perhaps take into account the effect of sawtoothing near the axis. In Ref. 1 this difficulty was addressed by flattening the p and q profiles within the $q = 1$ surface and matching the pressure gradient at $q = 1$ to the power deposited within it. Including a sawtooth region in this way would be expected to worsen the confinement.

In conclusion, we have pointed out that the average magnetic well in a tokamak can provide stability against ballooning modes until self-stabilization takes over. The marginal stability condition of Refs. 5 and 6, which contains both of these effects, thus provides an exact case study of a bifurcated S-α stability boundary, which had been previously examined by means of a generic model in Ref. 2.

The calculations in the present work indicate that steady-state high-beta self-stabilized equilibria exist, albeit at rather high heating powers. Whether the plasma can actually evolve from a stable low-beta state to such a high-beta self-stabilized state requires the solution of the temporal evolution problem. Work on this problem is reported elsewhere.[10]

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[1]J. W. Connor, J. B. Taylor, and M. F. Turner, Nucl. Fusion **24**, 642 (1984).
[2]G. Y. Fu and J. W. Van Dam, submitted to J. Plasma Phys.
[3]A. A. Ware and F. A. Haas, Phys. Fluids **9**, 956 (1966).
[4]V. D. Shafranov and E. I. Yurchenko, Zh. Eksp. Teor. Fiz. **53**, 1157 (1967) [Sov. Phys. JETP **26**, 682 (1968)].
[5]A. B. Mikhailovskii and E. I. Yurchenko, Plasma Phys. **24**, 977 (1982).
[6]A. B. Mikhailovskii, V. V. Demchenko, and A. Ya. Omelchenko, Fiz. Plazmy **9**, 351 (1983) [Sov. J. Plasma Phys. **9**, 204 (1983)].
[7]See, for example, R. J. Goldston, Plasma Phys. Controlled Nucl. Fusion **26**, 87 (1984).
[8]G. A. Navratil (private communication); see also G. A. Navratil and T. C. Marshall, Comments Plasma Phys. Controlled Fusion **10**, 185 (1986).
[9]M. Murakami, J. D. Callen, and L. A. Berry, Nucl. Fusion **16**, 347 (1976).
[10]G. Y. Fu, J. W. Van Dam, and M. N. Rosenbluth, in *Proceedings of the International School of Plasma Physics—Workshop on Theory of Fusion Plasmas* (International School of Plasma Physics, Varenna, Italy, in press).

Helicity injection with moving vacuum–plasma boundary with arbitrary flux surfaces

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If a toroidal plasma has arbitrary nested magnetic flux surfaces and a moving plasma–vacuum interface, then any helicity injected by modulating the magnetic fields is simply consumed by an increase in helicity dissipation due to the modulated fields.

It had been tacitly assumed[1] that the injection of helicity into a toroidal plasma is tantamount to driving dc currents in the plasma. However, Ref. 2 showed, for the special case of axisymmetric circular flux surfaces, that if there is a moving plasma–vacuum plasma interface (such as might occur when the velocity field is normal to the wall) then it is possible to inject helicity using oscillating fields (ac helicity injection) and not drive dc currents. Instead, what happens is that the oscillating fields themselves cause an increase in helicity dissipation which exactly consumes the injected helicity so that there is no net helicity available to balance the dissipation of dc currents.

In this Brief Communication we wish to extend the results of Ref. 2 to arbitrary flux surfaces. It was shown in Ref. 2 that the helicity conservation for a plasma with a moving vacuum–plasma interface is

$$\frac{d}{dt}\int_{V(t)} d^3r\,\mathbf{A}\cdot\mathbf{B} + \int_{S(t)} d\mathbf{S}\cdot[\mathbf{B}(\phi - \mathbf{A}\cdot\mathbf{U}) + \eta\mathbf{J}\times\mathbf{A}] = -2\int_{V(t)} \eta\mathbf{J}\cdot\mathbf{B}\,d^3r. \tag{1}$$

Consider now the situation where a steady state helicity is to be maintained:

$$\frac{d}{dt}\int_{V(t)} d^3r\,\mathbf{A}\cdot\mathbf{B} = 0 \tag{2}$$

and there are flux surfaces which move, and which are arranged, such that $\mathbf{B}\cdot d\mathbf{S} = 0$ at the plasma–vacuum interface (i.e., the plasma–vacuum interface does not cut a flux surface). In this case, Eq. (1) simplifies to

$$\int_{S(t)} \eta\mathbf{J}\times\mathbf{A}\cdot d\mathbf{S} = -2\int_{V(t)} \eta\mathbf{J}\cdot\mathbf{B}\,d^3r. \tag{3}$$

