MEAN REFLECTIVITIES

Fig. 2 summarizes the values of sea reflectivity for various conditions. These conditions are

sea state	1 to 5
grazing angle	6° to 47°
wind aspect	upwind, downwind, crosswind
polarization	horizontal and vertical
location	East or West Coast.

The general trends with respect to the grazing angle and sea state are in agreement with [4]. However, the measurements for vertical polarization are 3–10 dB higher than in the reference. This discrepancy is greater than can be accounted for by inaccuracies, which were estimated to be 1.6 dB rms. Further discussion may be found in [5].

REFERENCES

[1] Raytheon Company, "TAGSEA Program final report," vols. I, II, III, IV; BR-9254-1-4, August 27, 1976; AD-A036971-AD-A036974 (vol. I is a summary).

[2] G. V. Trunk, "Modification of radar properties of non-Rayleigh sea clutter," *IEEE Trans. Aerosp. Electron. Syst.*, p. 110, Jan. 1973.

[3] P. R. Sodergren, "A revised Ku-band sea clutter model," JHU/APL memo. MPD-72-U-033, July 19, 1972.

[4] F. E. Nathanson, *Radar Design Principles.* New York: McGraw-Hill, 1969.

[5] L. W. Brooks, F. E. Nathanson, and P. R. Brooks, "Final report TAGSEA clutter measurement verification," Tech. Serv. Corp. TSC-WO-284R, B50711, October 29, 1976, AD-A037249.

Wave Tilt Sounding of a Linearly Inhomogeneous Layered Half-Space

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***Abstract*—The wave tilt of a transverse electric (TE) electromagnetic wave over a linearly inhomogeneous lossy layer overlying a homogeneous half-space is studied. Two approaches are used: an exact formulation using solutions of Airy's equation and an approximate numerical solution using a large number of homogeneous layers with a linearly increasing dielectric constant. The numerical results of both solutions are practically identical as long as the thickness of the layers in the approximate model are somewhat smaller than a quarter-wavelength.**

I. INTRODUCTION

The electromagnetic wave tilt over a layered half-space has been studied in detail in connection with subsurface sounding (see [1]–[3]). In the real world, the electric properties in the subsurface are usually inhomogeneous. In this communication, we study the case of a linearly inhomogeneous lossy layer overlaying a homogeneous lossy half-space. The numerical results of the exact solution are compared to the results using a large number of homogeneous layers with an increasing dielectric constant. As a subsurface model, we used a linear approximation to the electric characteristics profile of permafrost rock in Alaska measured by Lytle *et al.* [4]. The numerical results show that the homogeneous layers approximation is satisfactory as long as the electric thickness of each layer in the model is somewhat smaller than one-quarter of the sounding wavelength.

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II. FORMULATION

The wave tilt due to a linearly inhomogeneous lossy layer on top of a homogeneous half-space can be derived in a straightforward fashion. Refering to Fig. 1, the expressions for the total transverse electric field E_y assuming harmonic time dependence of the form $e^{-i\omega t}$ are

—in the upper half-space ($z \leqslant 0$),

$$E_y = e^{ik_0(z\cos\theta + x\sin\theta)} + Re^{ik_0(-z\cos\theta + x\sin\theta)}; \quad (1)$$

—in the inhomogeneous layer ($0 \leqslant z \leqslant h$),

$$E_y = [Av(z) + Bu(z)]\, e^{ik_0 x \sin\theta}; \quad (2)$$

—in the lower half-space ($z \geqslant h$),

$$E_y = Ce^{ik_3 z + ik_0 x \sin\theta} \quad (3)$$

$$k_3 = k_0(\bar{\epsilon}_3 - \sin^2\theta)^{1/2} \quad (4)$$

where A, B, and C are constants; R is the reflection coefficient at the surface ($z = 0$); θ is the angle between the incident wave vector and the surface normal ($\theta \to \pi/2$); k_0 is the free-space wavenumber; and $v(z)$ and $u(z)$ are solutions of the wave equation with a linearly varying dielectric constant [1].

The continuity of the tangential electric and magnetic fields at the two boundaries allows us to determine R. The wave tilt for the transverse electric (TE) polarization is defined by

$$W \equiv -(H_z/H_x)_{z=0} = \tan\theta \, \frac{1+R}{1-R}, \quad (5)$$

which for the grazing incidence ($\theta = \pi/2$) gives

$$W = ik_0 \left[\frac{(v_0\dot{u}_1 - u_0\dot{v}_1) - ik_3(v_0u_1 - u_0v_1)}{(\dot{v}_0\dot{u}_1 - \dot{u}_0\dot{v}_1) - ik_3(\dot{v}_0u_1 - v_1\dot{u}_0)} \right] \quad (6)$$

