Single beam, single detector diagnostic for plasma density profiles across a magnetic field gradient

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A scheme is proposed whereby the density profile of a tokamak or mirror plasma is mapped to nonlinear sidebands of a diagnostic wave, so that the sideband frequency spectrum is an image of the density profile traversed by the wave. The diagnostic relies on a monochromatic, high-frequency ($\omega \gtrsim \omega_{ce}$), pulsed wave source to determine the density profile across a magnetic field gradient. In principle, the technique offers time-dependent direct density profiles having high spatial resolution. However, the feasibility of the diagnostic is limited by the intense power and the rapid rise time required of the pulsed diagnostic wave. For parameters relevant to fusion plasmas, and for the particular scattering geometry suggested here, the technique requires, roughly, 30 mJ pulses of 2 nsec duration. The required power can be reduced by a factor of roughly 10^4 by applying signal averaging techniques. The rise time of the pulsed source must be short in comparison to one period of the diagnostic wave.

I. INTRODUCTION

The electron density profile in tokamak and mirror plasmas is fundamental—it is required for understanding turbulence and transport, wave propagation, magnetohydrodynamic stability, and equilibrium. To date, four techniques have been developed to measure the density profile: They are (i) Thomson scattering of ruby laser light,[1–4] (ii) multichannel ordinary mode interferometry with Abel inversion or tomography,[1] (iii) Bragg scattering,[1,4–8] and (iv) electron cyclotron radiation measurements.[9–11] These techniques have certain limitations: Ordinary wave interferometry requires a separate channel for each location, and in addition, an Abel transform (requiring the assumption of azimuthal symmetry) must be performed. Bragg scattering provides only the Fourier transform of fluctuations in the density profile (useful for studying waves, but not equilibria), and again, spatial resolution is limited, in this case because of the poorly defined extent of the scattering volume. Determination of the electron density profile from the emission spectrum of an optically thin harmonic of the electron cyclotron frequency requires an independent measurement of the electron temperature profile.

We propose here a new diagnostic method whereby a single wave source and a single detector provide temporally and spatially resolved density profile measurements. A qualitative outline will be given first, followed by a detailed derivation.

As shown in Fig. 1, a high-power high-frequency ($\omega \gtrsim \omega_c$, where ω_c is the electron cyclotron frequency) pulsed extraordinary wave—the pump wave—is directed along the x axis (which might, for example, be the major axis of a tokamak). The inhomogeneous magnetic field is a function only of x: $\mathbf{B} = B(x)\hat{z}$. This pump wave perturbs the cyclotron trajectories of electrons in the plasma at each point along the major radius. The perturbation can be expressed as the sum of two terms: (i) a forced term at the frequency and wavenumber of the pump and (ii) a ballistic (or transient term), which is at ω_c and which (as will be shown later) has a wavenumber $k = \omega_c/c$. Nonlinear reevaluation of the electron motion, taking into account orbit perturbations that result from the ballistic term, provides nonlinear mixing between the pump and the ballistic wave. This mixing gives sidebands at $\omega \pm \omega_c$ with respective wavenumbers $(\omega \pm \omega_c)/c$. The sideband phase velocity is c so that the sideband is *coherently* forward scattered, yielding a strong signal (in contrast to Thomson scattering where only incoherent scattering occurs). The sideband intensity is propor-

