On the Near-Zone Inverse Doppler Effect

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Abstract—**Attention is invited to the recently discovered inverse Doppler effect which occurs in the near-zone field of an antenna emitting a continuous wave. On approaching the antenna, the received signal is blue-shifted in the far zone and then red-shifted in the near zone; and on receding from the antenna, the received signal is blue-shifted in the near zone and then red-shifted in the far zone. Calculations are presented for the case where the antenna is a simple dipole. It is shown that this effect gives not only the vector velocity of the moving receiver but also its range, i.e., its distance from the antenna.**

INTRODUCTION

IN FREE SPACE, a red shift indicates that a source of electromagnetic radiation is receding from an observer, and, conversely, a blue shift indicates that a source is approaching an observer. This shift in frequency, which is due to the relative motion between source and observer, is well known as the Doppler effect [1], [2].

However, a red shift (or blue shift) does not necessarily mean that the source is moving away from (or toward) the observer. Indeed, it has been demonstrated by Frank [3] and Lee [4] that in certain dispersive media there can occur effects resembling the inverse Doppler effect, wherein a receding source produces a blue shift and an approaching source produces a red shift. In support of the conjecture that an inverse Doppler effect can occur in the near zone of any source in free space, we shall show that such an effect occurs in the near zone of an oscillating dipole in free space and gives rise to a blue shift for a receding source and a red shift for an approaching one.

FIELD OF AN OSCILLATING DIPOLE

We take the oscillating dipole to be located at the origin of a spherical coordinate system (r, θ, ϕ) which is related to the Cartesian system (x, y, z) by $x = r \sin\theta \cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. Assuming that the dipole is oriented parallel to the z axis, we see [5] that in the dipole's equatorial plane, $\theta = \pi/2$, the electric vector has only a θ component E_θ and the magnetic vector has only a ϕ component H_ϕ.

That is, in the equatorial plane the components of the field emitted by the dipole are given by

$$E_\theta(r, t) = -\frac{p}{4\pi\epsilon r}\left(\frac{ik}{r} - \frac{1}{r^2} + k^2\right)e^{ikr}e^{-i\omega t} \tag{1}$$

$$H_\phi(r, t) = \frac{i\omega p}{4\pi r}\left(ik - \frac{1}{r}\right)e^{ikr}e^{-i\omega t} \tag{2}$$

Manuscript received June 15, 1979; revised December 7, 1979. This work was supported by the U.S. Army Research Office under Contract DAAG29-77-G-0130.

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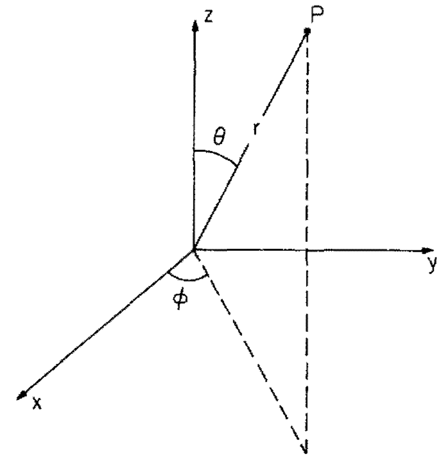

Fig. 1. Spherical and Cartesian coordinate systems for describing space surrounding an oscillating dipole located at their origin.

where p is the dipole moment, ϵ is the dielectric constant of free space, $k = \omega/c$, and c is the vacuum speed of light. For an observer at rest with respect to the dipole the frequency of the emitted field is ω (see Fig. 1).

FIELD MEASURED BY MOVING OBSERVER

Now we suppose that the observer is traveling at constant velocity v in the equatorial plane of the dipole, along a straight line passing through the dipole. More specifically, we suppose that the observer is traveling along the y axis from $y = -\infty$ to $y = \infty$ with velocity $\mathbf{v} = \mathbf{e}_y v$ where $\mathbf{e}_y$ denotes a unit vector in the y direction. Moreover, we take the speed of the observer to be moderate ($\beta = v/c \ll 1$).

