate dielectric with magnetic walls at the boundary of the stub, Vinding [1] has proposed for the reactance presented at the inner radius r_1

$$X_1 = \frac{h}{2\pi r_1} Z_0(r_1) \frac{360}{\alpha} \frac{\cos(\theta_1 - \psi_2)}{\sin(\psi_1 - \psi_2)} \tag{1}$$

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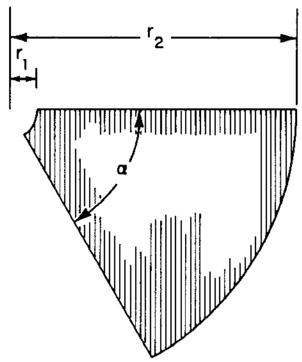

Fig. 1. Radial-line stub coordinates. (Substrate height = h.)

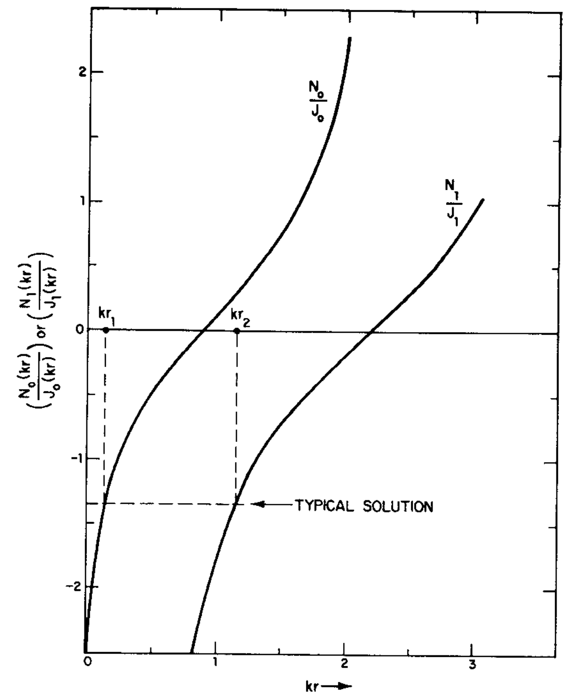

Fig. 2. Plot of (2). Broken line shows typical solution.

where

$$\tan\theta_1 = \frac{N_0(kr_1)}{J_0(kr_1)}$$

$$\tan\psi_i = -\frac{J_1(kr_i)}{N_1(kr_i)} \qquad (i = 1,2)$$

$$Z_0(r_1) = \frac{120\pi}{\sqrt{\epsilon_r}} \left[J_0^2(kr_1) + N_0^2(kr_1)\right]^{1/2} \cdot \left[J_1^2(kr_1) + N_1^2(kr_1)\right]^{-1/2}$$

$$k = 2\pi\sqrt{\epsilon_{re}}/\lambda_0.$$

In (1) above, $J_i(x)$ and $N_i(x)$ are Bessel functions of the first and second kinds, of ith order.

For a resonant stub we assume $X_1 = 0$ in (1), which leads to

$$\frac{N_1(kr_2)}{J_1(kr_2)} = \frac{N_0(kr_1)}{J_0(kr_1)}. \tag{2}$$

A plot of (2) is shown in Fig. 2. A solution of (2) is represented

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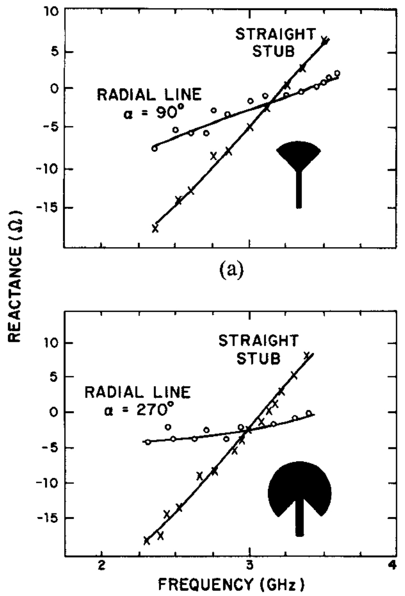