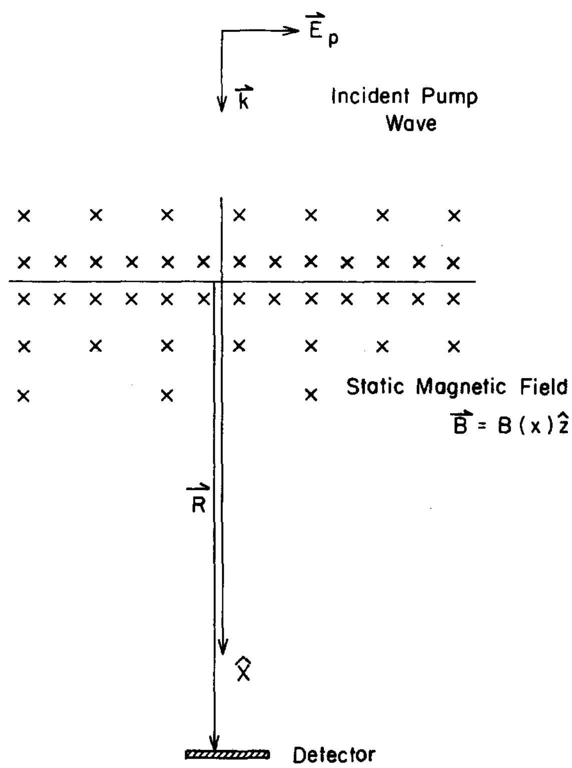

FIG. 1. Schematic representation of the density diagnostic method. A pulsed wave source with extraordinary polarization is incident on a nonuniformly magnetized plasma. The detector gathers radiation scattered in the forward direction.

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tional to the electron density at the position corresponding to ω_c, while the lifetime of the sideband reflects the rate at which parallel electron motion destroys the ballistic term. Repetitive pulsing of the incident wave is necessary to recreate the ballistic term. In order to follow the time evolution of the plasma, the repetition rate should be rapid enough to continuously recreate the ballistic term. However, the intense power required of the pump wave significantly limits the number of pulses per unit time that can be achieved in practice.

It may be helpful to think of the density diagnostic in terms of a mechanical analogy. Suppose that a harmonic oscillator of mass m and natural frequency ω_0, initially at rest, is subjected to a driving force $F(t) = H(t)F_0\cos(\omega t)$ [where $H(t)$ is the Heaviside function]. If the oscillator's position $y(t)$ is described by $\ddot{y} + \omega_0^2 y = F(t)/m$, then

$$y(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos(\omega t) - \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos(\omega_0 t)\,.$$

The first term of the right-hand side oscillates at the driving frequency, whereas the second term, the transient or ballistic term, oscillates at the natural frequency ω_0. The ballistic term represents the amount of natural-frequency oscillation needed to satisfy the initial condition, $y(0) = 0$. Any small nonlinearity included in the model [e.g., $\ddot{y} + \omega_0^2 y = F(t)/m + \epsilon y^2$] will result in a mixing between the ballistic term and the forced term, giving rise to oscillations of order ϵ at the sideband frequency, $\omega \pm \omega_0$. This mechanical system is analogous to the response of a magnetized plasma to a pulsed electromagnetic wave; the cyclotron frequency plays the role of the oscillator's natural frequency and the electric field of the pulsed pump wave plays the role of the driving force. Note that the equation of motion for a plasma electron in a uniform magnetic field $\mathbf{B}$ and a pulsed electric field $\mathbf{E}(\mathbf{x},t) = E_0 H(\omega t - \mathbf{k}\cdot\mathbf{x})\cos(\omega t - \mathbf{k}\cdot\mathbf{x})$ is

$$\left(m\frac{d^2}{dt^2} + \frac{q}{c}\mathbf{B}\times\frac{d}{dt}\right)\mathbf{x} = q\mathbf{E}(\mathbf{x},t)\,.$$

The linear operator on the left-hand side gives rise to cyclotron motion; the nonlinear driving force $q\mathbf{E}(\mathbf{x},t)$ is responsible for a mixing between the cyclotron motion and the forced oscillation of the electron.

In Sec. II we present a detailed calculation where the Vlasov equation is used to study the interaction of a pulsed electromagnetic wave with a magnetized plasma. A discussion of the feasibility of this technique appears in Sec. III.

