TA8 - 10:30

Control of Plants with Input Saturation Nonlinearities

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Abstract

This paper considers control design for systems with input magnitude saturation. Four examples, 2 SISO and 2 MIMO, are used to illustrate the properties of several existing schemes. A new method based on a modification of conventional antiwindup compensation is introduced. It is assumed that the reader is familiar with the problem of integral windup for saturating plants and conventional schemes for dealing with it.

Introduction

Nearly all systems have some type of control input saturation, and often it is the dominant nonlinearity. The focus of this paper is on otherwise linear systems which have an input magnitude saturation. We assume a linear controller already exists which produces "ideal" behavior on the unsaturated plant. The objective is to design additional compensation that leaves the original linear behavior unchanged but provides graceful degradation of system performance under saturation. There is very little formal theory that addresses this objective, but there are several somewhat ad hoc schemes and many working systems. This paper aims to begin the development of more systematic methods.

The existing schemes briefly reviewed here are Internal Model Control (IMC) [M], a method suggested by Horowitz (H) [H], and a version of conventional antiwindup (CAW). Adding no additional antiwindup compensation is, of course, a fourth alternative. IMC and H insure stability under saturation, but are not really intended to be antiwindup schemes and as such do not provide effective antiwindup. This can give poor performance, as is shown in the first SISO example. It must be emphasized that this is not a new result and neither method is intended to be an antiwindup scheme. They are considered here merely to provide a comparison with CAW. Because of space limitations, many details about the schemes discussed here will not be given.

CAW is much more effective than the alternatives on the first example, but can lead to instability, as shown in the second SISO example. CAW also performs very poorly in the MIMO examples, giving little improvement over doing nothing. The second MIMO example is unstable with or without CAW. A modified antiwindup scheme (MAW) is introduced which provides substantial improvement. A new nonlinear extension to μ analysis is used to prove the stability and robustness of MAW for the MIMO examples [D].

Motivation for this research comes from many practical problems, but particularly from a multivariable helicopter flight control problem where conventional antiwindup schemes were inadequate. The examples themselves are not directly based on any specific physical systems, but are intended to illustrate some issues that could arise in practical applications. They are simple enough that the behavior they exhibit should not require much comment in order to be understood. Many practical considerations that are essentially driven by implementation issues will not be discussed.

Siso Examples

The four example systems to be studied are all posed as disturbance rejection problems with input saturation as shown in Figure 1, where P is the plant and the controller K is given. The various methods will be described as the first example is introduced. It is assumed throughout that the saturation block represents, in each channel

$$\imath = \begin{cases} 1, & \text{if } c > 1; \\ \text{c}, & \text{if } -1 \le c \le 1; \\ \text{-1}, & \text{if } c < -1; \end{cases}$$

that the output of the saturation is available as a measurement, and that there is no uncertainty in the saturation itself. This arrangement greatly simplifies the exposition and is not a practical limitation, but space precludes a full justification. Skeptics are encouraged to think of the saturation as occuring internally in the controller as part of an antiwindup scheme against a saturating element in the plant.

For the first controller,

$$P = (.1+s)/2s, \quad \text{and} \quad K = 2/(.1+s),$$

giving a loop transfer function $L = PK = 1/s$. The response of e and c to a unit step in d is shown in Figure 5a. The error e has an overshoot not present in the linear response because c exhibits classical windup. As can be seen here, K need not have integrators to produce windup. It merely needs relatively slow dynamics that are driven by the error when the system is in saturation.

The method of Horowitz is based on Figure 2. He emphasizes the role of the the loop transfer function $L_n = H + GP$ around the nonlinear saturation element and suggests, among other things, designing L_n to have large gain and bandwidth relative to L. G and H are chosen to produce the desired L and L_n.For $L_n = 10/s$, we get

$$G = \frac{2(10+s)}{(.1+s)(1+s)}, \quad \text{and} \quad H = \frac{-10}{s}.$$

This does not stop windup or overshoot as is shown in the response in Figure 5b. Larger L_n do not improve the response.

The IMC method is shown in Figure 3. In practice the input to P_o is c and P_o contains a model of the saturation. The response of the IMC method is shown in Figure 5c. Translation of the original system into the IMC framework gives

$$Q = \frac{2s}{(.1+s)(1+s)}$$

and $P_o = P$. The controller output c is identical to the linear case, a characteristic of IMC. In this case an error offset exists.