Fig. 3. Reactance of λ/4 microstrip stub compared with (a) radial-line stub with sectoral angle $\alpha = 90°$ and (b) radial stub with sectoral angle $\alpha = 270°$.

by the horizontal broken line in this figure. Normally r_1 will be chosen to be a small fraction of a wavelength, much smaller than r_2. In the range $0.1 < kr_1 < 0.5$, it can be seen from Fig. 2 that, approximately, $|kr_1 - kr_2| \cong 1$. Within this approximation

$$r_2 - r_1 \cong \lambda_0/(2\pi\sqrt{\epsilon_{re}}). \tag{2a}$$

Thus the outer radius of the stub is of the order of 1/6 (or, $1/2\pi$) of the wavelength in the dielectric. Accurate values may be found graphically from Fig. 2. The factor ϵ_{re} in (2a) is defined as the effective relative dielectric constant for wave propagation on the transmission system. This definition permits a transmission system to be characterized as a quasi-TEM system. The electrical length of a line element is then given by

$$\theta = \sqrt{\epsilon_{re}}(\omega l/c) \tag{3}$$

where l is the physical length of the element, c is the velocity of light in a vacuum, and ω is the angular frequency.

If there were no fringing fields at the edges of the stub, ϵ_{re} would be chosen to be the relative dielectric constant ϵ_r of the substrate material. It has been proposed that the effective dielectric constant of a microstripline of width $\alpha(r_2 + r_1)/4$ be used for the ϵ_{re} of the radial stub (α rad) [2]. The latter proposal may be expected to be limited to stubs of small sectoral angles α, probably less than 30°. For sectoral angles up to 90° or larger, however, it can be assumed that the bulk dielectric constant ϵ_r of the substrate material is a good approximation for the ϵ_{re} of the stub. In the present work, no open-end correction is applied to the radial-line stub, although it may be assumed that a correction equal to that for a microstripline of width equal to that of the outer perimeter at r_2 is appropriate.

When the sectoral angle α of the stub is increased to 180° and larger, the advantage of a salient inner terminal point is no longer achieved. A compensating advantage of increasing frequency bandwidth is gained with increasing α, however, as is demonstrated by measurement. The radial-line stub of angle 90° or greater is more broad-banded than a conventional quarter-wavelength open stub of straight microstripline of similar resonance frequency, in the sense that its dispersion is smaller. Fig. 3(a) and (b) shows the measured reactances of radial stubs of angle 90° and 270°, shown in comparison with the measured reactance of a straight quarter-wavelength open-ended stub constructed on the same 25-mil alumina substrate material as was used for the radial-line stubs. These stubs were driven by microstripline sections symmetrically located relative to the stub, as indicated in the insets in Fig. 3. The impedances plotted are referred to the junction of the microstripline with the stub in each case.

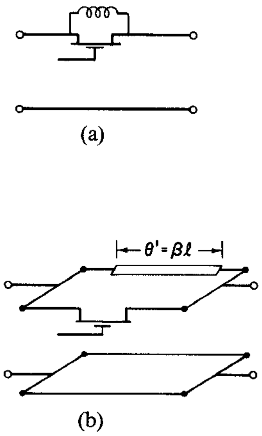

Fig. 4. (a) Parallel-tuned FET. (b) Microstripline section in parallel with FET.