II. INTERACTION OF A PULSED ELECTROMAGNETIC WAVE WITH A MAGNETIZED PLASMA

At $t = -\infty$ we envision an equilibrium plasma immersed in a nonuniform magnetic field, as shown in Fig. 1. The equilibrium electron distribution function $f_0(\mathbf{x},\mathbf{v})$ is assumed to satisfy

$$\mathbf{v}\cdot\frac{\partial f_0}{\partial \mathbf{x}} + \frac{q}{mc}[\mathbf{v}\times\mathbf{B}(\mathbf{x})]\cdot\frac{\partial f_0}{\partial \mathbf{v}} = 0\,.$$

For simplicity, motion along the magnetic field is ignored. The electric field of the pump wave is given by

$$\mathbf{E}_p(\mathbf{x},t) = \hat{y}E_0 H\left(t - \frac{x}{c}\right)\cos\left[\omega\left(t - \frac{x}{c}\right)\right],$$

where H is the Heaviside function. The resulting distribution function at time t is then the solution of the Vlasov equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E}_p + \frac{\mathbf{v}\times\mathbf{B}(\mathbf{x})}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{x},\mathbf{v},t) = 0\,, \tag{1}$$

with boundary conditions

$$f(\mathbf{x},\mathbf{v},t = -\infty) = f_0(\mathbf{x},\mathbf{v})\,.$$

The acceleration term in Eq. (1) contains only the externally prescribed fields $\mathbf{E}_p$ and $\mathbf{B}(\mathbf{x})$; the fields produced by the plasma particles are not included. This is a reasonable approximation if $\omega \gg \omega_{ce}, \omega_{pe}$.

It is convenient to define the following list of dimensionless variables:

$$\omega_{c0} = (q/mc)B_z(\mathbf{x}=0)\,,\quad \Omega_c = (1/\omega_{c0})[qB_z(\mathbf{x})/mc]\,,$$
$$\boldsymbol{\xi} = \omega_{c0}\mathbf{x}/c\,,\quad \boldsymbol{\beta} = \mathbf{v}/c\,,\quad \tau = \omega_{c0}t\,,\quad \epsilon = E_0/B_z(\mathbf{x}=0)\,,$$
$$g(\xi_x,\tau) = H(\tau - \xi_x)\cos[b(\tau - \xi_x)]\,,\quad b = \omega/\omega_{c0}\,.$$

In this list of definitions, ω_{c0} is the cyclotron frequency evaluated at $\mathbf{x} = 0$; Ω_c is the normalized cyclotron frequency; $\boldsymbol{\xi}$, $\boldsymbol{\beta}$, and τ represent the normalized position, velocity, and time, respectively; ϵ is the ratio of the pump electric field to the static magnetic field; $g(\xi_x,\tau)$ is the normalized electric field of the pump, and b is the ratio of the pump frequency to the cyclotron frequency. With this set of variables, Eq. (1) becomes

$$\left(\frac{\partial}{\partial \tau} + \boldsymbol{\beta}\cdot\frac{\partial}{\partial \boldsymbol{\xi}} + [\epsilon\hat{y}g(\xi_x,\tau) + \Omega_c(\xi_x)\boldsymbol{\beta}\times\hat{z}]\cdot\frac{\partial}{\partial \boldsymbol{\beta}}\right) \times F(\boldsymbol{\xi},\boldsymbol{\beta},\tau) = 0\,, \tag{2}$$

subject to the initial conditions $F(\boldsymbol{\xi},\boldsymbol{\beta},\tau = -\infty) = F_0(\boldsymbol{\xi},\boldsymbol{\beta})$, where F_0 satisfies

$$\boldsymbol{\beta}\cdot\frac{\partial F_0}{\partial \boldsymbol{\xi}} + \Omega_c(\boldsymbol{\xi})(\boldsymbol{\beta}\times\hat{z})\cdot\frac{\partial F_0}{\partial \boldsymbol{\beta}} = 0\,. \tag{3}$$

A dimensionless current density and number density are defined by

$$n(\boldsymbol{\xi},\tau) = \int F(\boldsymbol{\xi},\boldsymbol{\beta},\tau)\,d\boldsymbol{\beta}\,,\quad \mathbf{J}(\boldsymbol{\xi},\tau) = \int \boldsymbol{\beta}F(\boldsymbol{\xi},\boldsymbol{\beta},\tau)\,d\boldsymbol{\beta}\,.$$

In solving Eq. (2), our goal is to find current fluctuations in the perturbed plasma which oscillate at the sideband frequencies $(b \pm 1)$. In terms of the original units, the y component of the scattered radiation is then given by

$$E_y^{\text{scatt}} = -\frac{1}{c^2 R}\int \left.\frac{\partial J_y}{\partial t}\right|_{\text{ret}} d^3x'\,, \tag{4}$$

where R is the distance to the observation point and where the integration extends over the scattering volume.