Figure 4 shows a version of the CAW scheme. There are many alternative configurations which achieve the same effect, but this one is particularly easy to understand and compare with the other approaches. The difference between the controller output and the plant input is fed back through X into some coprime factorization $K = V^{-1}U$. If the associated loop transfer function $L_X = V^{-1}X$ has gain and bandwidth much higher than that of L, this prevents the windup. For $X = 10$ and $V^{-1} = K$, the response is shown in Figure 5d. There is neither windup nor overshoot.

On this example, CAW is clearly superior to IMC and H, and would continue to be so on other inputs. Of course, this is not surprising as neither IMC nor H are antiwindup schemes. Indeed, this phenomena is well-known to IMC aficianados and they suggest the addition of CAW to IMC, which is easily done. CAW cannot be added to H as they are fundamentally incompatible. Since, $L_X = (L - L_n)/(I + L_n)$ and $L_n = (L - L_X)/(I + L_X)$ it is not possible for both L_X and L_n to have higher gain and bandwidth than L. To be fair, Horowitz might object that he would never design L_n as we have done here and that we have totally misunderstood his methodology. While this is likely, [H] seems pretty clear about problems of this type.

One obvious advantage that IMC and H have over CAW is that for stable P, they guarantee closed-loop stability even under saturation. Horowitz emphasizes that stability under saturation is determined by L_n, and designs it accordingly. IMC actually produces $L_n = 0$, so it cannot be unstable under saturation. Since CAW focuses on L_X it may produce an L_n that leads to instability. CAW as implemented here is also highly sensitive to uncertainty in the saturating element. Alternative implementations can eliminate this sensitivity, but space precludes further treatment here.

Although CAW is stable and yields good performance on the first example, naive application of CAW to the second example leads to instability. The plant is a fourth order lag-lead butterworth,

$$P = 0.2\left(\frac{s^2 + 2\varsigma_1\omega_1 + \omega_1^2}{s^2 + 2\varsigma_1\omega_2 + \omega_2^2}\right)\left(\frac{s^2 + 2\varsigma_2\omega_1 + \omega_1^2}{s^2 + 2\varsigma_2\omega_2 + \omega_2^2}\right).$$

where $\omega_1 = .2115, \omega_2 = .0473, \varsigma_1 = .3827$ and $\varsigma_2 = .9239$. The controller is $K = 5/s$. This system has substantially greater low-frequency disturbance rejection than one with $L = 1/s$, which has the same bandwidth. The price paid for this is conditional stability and poorer disturbance rejection in frequencies just below crossover. The linear (no saturation) response to a step input of unit amplitude is oscillatory and is shown in Figure 6a. When the plant input saturation is included the system is driven into a limit cycle as shown in Figure 6b.

When CAW with $X = 10$ and $V^{-1} = K$ is applied the step response is stable and well behaved as shown in Figure 6c. Unfortunately, the system is driven to a limit cycle by a disturbance input with a unit step at time $t = 0$ and a switch to -1 at $t = 4$ (Figure 6d). The IMC and H schemes will stabilize this plant but with much poorer step responses than CAW. The ideal saturation compensation scheme is probably some tradeoff between these methods, but at present there is no formal methodology for optimizing performance while guaranteeing system stability. Examination of L and L_n using standard describing function and small gain ideas correctly predicts the stability characteristics for this example so it seems likely that with some work, improvements can be made. While this example is admittedly a little weird, a general methodology should be developed which can handle such cases.

Mimo Examples

The first MIMO example has

$$P = \frac{4(0.1+s)}{s}R^{-1} \quad \text{and} \quad R = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$$

The controller $K = kR$ with $k = \frac{1}{4(0.1+s)}$ gives $L = PK = (1/s)I$ and a nominally decoupled response $e/d = (I+L)^{-1} = s/(s+1)I$. The response (with saturation) to a worst-case direction step disturbance of amplitude $d = (.61\ .79)^T$ is shown in Figure 7a. The response for CAW with $X = 100I$, $U = R$, and $V^{-1} = kI$ is shown in Figure 7b. The overshoot, characteristic of windup, is no longer present but the error is still large.