To proceed further, we define the fluxes[3] ψ,χ such that

$$\mathbf{A} = \psi\nabla\theta + \chi\nabla\phi, \quad \mathbf{B} = \nabla\psi\times\nabla\theta + \nabla\chi\times\nabla\phi. \tag{4}$$

Then, Faraday's law and the magnetohydrodynamic Ohm's law $\mathbf{E} + \mathbf{U}\times\mathbf{B} = \eta\mathbf{J}$ together with Eq. (4) give

$$\eta\mathbf{J} = -\nabla\Phi - \frac{d\psi}{dt}\nabla\theta - \frac{d\chi}{dt}\nabla\phi + \nabla\psi\mathbf{U}\cdot\nabla\theta + \nabla\chi\mathbf{U}\cdot\nabla\phi, \tag{5}$$

where Φ is the electrostatic potential.

If it is now assumed that ψ,χ form flux surfaces, i.e., $\psi = \psi(\xi)$, $\chi = \chi(\xi)$, where ξ is the effective radius of the flux surface so that $\xi = 0$ at the magnetic axis and increases on moving away from the magnetic axis, then the following relations hold[3]:

$$\nabla\psi\times\nabla\chi = 0, \quad d^3r = d\xi\, d\theta\, d\phi / |\nabla\xi\cdot\nabla\theta\times\nabla\phi|,$$
$$d\mathbf{S} = \nabla\xi\, d\theta\, d\phi / |\nabla\xi\cdot\nabla\theta\times\nabla\phi|. \qquad (6)$$

As in the Ref. 2, the most general forms for bounded fluxes ψ,χ are

$$\psi = C_\psi(\xi) + D_\psi(\xi)\cos[\omega t - \alpha(\xi)],$$
$$\chi = C_\chi(\xi) + D_\chi(\xi)\sin[\omega t - \alpha(\xi)], \qquad (7)$$

where

$$\xi(r,t) = r - \int_0^t U(r(t'),t')dt'$$

is a constant of the motion (i.e., $d\xi/dt = 0$) and $D_\chi = 0$ at $\xi = 0$. If $\Phi = \Phi(\xi)$ also, then the time average (denoted by $\langle\ \rangle$) of the lhs (injection rate) of Eq. (3) becomes

$$\left\langle \int_{S(t)} d\mathbf{S}\cdot\eta\mathbf{J}\times\mathbf{A}\right\rangle = \left\langle \int d\theta\, d\phi\left(-\chi\frac{d\psi}{dt} + \psi\frac{d\chi}{dt}\right)\right\rangle = \int d\theta\, d\phi\, \omega D_\chi D_\psi. \qquad (8)$$

Similarly the time average of the rhs of Eq. (3) (dissipation rate) becomes

$$\left\langle -2\int \eta\mathbf{J}\cdot\mathbf{B}\, d^3r\right\rangle = \left\langle -2\int d\theta\, d\phi\, d\xi\left(-\frac{\partial\chi}{\partial\xi}\frac{d\psi}{dt} + \frac{\partial\psi}{\partial\xi}\frac{d\chi}{dt}\right)\right\rangle = \int d\theta\, d\phi\, d\xi\,\frac{\partial}{\partial\xi}(D_\chi D_\psi) = \text{lhs of Eq. (3)}. \qquad (9)$$

Thus the helicity dissipation resulting from *oscillating fields* [rhs of Eq. (3)] exactly consumes all the helicity flux injected by those same oscillating fields [lhs of Eq. (3)] and so there is no helicity flux left to balance the helicity dissipation because of dc currents and fields.

Hence, to obtain useful ac helicity injection it is necessary to have no moving plasma–vacuum interface (which is equivalent to maintaining constant inductance despite forces which are trying to alter the plasma geometry) or not have flux surfaces at all.

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[1]T. H. Jensen and M. S. Chu, Phys. Fluids **27**, 2881 (1984).
[2]P. M. Bellan, Phys. Rev. Lett. **57**, 2383 (1986).
[3]A. H. Boozer, Phys. Fluids **29**, 4123 (1986).