The parameter $\epsilon = E_0/B_z$ will have a small value. (For a tokamak toroidal field of ~ 50 kG and for 30 mJ pulses of 2 nsec duration spread over a 100 cm^2 area, the value of ϵ is roughly 7×10^{-4}. As described in Sec. III, signal averaging can reduce the required power by a factor of roughly 10^4, yielding $\epsilon \sim 7\times10^{-6}$.) Since ϵ is small, one can expand F in powers of ϵ, whereby Eq. (2) becomes

$$LF_k = -g(\xi_x,\tau)\frac{\partial F_{k-1}}{\partial \beta_y} \quad (k\geqslant 1), \tag{5}$$

with L defined by

$$L = \frac{\partial}{\partial \tau} + \boldsymbol{\beta}\cdot\frac{\partial}{\partial \boldsymbol{\xi}} + \Omega_c(\boldsymbol{\xi})[\boldsymbol{\beta}\times\hat{z}]\cdot\frac{\partial}{\partial \boldsymbol{\beta}}.$$

Equation (5) can be solved by the method of characteristics.[12] For $k = 1$, Eq. (5) gives

$$F_1 = -\int_{-\infty}^{\tau} d\tau' \, H(\tau' - \xi'_x)\left(\frac{\partial F_0}{\partial \beta_y}\right)' \cos[b(\tau' - \xi'_x)], \tag{6}$$

where the primed variables satisfy the orbit equations

$$\frac{d\boldsymbol{\beta}'}{d\tau'} = \Omega_c(\xi'_x)\boldsymbol{\beta}'\times\hat{z}, \tag{7a}$$

$$\frac{d\boldsymbol{\xi}'}{d\tau'} = \boldsymbol{\beta}', \tag{7b}$$

subject to the boundary conditions

$$\boldsymbol{\xi}'(\tau'=\tau) = \boldsymbol{\xi}, \quad \boldsymbol{\beta}'(\tau'=\tau) = \boldsymbol{\beta}.$$

The zero-order distribution F_0 is a solution of the stationary Vlasov equation (3) which may include spatial gradients in temperature and/or density. However, we will assume that spatial gradients in the equilibrium plasma are weak $[(\partial F_0/\partial x)(r_L/F_0) \ll 1$, where r_L is the electron Larmor radius$]$ so that the dominant behavior of the distribution function, in any local region, can be found by ignoring spatial gradients in F_0 and $\mathbf{B}$. In view of this approximation, we set $\Omega_c(\xi_x) = 1$ in Eq. (7a) and assume further that F_0 is a function only of β^2. The solution to Eq. (7) is then

$$\beta'_x(\tau') = \beta_\perp \cos[\varphi - (\tau' - \tau)], \tag{8a}$$

$$\beta'_y(\tau') = \beta_\perp \sin[\varphi - (\tau' - \tau)], \tag{8b}$$

$$\xi'_x(\tau') = -\beta_\perp \sin[\varphi - (\tau' - \tau)] + \beta_\perp \sin\varphi + \xi_x, \tag{8c}$$

where $\beta_x = \beta_\perp \cos\varphi$ and $\beta_y = \beta_\perp \sin\varphi$. Inserting Eq. (8) into Eq. (6) and carrying out the required integration gives the first-order distribution,

$$F_1 = -2\frac{\partial F_0}{\partial \beta_\perp^2}\beta_\perp \sum_{m=-\infty}^{m=\infty} \frac{J'_m(b\beta_\perp)}{m-b} \times\{\cos[b(\tau - \xi_x) + m\varphi - b\beta_\perp \sin\varphi] - \cos[m\tau + m\varphi - b\beta_\perp\sin\varphi - b\xi_x + (b-m)L]\}, \tag{9}$$

where the parameter L is defined implicitly by

$$L + \beta_\perp \sin(\varphi - L + \tau) - \beta_\perp \sin\varphi - \xi_x = 0. \tag{10}$$