where the dot corresponds to differentiation with respect to z, the subscript 0 means value at $z = 0$, and the subscript 1 means value at $z = h$. This can be rewritten in terms of linearly independent solutions of Airy's equation, $U(\eta)$ and $V(\eta)$ (5):

$$W = -i\alpha \left[\frac{(V_0U_1' - U_0V_1') + i\beta(V_0U_1 - U_0V_1)}{(V_0'U_1' - U_0'V_1') + i\beta(V_0'U_1 - V_1U_0')} \right] \quad (7)$$

where

$$\beta = (\bar{\epsilon}_3 - 1)^{1/2}\alpha$$
$$\alpha = k_0/[(\bar{\epsilon}_2 - \bar{\epsilon}_1)k_0^2/h]^{1/3}$$
$$\bar{\epsilon} = \epsilon_r(1 + i\tan\delta),$$

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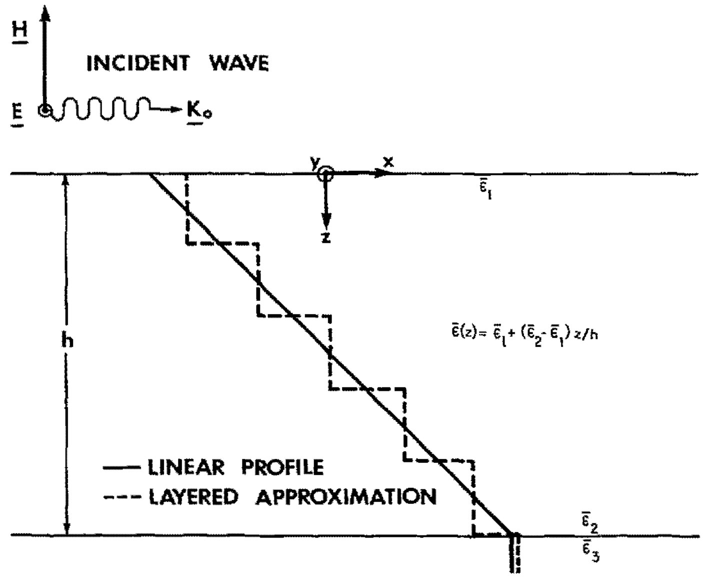

Fig. 1. Dielectric profiles. Complex dielectric constant $\bar{\epsilon}(z)$ varies linearly from value $\bar{\epsilon}_1$ at surface to $\bar{\epsilon}_2$ at homogeneous half-space. In examples under consideration, $\bar{\epsilon}_2 = \bar{\epsilon}_3$, which is value in homogeneous half-space.

$\bar{\epsilon}_1$ is the complex dielectric constant at the top of the inhomogeneous layer,
$\bar{\epsilon}_2$ is the dielectric constant at the bottom of the inhomogeneous layer,
$\bar{\epsilon}_3$ is the complex dielectric constant in the half-space, and
h is the thckness of the inhomogeneous layer.

The prime means differentiation with respect to η, the subscript zero corresponds to the value at $\eta = \eta_0$, and the subscript one corresponds to the value of $\eta = \eta_1$ where

$$\eta_0 = (1 - \bar{\epsilon}_1)[k_0{}^2h^2/(\bar{\epsilon}_2 - \bar{\epsilon}_1)^2]^{1/3}$$

$$\eta_1 = (1 - \bar{\epsilon}_2)[k_0{}^2h^2/(\bar{\epsilon}_2 - \bar{\epsilon}_1)^2]^{1/3}.$$

For the numerical calculation we use the solutions

$$U(z) = H_i{}^{(1)}(z) = \text{Bi}\,(z) + i\,\text{Ai}\,(z)$$

$$V(z) = H_i{}^{(2)}(z) = \text{Bi}\,(z) - i\,\text{Ai}\,(z) \tag{8}$$

where Ai(z) and Bi(z) are the Airy functions [5]. The functions defined in (8) are analogous to Hankel functions. In terms of these functions the general solution of Airy's equation is

$$C_1H_i{}^{(1)}(z) + C_2H_i{}^{(2)}(z). \tag{9}$$

However, for complex arguments one term in (9) blows up exponentially, while the other decays exponentially. As a digital computer has only finite accuracy, often one term will be lost from the general expression (9). The choice (8) affords the two linearly independent solutions which are generally necessary for the numerical evaluation of (7).

III. NUMERICAL MODEL

The exact wave tilt expression (7) was plotted and compared to the numerical solution using an approximate model composed of a large number of homogeneous layers [1], [3]. In this model, the dielectric constant of each homogeneous layer was set equal to the exact value of the dielectric constant at the middle plane of the layer. As a dielectric profile, we used two models.