The MAW design (Figure 8) gives the response in Figure 7c. The feedback element α is a scalar multiplying the controller output vector c given by

$$\alpha = \begin{cases} 0, & \text{if } \|c\|_\infty \leq 1-\epsilon \\ 10 & \text{if } \|c\|_\infty > 1-\epsilon \end{cases}$$

with $\epsilon = .02$. It is equivalent to a version of CAW for the scalar case. The operation of this type of antiwindup scheme can easily be seen by comparing the first two seconds of the CAW response (Figure 7d) to the MAW response (Figure 7e). The linear responses are, of course, identical until c_1 saturates at 1.12 seconds. With CAW this saturation produces a change in the direction of c which interacts with the asymmetry in P to produce a large error. Analysis using μ shows that while this system is robustly stable for large independent gain changes in each channel as are produced by saturation, the performance suffers dramatically. This is entirely consistent with the simulations. Robust stability problems would also arise for uncertainty at the outputs. The MAW controller prevents this direction change in c while still preventing windup.

A slightly different example provides an even more dramatic contrast. Here $P = P_1P_0$,

$$P_0 = \frac{4(0.1+s)}{s}R^{-1}, \quad P_1 = \begin{pmatrix} \dfrac{2(10-s)}{(10+s)} & 0 \\ 0 & \dfrac{(5-s)}{(5+s)} \end{pmatrix},$$

and K is as above. Note that $L = (1/s)P_1$ is still diagonal but with different elements on the diagonal. The system is unstable following a step disturbance of amplitude $d = (.36\ .93)^T$ (Figure 9a). With CAW implemented as in Example 1 the system is also unstable (Figure 9b). Simple μ analysis shows that this system has serious robust stability problems for diagonal input uncertainty so this is not surprising.

The system can be stabilized with MAW implemented as in the first example. Figure 9c shows the well-behaved response of this scheme to the above step disturbance. Intuitively, this is expected, because the "gain change" at the input produced by MAW is effectively the same in both channels and the system is robustly stable with respect to input uncertainty of this type. Of course, MAW is nonlinear so standard μ analysis does not, strictly speaking, apply, and conventional nonlinear small gain theory cannot exploit the structure inherent in MAW. Fortunately, a new nonlinear extension to μ analysis can treat this problem [D].

Space limitations preclude including the details from [D], so the results are summarized. The nominal system with MAW is shown to be stable under saturation. MAW also insures graceful degradation of robustness. If a full block of uncertainty $w\Delta$ with $\bar{\sigma}(\Delta) < 1$ is connected from d to e, the smallest w, say w_0, that produces instability is a measure of multivariable stability margins at the output. For no saturation $w_0 = .72$ and under saturation, $w_0 = .69$. Thus MAW preserves not only stability, but also the robustness to uncertainty at the output. If the system were LTI, this would also have a robust performance interpretation as $1/w_0$ is the worst-case "sensitivity" $\|S\|_\infty$, $S = (I+L)^{-1}$. Because the system is nonlinear, such an interpretation is not entirely correct, but the simulatio are consistent with it.

Conclusions

The problem of plant input saturation has been considered through the study of four simple examples. The examples are somewhat extreme but do highlight the deficiencies in existing schemes and point to directions for future research. The MIMO examples presented here illustrate in this context the recurring theme that, unlike SISO systems, MIMO systems have additional difficulties created by plant directionality.

Several schemes exist for enhancing the stability and performance of systems subject to saturation. Unfortunately, there is no formal methodology for guaranteeing stability and at the same providing graceful degradation of performance. Conventional antiwindup (CAW) is effective for many systems, but naive application can lead to instability in some situations.

For multivariable plants exhibiting high directionality, a naive application of CAW can have poor performance or even be unstable. This was illustrated by the two MIMO examples. A modified antiwindup (MAW) scheme was introduced which was very effective for these two examples, and could be proven so using a new nonlinear extension to μ analysis [D]. While this suggests some possibilities for a general methodology, there remains many unanswered questions. It should not be difficult to construct examples where naive use of the MAW scheme presented here would also fail.

Many practicing engineers have developed good intuition on the effects of saturation and methods for handling it. This intuition is typically based on simple SISO problems for which CAW works well. Practical multivariable control problems are becoming more common and more complex. While much progress has been made in the development of applicable multivariable control theory, some important issues have been neglected. The techniques for handling input saturation will have to keep pace with the other developments in multivariable control theory if the new theory is to be generally applicable. It is hoped that this paper, along with [D], will provide the beginning

Acknowledgements

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[H] I.Horowitz, "A Synthesis Theory for a Class of Saturating Systems". INT.J.CONTROL, Vol. 38, No. 1, pp169-187. 1983.

[M] M.Morari, E.Zafirou, & C. Economou, "An Introduction to Internal Model Control", Springer-Verlag, 1987, to appear.