III. Microstripline Resonating Element

GaAs FET's with zero dc drain bias have recently been employed as switches in microstrip circuits [3], [4]. The source–drain impedance of the FET is placed in series with the signal path and the impedance magnitude is varied with gate bias. Inductance may be placed in the parallel with the FET to resonate its capacitance in the depleted state in order to improve the off-state isolation of the switch (Fig. 4(a)). A suitably chosen section of microstripline may be used to resonate the FET capacitance. No procedure has been given in the literature for the determination of the parameters of the microstripline segment required to resonate a given FET source–drain capacitance. A suitable procedure is therefore given below. A microstripline segment is a distributed element and cannot be given a two-terminal (i.e., one-port) characterization solely in terms of its inductance per unit length. The required characteristic impedance Z_C and electrical length θ of the resonating line section may be found from an admittance representation of the series-connected FET impedance and the line section as two ports connected in parallel (Fig. 4(b)). For parallel-connected two-ports, the admittance matrices add

$$Y_{\text{ser.}} + Y_{\text{microstrip}} = Y_{\text{resultant}}. \tag{4}$$

For an assumed lossless transmission line section, the admittance matrix is

$$Y_{\text{microstrip}} = \begin{bmatrix} -jY_C\cot\theta & jY_C\csc\theta \\ jY_C\csc\theta & -jY_C\cot\theta \end{bmatrix} \tag{5}$$

where $Y_C = 1/Z_C$ is the characteristic admittance of the line and θ is its electrical length.

The admittance matrix of a series-connected impedance $Z_S = 1/Y_S$ is

$$Y_{\text{ser.}} = \begin{bmatrix} Y_S & -Y_S \\ -Y_S & Y_S \end{bmatrix}. \tag{6}$$

For the resonance condition in which the transmission through the combined two-ports is minimized, we adopt the convention that the Y_{21} element of (4) vanishes. This condition corresponds also to the vanishing of the forward-scattering coefficient S_{21} of the combination. This leads to the requirement

$$Y_S = jY_C \csc\theta. \tag{7}$$

For a lossless microstripline section in shunt with a series-connected capacitor C_S, (7) becomes

$$\frac{1}{\omega C_S} = Z_C \sin\theta \tag{8}$$

where θ is the electrical length of the resonating microstrip loop. Then if a suitable characteristic impedance value Z_C is chosen, the necessary length of microstripline may be determined from (8), with use of (3). For a purely reactive series impedance element Z_S and a lossless microstrip tuning segment, the insertion loss at resonance of the tuned combination is nominally infinite. For a Z_S containing a resistive component, the tuned insertion loss is finite. For resistive Z_S, the capacitive admittance ωC_S of $G_S + j\omega C_S = 1/Z_S$ is used in (8). It may also be shown that, of the two solutions of (8) for a given Z_C: θ and $(180-\theta)$ degrees (where θ is the first-quadrant solution), a larger resultant insertion loss at resonance is obtained for a Z_S with a real part by use of the longer microstripline segment of electrical length $(180-\theta)$ degrees.

IV. Open-Ended Coupled-Microstrip Stub

When a p-i-n diode is used as a switch in series with the microstripline, a shunting inductor or microstripline section cannot be employed to resonate the off-state capacitance of the diode, due to its shunting of the diode bias current. A convenient circuit element for this application is the open-ended coupled-line stub. The general $(ABCD)$ circuit matrices of coupled-line stubs have been obtained by Zysman and Johnson [5]. These may be converted to admittance matrices for use in the same procedure as followed above to find the parameters of resonating stubs for a series-connected impedance Z_S. From [5, fig. 5], the Y_{21} component of the admittance matrix of an open-ended coupled-microstrip stub is

$$Y_{21} = -\frac{j}{2}\left[Y_{0o}\tan\theta_o - Y_{0e}\tan\theta_e\right] \tag{9}$$

where Y_{0o} and Y_{0e} are the reciprocals of the odd- and even-mode characteristic impedances of the coupled microstriplines [6], and θ_o and θ_e are the electrical lengths of the stub in the odd- and even-modes, respectively. If the stub, assumed lossless, is placed in parallel with a series reactance X_S (Fig. 5(a)), the vanishing of the Y_{21} element of the combined admittance matrices leads to

$$\frac{1}{X_S} = \frac{1}{2}\left(Y_{0o}\tan\theta_o - Y_{0e}\tan\theta_e\right) \text{ (Open Stub)}. \tag{10}$$