According to the Lorentz transformation of fields, the electric field in the rest frame K' of the observer is given, to first order in β, by

$$\mathbf{E}' = \mathbf{E} + \mu\mathbf{v}\times\mathbf{H} \tag{3}$$

where μ is the permeability of free space and where E and H are the fields in the rest frame K of the dipole. Since $v/c \ll 1$, the Lorentz transformation of coordinates reduces to

$$y = vt' \qquad t = t' \tag{4}$$

where t' denotes time in K'. Accordingly, when $t' < 0$ the observer is approaching the dipole, when $t' = 0$ the observer is at the dipole, and when $t' > 0$ the observer is receding from the dipole (see Fig. 2).

From (1)–(4) it follows that the z component of the electric field measured by the moving observer is given by

$$E_z' = p\frac{e^{i(kv-\omega)t'}}{4\pi\epsilon vt'}\left[\frac{ik}{vt'} - \frac{1}{(vt')^2} + k^2 + ik\beta\left(ik - \frac{1}{vt'}\right)\right] \tag{5}$$

for $t' > 0$, that is, for the observer moving away from the dipole.

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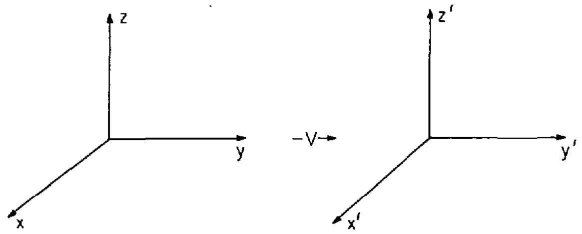

Fig. 2. Coordinate system x, y, z for rest frame K of dipole and coordinate system x', y', z' for rest frame K' of observer with V denoting relative velocity of K' with respect to K.

Similarly, we obtain

$$E_z' = p\,\frac{e^{-i(kv+\omega)t'}}{4\pi\epsilon vt'}\left[\frac{ik}{vt'}+\frac{1}{(vt')^2}-k^2+ik\beta\left(ik+\frac{1}{vt'}\right)\right] \tag{6}$$

for $t' < 0$, that is, for the observer moving toward the dipole.

THE PHASE OF THE MEASURED FIELD

The measured electric field can be expressed as

$$E_z' = Ae^{i\psi}e^{ikvt'}e^{-i\omega t'}, \qquad t'>0 \tag{7}$$

and

$$E_z' = Ae^{i\psi}e^{-ikvt'}e^{-i\omega t'}, \qquad t'<0 \tag{8}$$

where the amplitude A and the phase ψ are real functions of t'. From (5) and (7) we obtain

$$\tan\psi = \frac{kvt'(1-\beta)}{(kvt')^2(1-\beta)-1}, \qquad t'>0 \tag{9}$$

and from (6) and (8)

$$\tan\psi = \frac{-kvt'(1+\beta)}{(kvt')^2(1+\beta)-1}, \qquad t'<0. \tag{10}$$

These equations determine ψ as a multibranched function of kvt'. Choosing the branch of ψ that lies within the bounds $0 \le \psi \le \pi$, we see that $\psi = \pi$ when $t' = 0$ and then approaches zero as $t' \to \pm\infty$ (see Fig. 3).

The phase ψ in the rest frame K of the dipole is given by

$$\tan\psi = \frac{ky}{(ky)^2-1}, \qquad y>0 \tag{11}$$

$$\tan\psi = \frac{-ky}{(ky)^2-1}, \qquad y<0. \tag{12}$$

Clearly, from (9) and (11) and from (10) and (12) we see that, with respect to the transformation from K to K', ψ is an invariant in the far zone of the dipole ($kvt' \gg 1$, $ky \gg 1$; $kvt' \ll -1$, $ky \ll -1$) but not in its near zone ($|kvt'| \ll 1$, $|ky| \ll 1$). Thus for the near-zone field there is no phase invariance.

THE DOPPLER FREQUENCY FOR THE TRANSVERSE ELECTRIC FIELD

As the observer is receding from the dipole ($t' > 0$) we see from (7) that the total phase Φ measured by the observer is

$$\Phi = \psi + kvt' - \omega t'. \tag{13}$$

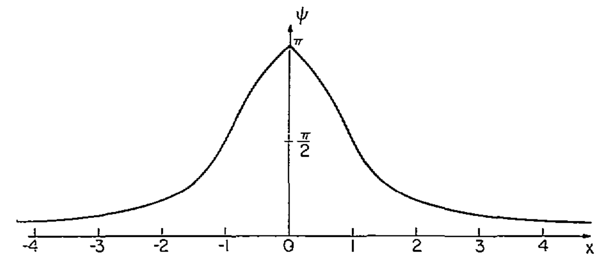

Fig. 3. Function $\psi(X)$ as measured by an observer moving at velocity v with respect to dipole oscillating at frequency $\omega = kc$ in K, where $X \equiv kvt' = kvt$ and $\beta = 1/100$.