Figure 1. System Schematic Diagram

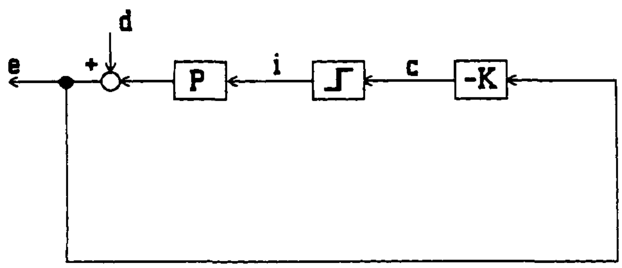


Figure 2. Horowitz Structure for Saturating System

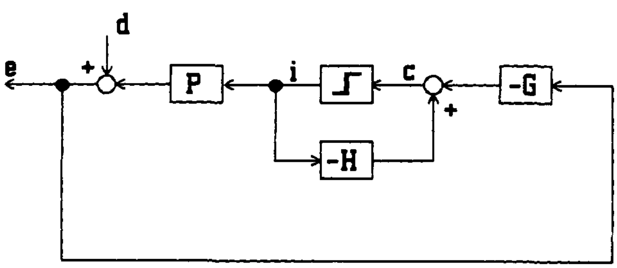


Figure 3. IMC Structure for Saturation Compensation

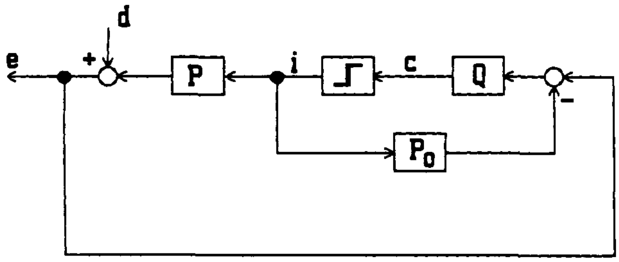
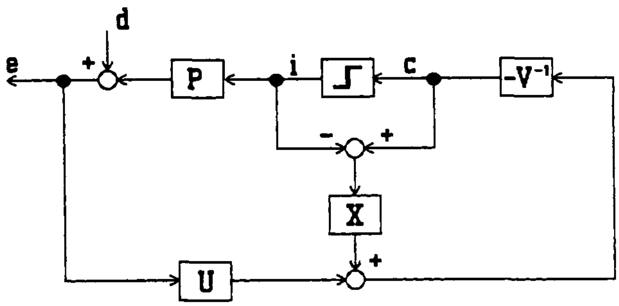
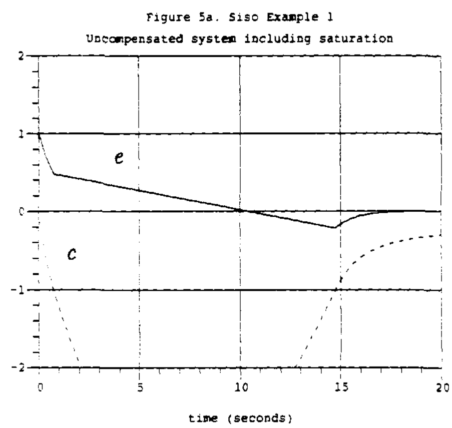