In practical coupled-line stub tuner design, a line width W and separation S may be assigned, thereby fixing Y_{0o} and Y_{0e}. Normally a small line separation is desirable to localize the fields about the stub. Values of the characteristic admittances of coupled lines may be obtained from the computer program (MSTRIP) of Bryant and Weiss [6], [7], or by means of closed-form approximations [8], [9]. Equation (10) may be reduced to an equation in the single unknown θ_o by use of the relation

$$\theta_e = \sqrt{\epsilon_{re}^e/\epsilon_{re}^o}\,\theta_o \tag{11}$$

where ϵ_{re}^e and ϵ_{re}^o are the effective relative dielectric constants for the even- and odd-modes of propagation on coupled microstriplines [6]–[9]. Then (10) may be readily solved by means of trial-and-error methods. It should be noted that, for capacitive

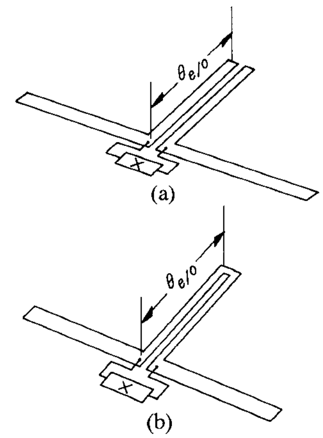

Fig. 5. Series-connected reactance X parallel-tuned with (a) open-ended coupled-line stub, and (b) shorted coupled-line stub.

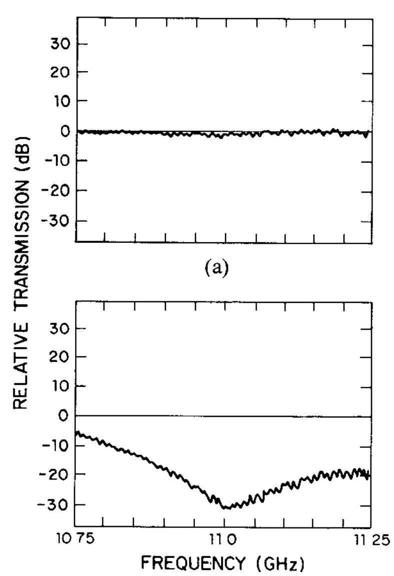

Fig. 6. Relative transmission by tuned p-i-n diode switch. (a) On-state. (b) Off-state.

reactance, $X_S = -1/C_S\omega$ is a negative number, in (10). With θ_o known, the physical length of the stub is obtained by use of the equation analogous to (3) for the odd mode of propagation.

An example of the use of a coupled-line stub to resonate the capacitance of a p-i-n diode used as a switch connected in series with the microstripline is shown in Fig. 6. The isolation in the off-state is seen to be significantly greater at the stub-tuned resonance at 11 GHz than at off-resonance frequencies.

V. Shorted Coupled-Microstrip Stub

The same procedure as above may be followed to find the resonance condition for a shorted coupled-line stub, used to resonate a series-connected impedance in microstripline (Fig. 5(b)). This stub may be used where dc isolation between the stub terminals is not required. The resonance condition $(Y_{21} = 0 = S_{21})$ is found from [5, fig. 4] to be

$$\frac{1}{X_S} = -\frac{1}{2}\left(Y_{0o}\cot\theta_0 + Y_{0e}\tan\theta_e\right) \text{ (Shorted Stub)}. \tag{12}$$

The notation and method of solution of (12) are similar to that of (10) above.

VI. Conclusion

Information has been given for the utilization of reactive microstrip circuit elements: the radial-line stub, a microstripline loop paralleling a series reactance, and coupled microstrip stubs. With the exception of the radial-line stub, these components have been treated as two-port elements in parallel connection with a lumped impedance, assumed to be connected in series with the microstripline. Specifically, the element parameters have been derived for the case in which the combination of reactive element and lumped impedance effectively form a blocking filter at the design frequency. This configuration is that required for high isolation in the design of a microstrip switch using a switchable semiconductor element in series with the line.