1) A linear approximation to the actual profile measured by Lytle *et al.* [4] in permafrost rock in northern Alaska. Their borehole measurement resulted in a dielectric constant (ϵ_r) and a loss tangent (tan δ) monotonically increasing with depth from ($\epsilon_r = 3$, tan $\delta = 0.017$) at the surface to ($\epsilon_r = 25$, tan $\delta = 0.052$) at a depth of 150 m. Below 150 m, we assumed in our model that the electric properties are constant and equal to the ones at 150 m.

2) The second model is the same as the previous model with a loss tangent lower by a factor of 10. The measurements of Lytle *et al.* [4] were made at 23 MHz. We assumed these same values in the 1–10-MHz region where we made our analysis.

Figs. 2 and 3 represent the magnitude and phase of the wave tilt for the two above models, respectively. In Fig. 2 we plotted the exact solution and the approximate solution using 100 layers. For all practical purposes the two solutions give identical results. Notice the oscillations present in the lower frequency region. These oscillations are a result of interference from the reflection at the bottom of the inhomogeneous layer where there is a discontinuity in the slope of the electric properties. The period of the oscillation should be very close to

$$\Delta f = c/[2h(\langle\epsilon_r\rangle - 1)^{1/2}] \tag{10}$$

where $\langle\epsilon_r\rangle$ is the average dielectric constant in the inhomogeneous layer and c is the velocity of light in a vacuum. The above expression gives $\Delta f = 277$ kHz, which is very close to the value observed in Figs. 2 and 3.

In Fig. 3 we show the curves corresponding to case 2, where the loss tangent is smaller by a factor of ten. The oscillations still have the same period, but their amplitude is larger because of the reduced attenutation. For the approximate solution we plotted the results for 50- and 100-layer approximations. It is clear that in the case of 50 layers (i.e., each layer has a 3-m thickness), the numerical solution starts to deviate from the

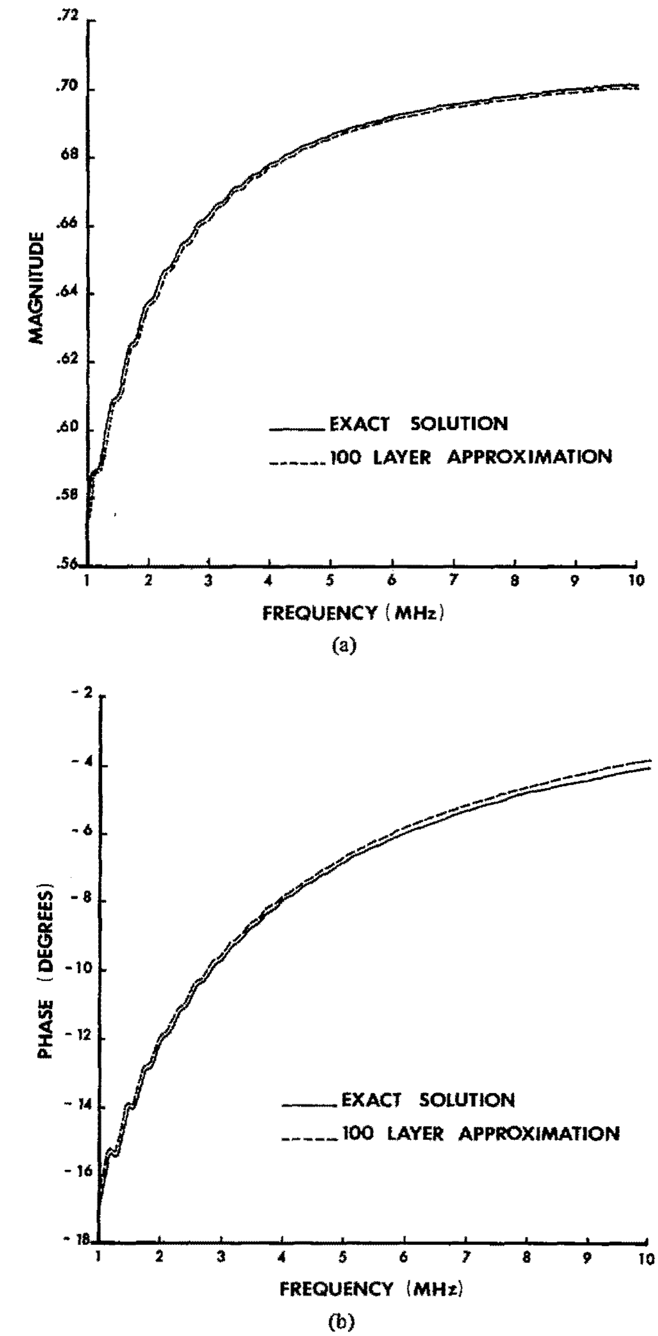

Fig. 2. (a) Wave tilt magnitude as function of frequency for dielectric profile in which (ϵ_{r1} = 3, tan δ_1 = 0.017) and ($\epsilon_{r2} = \epsilon_{r3}$ = 25, tan δ_2 = tan δ_3 = 0.052). (b) Corresponding wave tilt phase. h = 150 m.