The frequency ω' of the observed field is defined by [6]

$$\omega' = -\frac{d}{dt'}\,\Phi. \tag{14}$$

From (13) and (14) we find that

$$\omega' = \omega - kv - \frac{d\psi}{dt'}. \tag{15}$$

Introducing the parameter η we can write (15) as

$$\omega' = \omega - kv\eta, \qquad t'>0 \tag{16}$$

where

$$\eta = 1 + \frac{1}{kv}\frac{d\psi}{dt'}, \qquad t'>0. \tag{17}$$

When $\eta > 0$ we have a red shift, and when $\eta < 0$ we have a blue shift.

Similarly, as the observer is approaching the dipole ($t' < 0$), we see from (8) that the total phase measured by the observer is

$$\Phi = \psi - kvt' - \omega t'. \tag{18}$$

Then, according to (14), we have

$$\omega' = \omega + kv - \frac{d\psi}{dt'}. \tag{19}$$

That is,

$$\omega' = \omega - kv\eta, \qquad t'<0 \tag{20}$$

where now we define η by

$$\eta = -1 + \frac{1}{kv}\frac{d\psi}{dt'}, \qquad t'<0 \tag{21}$$

so that, again, $\eta > 0$ yields a red shift and $\eta < 0$ yields a blue shift.

From (9) and (17) we find that

$$\eta = \frac{[(kvt')^2(1-\beta)-1]^2-(1-\beta)}{[(kvt')^2(1-\beta)-1]^2+[kvt'(1-\beta)]^2}, \qquad t'>0 \tag{22}$$

and from (10) and (21) we find that

$$\eta = -\frac{[(kvt')^2(1+\beta)-1]^2-(1+\beta)}{[(kvt')^2(1+\beta)-1]^2+[kvt'(1+\beta)]^2}, \qquad t'<0. \tag{23}$$

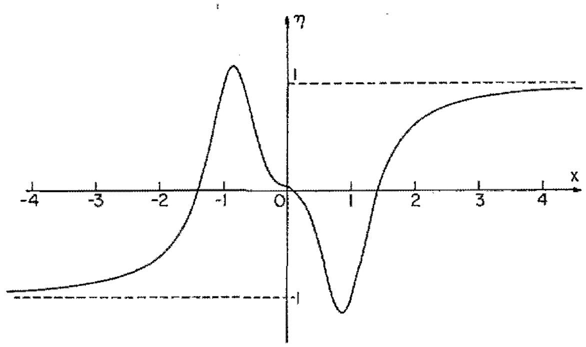

Fig. 4. Function $\eta(X)$ for transverse electric field as measured by an observer moving at velocity v with respect to dipole oscillating at frequency $\omega = kc$ in K, where $X \equiv kvt' = kvt$ and $\beta = 1/100$.

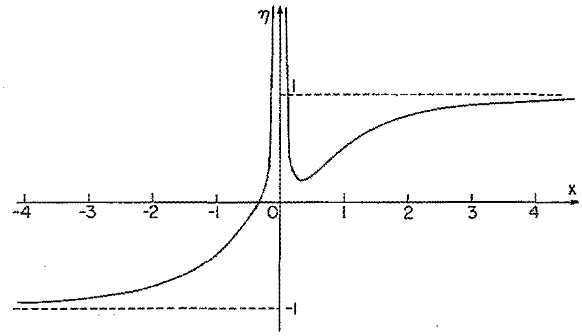

Fig. 5. Function $\eta(X)$ for transverse magnetic field as measured by an observer moving at velocity v with respect to dipole oscillating at frequency $\omega = kc$ in K, where $X \equiv kvt' = kvt$ and $\beta = 1/100$. In general, $\eta(0) = 1/\beta$.