Figure 4. Conventional Antiwindup Structure





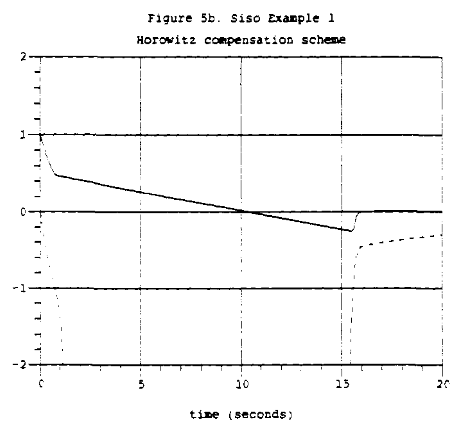


Figure 5c. Siso Example 1
IMC compensation scheme

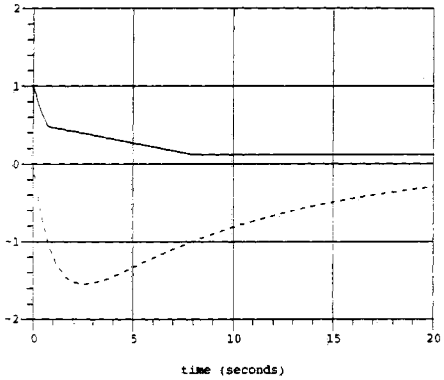


Figure 5d. Siso Example 1
Conventional antiwindup scheme

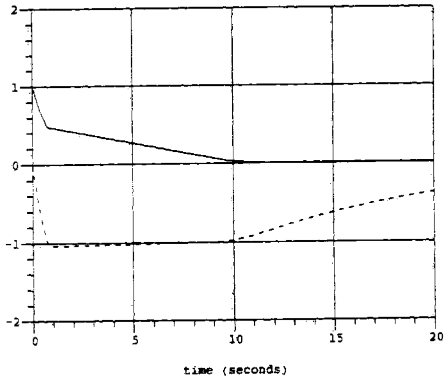


Figure 6a. Siso Example 2
System without saturation

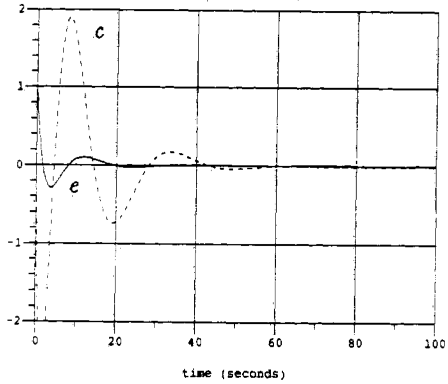


Figure 6b Siso Example 2
Uncompensated system with saturation

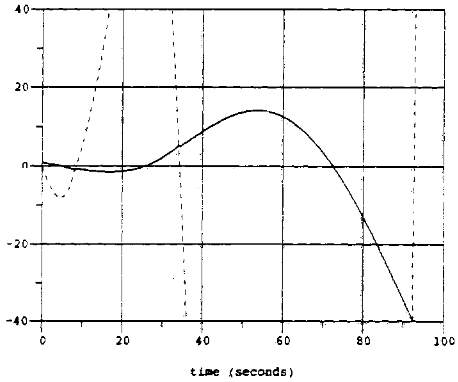


Figure 6c Siso Example 2
Conventional antiwindup scheme

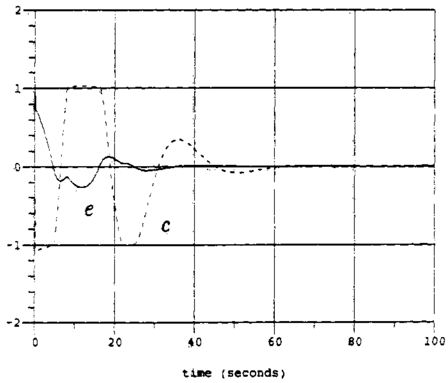


Figure 6d. Siso Example 2
Conventional antiwindup scheme

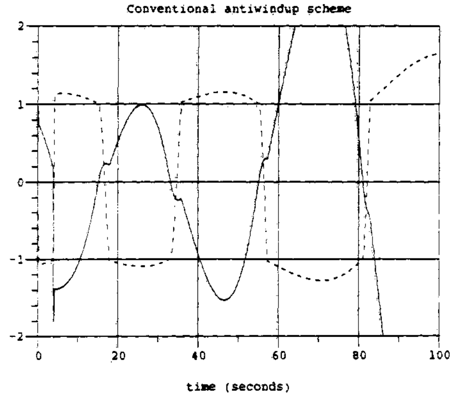


Figure 7a Mimo Example 1
Uncompensated system response

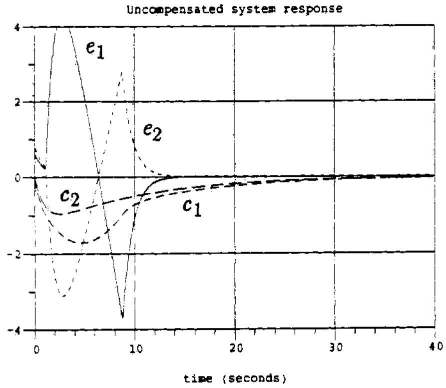