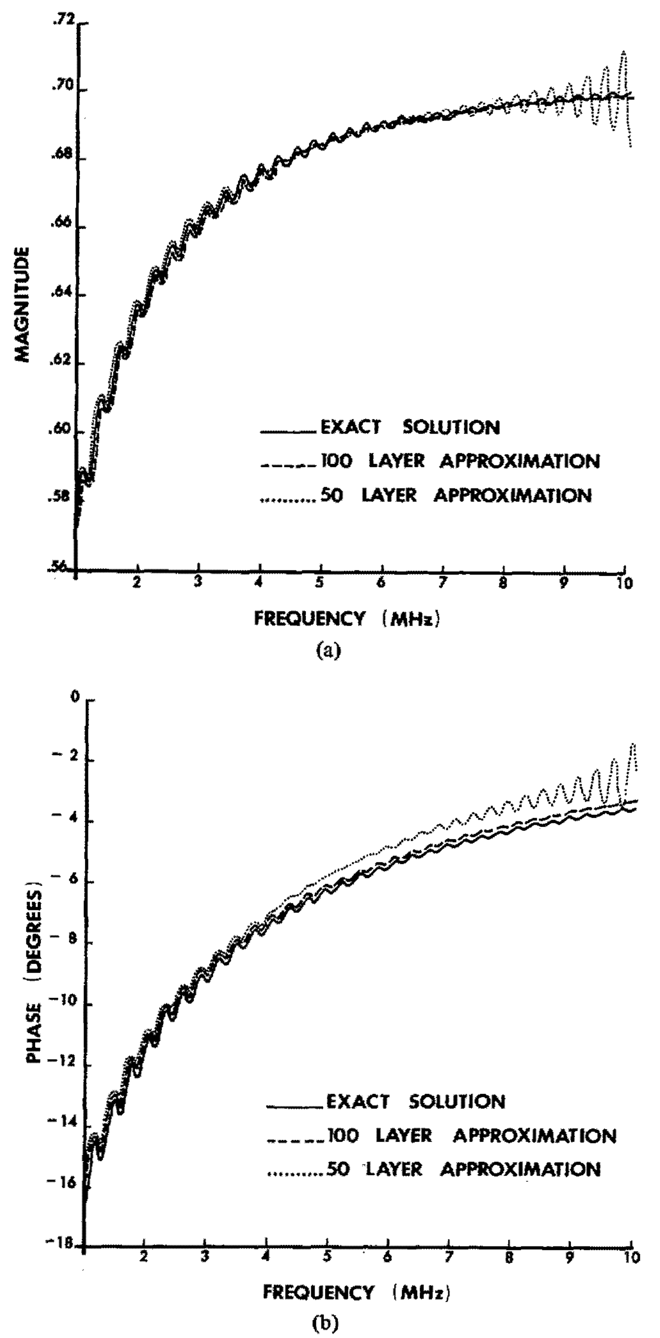

Fig. 3. (a) Wave tilt magnitude for dielectric profile in which (ϵ_{r1} = 3, tan δ_1 = 0.0017) and ($\epsilon_{r2} = \epsilon_{r3}$ = 25, tan δ_2 = tan δ_3 = 0.0052). (b) Corresponding wave tilt phase. h = 150 m.

exact solution for frequencies higher than 4 MHz. At this frequency the wavelength at the bottom of the inhomogeneous layer is 15.3 m. This gives a quarter-wavelength, which is slightly larger than the layer thickness in the approximation. Thus the oscillations in the result from the approximate layered model are due to fictitious resonance effects generated by the nature of the stratified model. In the case of 100 layers (i.e., 1.5-m layer thickness), the deviation in the approximate solution starts to appear at a frequency of 8 MHz as expected and increases at higher frequencies.

REFERENCES

[1] J. Wait, *Electromagnetic Waves in Stratified Media.* New York: MacMillan, 1962.
[2] R. King, "Wave tilt measurements," *IEEE Trans. Anten. Propagat.*, 1976.
[3] R. Lytle, D. Lager, and E. Laine, "Subsurface probing by high-frequency measurements of the wave tilt of electromagnetic surface waves," *IEEE Trans. Geosci. Electron.*, vol. *GE-14*, 1976.
[4] R. Lytle *et al.*, "Determination of the *in situ* high-frequency properties of permafrost rock," *Radio Sci.*, vol. 11, p. 285, 1976.
[5] N. Abromowitz and I. Stegun, *Handbook of Mathematical Functions.* National Bureau of Standards, 1965.