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An Optimized *L*-Band Eight-Way Gysel Power Divider–Combiner

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Abstract —**Theoretical analysis of a general *n*-way power divider-combiner in a Gysel configuration has been carried out to derive relationships among fundamental parameters. On the basis of its symmetries, an eight-way network has been described, analyzed, and optimized over a 25-percent bandwidth by the computer program COMPACT™.**

Experimental data is reported and compared with theoretical predictions.

I. Introduction

The most popular *n*-way in-phase power divider–combiner (PDC) was proposed by Wilkinson in 1960 [1]. With the advent of microwave integrated circuits (MIC's) in the late 1960's it was immediately clear that, for $n > 2$, this kind of network is impractical to develop due to the nonplanarity introduced by the internal resistor star. Furthermore, as was pointed out by many authors [2], [3], the internal resistors are not connected to ground, so that heat-sinking and tuning of parasitic capacitances is difficult to achieve.

Gysel introduced in 1975 a modification of the concept [3] which from the practical standpoint has proven until now to be more suitable than others for high-power applications. In fact, the inconveniences presented by the Wilkinson topology are overcome.

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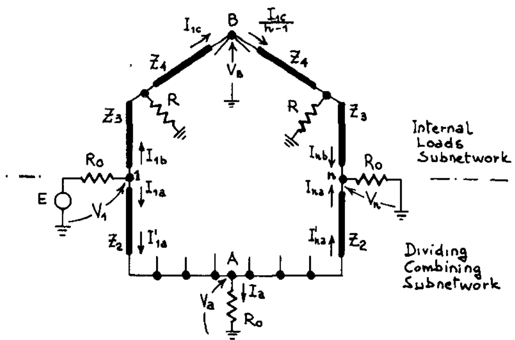

Fig. 1. *n*-way Gysel PDC.

As it will be shown later, although it is not realizable in a single plane for $n > 2$ in the same way as its forefather, the Gysel structure lends itself to being easily realized in microstrip or stripline also, for moderately high n.

In spite of that, the use of Gysel PDC has not become widespread, probably due to the lack of theoretical analysis on the subject and the complexity of optimizing the network on medium-to-wide bandwidth for an increasing splitting factor n.

In this paper it is intended to clarify some points of the matter, introducing at the same time an approach to eliminate node limitations of the program COMPACT™. The methodology is then applied to the practical case of an eight-way PDC designed for solid-state transmitters in the 960–1215-MHz Navaids frequency range (DME or TACAN).

II. Theoretical Analysis

To derive the exact relationship for the Gysel configuration, we can follow the same reasoning adopted by Wilkinson [1]. Let us consider Fig. 1 with all lines a quarter-wavelength long.

Then

$$V_1 = jI'_{1a} Z_2$$
$$I_{1a} = j\frac{V_a}{Z_2} \tag{1}$$
$$V_a = jI_{na} Z_2$$
$$I'_{na} = j\frac{V_n}{Z_2}. \tag{2}$$

On the other hand, since all branches are identical, we have

$$I'_{1a} = \frac{V_a}{Ro} - (n-1) I'_{na} \tag{3}$$

and it is also true that

$$I_{na} = \frac{Vn}{Ro} = I_{nb} \qquad I_{1a} = \frac{E - V_1}{Ro} - I_{1b}. \tag{4}$$

Combining (1)–(4), we can eliminate the currents (see Appendix) leaving the system of the three unknowns V_a, V_1, and V_n

$$V_1 = jV_a \frac{Z_2}{Ro} + (n-1) V_n$$
$$V_a = jV_n \frac{Z_2}{Ro} - jZ_2 \frac{R}{nZ_3^2}(V_1 - V_n)$$
$$\frac{E - V_1}{Ro} = j\frac{Va}{Z_2} + \frac{n-1}{n}\frac{R}{Z_3^2}(V_1 - V_n). \tag{5}$$