Figure 7b. Mimo Example 1
Conventional Antiwindup Scheme

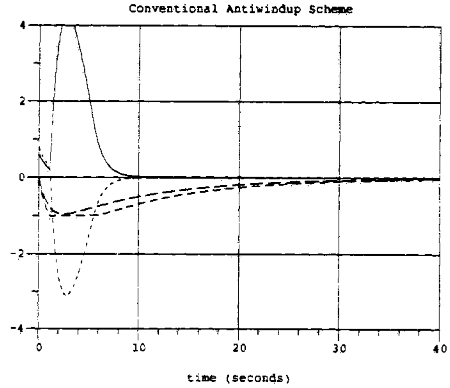


Figure 7c Mimo Example 1
Modified Antiwindup Scheme

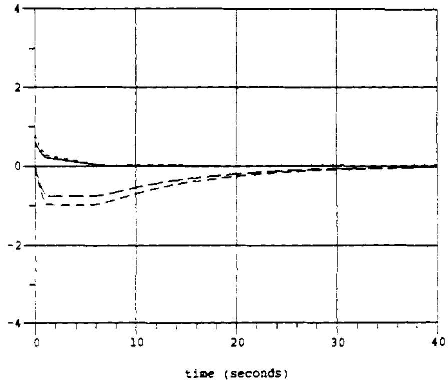


Figure 7d. Mimo Example 1
Conventional Antiwindup - detail

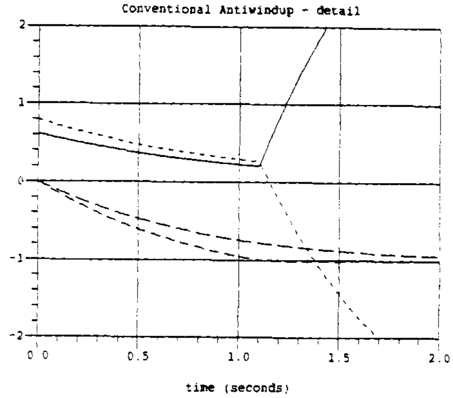


Figure 7e. Mimo Example 1
Modified Antiwindup - detail

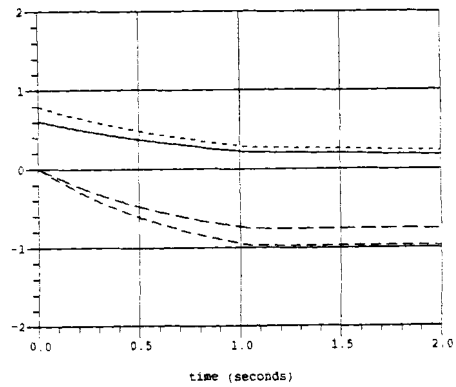


Figure 9a. Mimo Example 2
Uncompensated system response

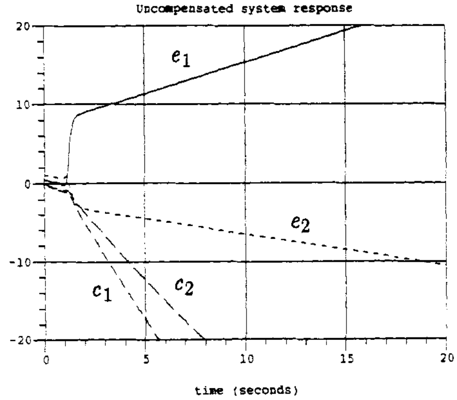


Figure 8. Modified Antiwindup Structure for MIMO Systems

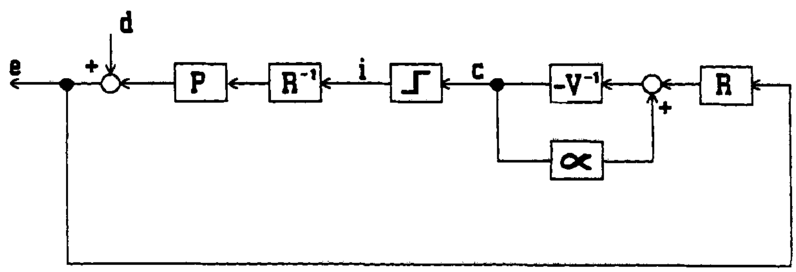


Figure 9b. Mimo Example 2
Conventional Antiwindup scheme

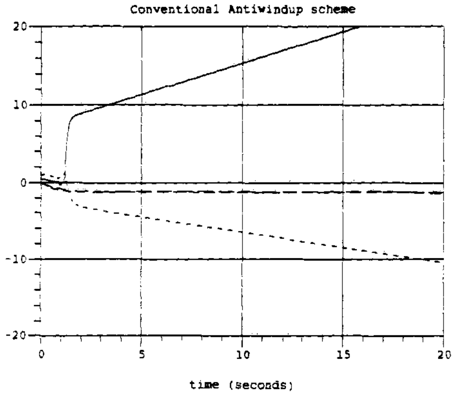


Figure 9c. Mimo Example 2
Modified Antiwindup scheme