In Eq. (9), J'_m is the derivative of the Bessel function of order m. The first term in the curly brackets, the forced term, oscillates at the same frequency as the pump wave. The second term in the curly brackets, the ballistic term, is somewhat more complicated. Upon inspection of Eq. (10), one finds that the parameter L is a periodic function of τ with period 2π. Using this fact, one sees that the entire ballistic term of Eq. (9) is a periodic function of τ with period 2π. It follows that the ballistic term can be written as a sum of Fourier components, at the cyclotron frequency, and at all harmonics of the cyclotron frequency. It is important to notice that F_1 does not contain frequency components at the mixed frequency, $b\pm 1$. Thus it is necessary to carry the calculation to second order in ϵ to obtain mixing between the cyclotron motion of the electrons and the incident pump. [It was found that non-Vlasov treatments—e.g., single particle models—which do not have the "quiet start" of Eq. (3) would give a spurious mixed-frequency term of order ϵ.]

From Eq. (5), the second-order distribution is given by

$$F_2 = -\int_L^{\tau} d\tau'\left(\frac{\partial F_1}{\partial \beta_y}\right)' \cos[b(\tau' - \xi'_x)]. \tag{11}$$

However, only the component of F_2 which oscillates at the mixed frequency $(b\pm 1)$ is of interest here. Thus it is only necessary to retain terms in F_1 which are proportional to $\cos(\tau - \xi_x)$ or $\sin(\tau - \xi_x)$ (i.e., the ballistic term). If the ballistic term of Eq. (9) is expanded in powers of $\beta_\perp$, one finds

$$F_1^{\text{ball}} = -2\frac{\partial F_0}{\partial \beta_\perp^2}\left(\frac{\beta_\perp\cos(\tau - \xi_x + \varphi)}{b^2-1} + \frac{\beta_\perp^2\sin(\varphi)\sin(\tau - \xi_x + \varphi)}{b^2-1} - \tfrac{1}{2}\beta_\perp^2\cos[2(\tau - \xi_x + \varphi)] + \cdots\right). \tag{12}$$

It is unnecessary to include higher-order terms in $\beta_\perp$ because these will ultimately yield small thermal corrections, which are not of interest here. Furthermore, the second-harmonic term in Eq. (12) can be dropped because it will not give sidebands at $b\pm 1$. Writing the relevant part of Eq. (12) in terms of $\beta_x = \beta_\perp\cos\varphi$ and $\beta_y = \beta_\perp\sin\varphi$ yields

$$F_1^{\text{ball}} = \frac{2}{1-b^2}\frac{\partial F_0}{\partial \beta_\perp^2}[\beta_x\cos(\tau - \xi_x) - \beta_y\sin(\tau - \xi_x) + \beta_y^2\cos(\tau - \xi_x) + \beta_x\beta_y\sin(\tau - \xi_x)]. \tag{13}$$

Using Eq. (11), the y component of the second-order current density becomes

$$J_{2y} = -\int d\beta_x \int d\beta_y\,\beta_y \int^{\tau} d\tau'\left(\frac{\partial F_1^{\text{ball}}}{\partial \beta_y}\right)' \times\cos[b(\tau' - \xi'_x)]. \tag{14}$$

The lower limit of the τ' integral has been omitted since only the upper limit gives sidebands. From Eq. (8) one can deduce the transformations

$$\beta_x = \beta'_x\cos(\tau' - \tau) - \beta'_y\sin(\tau' - \tau), \tag{15a}$$

$$\beta_y = \beta'_x\sin(\tau' - \tau) + \beta'_y\cos(\tau' - \tau), \tag{15b}$$

$$\xi'_x = \xi_x + \beta'_y[\cos(\tau' - \tau) - 1] + \beta'_x\sin(\tau' - \tau). \tag{15c}$$