The behavior of η as a function of kvt' is shown in Fig. 4. The function $\eta(kvt')$ has three zeros, one at $kvt' = -x_1$, another at $kvt' = x_2$, and the third at $kvt' = x_3$, where

$$x_1 = \left[\frac{1+(1+\beta)^{1/2}}{1+\beta}\right]^{1/2} \simeq \sqrt{2} \tag{24}$$

$$x_2 = \left[\frac{1-(1-\beta)^{1/2}}{1-\beta}\right]^{1/2} \simeq \sqrt{\frac{\beta}{2}} \tag{25}$$

$$x_3 = \left[\frac{1+(1-\beta)^{1/2}}{1-\beta}\right]^{1/2} \simeq \sqrt{2}. \tag{26}$$

From this we note that, as the observer travels from $y = -\infty$ ($kvt' = -\infty$) to $y = \infty$ ($kvt' = \infty$), there is a blue shift for $-\infty \leq kvt' \leq -x_1$, then a red shift for $-x_1 \leq kvt' \leq x_2$, then a blue shift for $x_2 \leq kvt' \leq x_3$, and finally a red shift for $x_3 \leq kvt' \leq \infty$. Thus we see that, for the transverse electric field, there is an inverse Doppler effect in the vicinity of the dipole.

THE DOPPLER FREQUENCY FOR THE TRANSVERSE MAGNETIC FIELD

The magnetic field $\mathbf{H}'$ in K' is related to the fields $\mathbf{E}$ and $\mathbf{H}$ in K by the Lorentz transformation

$$\mathbf{H}' = \mathbf{H} - \epsilon \mathbf{v} \times \mathbf{E} \tag{27}$$

where ϵ is the dielectric constant of free space. Since, in the equatorial plane of the dipole, $\mathbf{H}$ has only a ϕ component, $\mathbf{E}$ has only a θ component, and $\mathbf{v} = \mathbf{e}_y v$, the observed transverse magnetic field is given by

$$H_\phi' = H_\phi - \epsilon v E_\theta, \qquad t' > 0 \tag{28}$$

$$H_\phi' = H_\phi + \epsilon v E_\theta, \qquad t' < 0. \tag{29}$$

From a knowledge of E_θ and H_ϕ, as given by (1) and (2), we find from (28) and (29) that

$$H_\phi' = p\,\frac{e^{ikvt'}}{4\pi vt'}\left[i\omega\left(ik - \frac{1}{vt'}\right) + v\left(\frac{ik}{vt'} - \frac{1}{(vt')^2} + k^2\right)\right] e^{-i\omega t'}, \qquad t' > 0 \tag{30}$$

$$H_\phi' = p\,\frac{e^{-ikvt'}}{4\pi vt'}\left[-i\omega\left(ik + \frac{1}{vt'}\right) - v\left(\frac{ik}{vt'} + \frac{1}{(vt')^2} - k^2\right)\right] e^{-i\omega t'}, \qquad t' < 0. \tag{31}$$

In terms of the amplitude A and phase ψ, (30) can be written as

$$H_\phi' = A e^{i\psi} e^{ikvt'} e^{-i\omega t'}, \qquad t' > 0 \tag{32}$$

where

$$\tan\psi = \frac{(kvt')(1-\beta)}{(kvt')^2(1-\beta)+\beta}, \qquad t' > 0. \tag{33}$$

Similarly, (31) can be written as

$$H_\phi' = A e^{i\psi} e^{-ikvt'} e^{-i\omega t'}, \qquad t' < 0 \tag{34}$$

where now

$$\tan\psi = \frac{-(kvt')(1+\beta)}{(kvt')^2(1+\beta)-\beta}, \qquad t' < 0. \tag{35}$$

As in the case of the transverse electric field, the observed frequency ω' of the transverse magnetic field can be written as

$$\omega' = \omega - kv\eta \tag{36}$$

for all t', where now

$$\eta = \frac{[(kvt')^2(1-\beta)+\beta]^2 + \beta(1-\beta)}{[(kvt')^2(1-\beta)+\beta]^2 + [kvt'(1-\beta)]^2}, \qquad t' > 0 \tag{37}$$

and

$$\eta = -\frac{[(kvt')^2(1+\beta)-\beta]^2 - \beta(1+\beta)}{[(kvt')^2(1+\beta)-\beta]^2 + [kvt'(1+\beta)]^2}, \qquad t' < 0. \tag{38}$$

This η function is shown in Fig. 5. Accordingly, as the observer travels from $y = -\infty$ ($kvt' = -\infty$) to $y = \infty$ ($kvt' = \infty$) and measures the frequency of the transverse magnetic field, there is a blue shift for $-\infty \leq kvt' \leq -x_0$ and a red shift for $-x_0 \leq kvt' \leq \infty$, where

$$x_0 = \left[\frac{\beta + [\beta(1+\beta)]^{1/2}}{1+\beta}\right]^{1/2}. \tag{39}$$

Clearly, we see that also for the transverse magnetic field there is an inverse Doppler effect in the vicinity of the dipole. We note that at the origin the value of η is $1/\beta$.

THE DOPPLER FREQUENCY FOR THE RADIAL ELECTRIC FIELD

The above calculations refer to the fields in the equatorial plane of the dipole. Since the radial component E_r of the electric field is identically zero on the equatorial plane, to learn about the Doppler

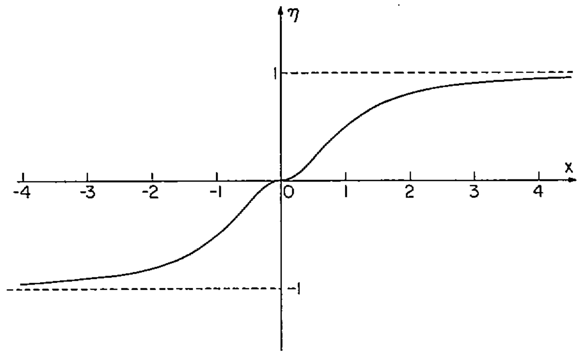

Fig. 6. Function $\eta(X)$ for radial electric field as measured by an observer moving a velocity v with respect to dipole oscillating at frequency $\omega = kc$ in K, where $X \equiv kvt' = kvt$.

frequency for E_r, let us examine the Doppler frequency for an observer moving along the z axis where E_r is most pronounced (and E_θ and H_ϕ are identically zero).

Following the procedure used for the transverse fields, we can show that the observed frequency ω' is again given by

$$\omega' = \omega - kv\eta, \tag{40}$$

where v is the constant speed at which the observer travels from $z = -\infty$ to $z = \infty$ and where now the η function is given by

$$\eta = \frac{(kvt')^2}{(kvt')^2+1}, \qquad t'>0 \tag{41}$$

and

$$\eta = \frac{-(kvt')^2}{(kvt')^2+1}, \qquad t'<0. \tag{42}$$

Thus as t' increases from $-\infty$ to ∞, η goes smoothly from -1 to 1 and is zero when $t' = 0$ (see Fig. 6). There is no inverse Doppler effect for E_r.

PRACTICAL SUMMARY

To summarize our results qualitatively let us consider a situation where an aircraft is flying toward a known primary source such as a transmitting antenna or toward a known secondary source such as a scatterer. The aircraft antenna will first sense a blue-shifted signal from which the velocity of the aircraft can be determined. Then, as the aircraft enters the near zone of the antenna or scatterer, the aircraft antenna will detect not one but three different signals, one for each component of the field. From these three signals the distance of the aircraft from the source can be determined. Thus from the ordinary Doppler effect (far zone) one can obtain velocity information, and from the inverse Doppler effect (near zone) one can obtain, in addition, range information. The inverse effect becomes most practical at low frequencies where the near-zone field has a relatively large spatial extent. It is independent of the power level of the transmitter or scatterer.

CONCLUSION

The Doppler effect has been calculated for an observer traveling at constant velocity along a straight line passing through an oscillating dipole. The calculations show that as long as the observer is in the dipole's far-zone field the Doppler effect is quite normal, viz., the Doppler shift is the same for all field components, is independent of distance from the dipole, and is blue on approaching the dipole and red on receding from it. However, when the observer is in the dipole's near-zone field the Doppler effect is anomalous, for then

a) there are several Doppler shifts, one for each field component;
b) the Doppler shifts are functions of distance from the dipole; and
c) the Doppler shifts for the transverse field components are inverse, i.e., blue on receding from the dipole and red on approaching it.

The near-zone Doppler effect is more informative than the far-zone Doppler effect: it gives range, polarization, and velocity.

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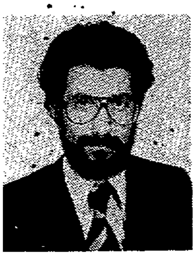

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