From Eqs. (15a) and (15b) it follows that

$$d\beta_x\,d\beta_y = d\beta'_x\,d\beta'_y. \tag{16}$$

Using Eqs. (15) and (16) in Eq. (14), and interchanging the order of integration gives

$$J_{2y} = -\int^{\tau} d\tau' \int d\beta'_x \int d\beta'_y \left(\frac{\partial F_1^{\text{ball}}}{\partial \beta_y}\right)' \times [\beta'_x \sin(\tau' - \tau) + \beta'_y \cos(\tau' - \tau)] \times \cos[b(\tau' - \xi'_x)] . \quad (17)$$

Finally, Eq. (13) is inserted into Eq. (17) and the indicated integrations are performed. It is necessary to use Eq. (15c) to expand the argument of the trigonometric functions which appear within the integral in Eq. (17). The remaining calculation is straightforward, but tedious, and we only list the results here:

$$J_{2y} = \frac{\chi_+(b)}{1+b} \sin[(1+b)(\tau - \xi_x)] + \frac{\chi_-(b)}{1-b} \times \sin[(1-b)(\tau - \xi_x)] , \quad (18)$$

where

$$\chi_{\pm}(b) = \frac{1 \pm b + b^2}{b(b \pm 2)(b \mp 1)^2} .$$

Inserting Eq. (18) into Eq. (4), and noting that $t_{\text{ret}} = t - (R - x)/c$, one finds

$$\frac{E_y^{\text{scatt}}}{E_0} = \frac{r_e}{R} N \frac{E_0}{B_0} \left\{ \chi_+(b) \cos\left[(1+b)\omega_c\left(t - \frac{R}{c}\right)\right] + \chi_-(b) \cos\left[(1+b)\omega_c\left(t - \frac{R}{c}\right)\right]\right\} , \quad (19)$$

where $r_e = e^2/mc^2$ is the classical electron radius, N is the total number of electrons in the scattering volume, E_0/B_0 is the ratio of the pump electric field to the static magnetic field, and R is the distance to the observation point. A plot of $\chi_{\pm}(b)$ appears in Fig. 2.

III. FEASIBILITY

A rough estimate of the fraction of the incident power scattered into the sidebands can be obtained using Eq. (19). If the scattering volume has area A and is located a distance R from the detector, as pictured in Fig. 3, then

$$\frac{P_{\text{scatt}}}{P_i} = 8\pi N^2 \chi^2 \left(\frac{r_e}{R}\right)^2 \left(\frac{P_i}{cAB_0^2}\right) . \quad (20)$$

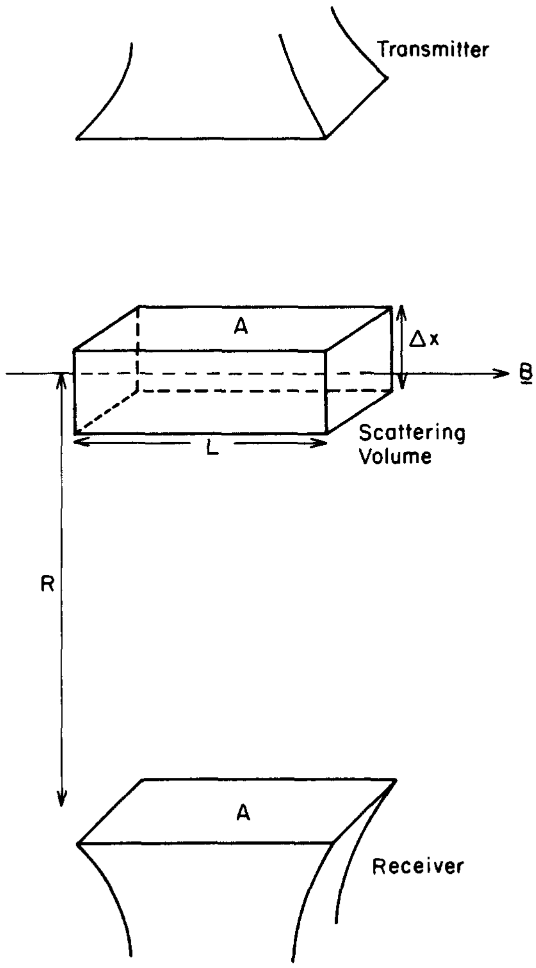

FIG. 3. Scattering geometry.

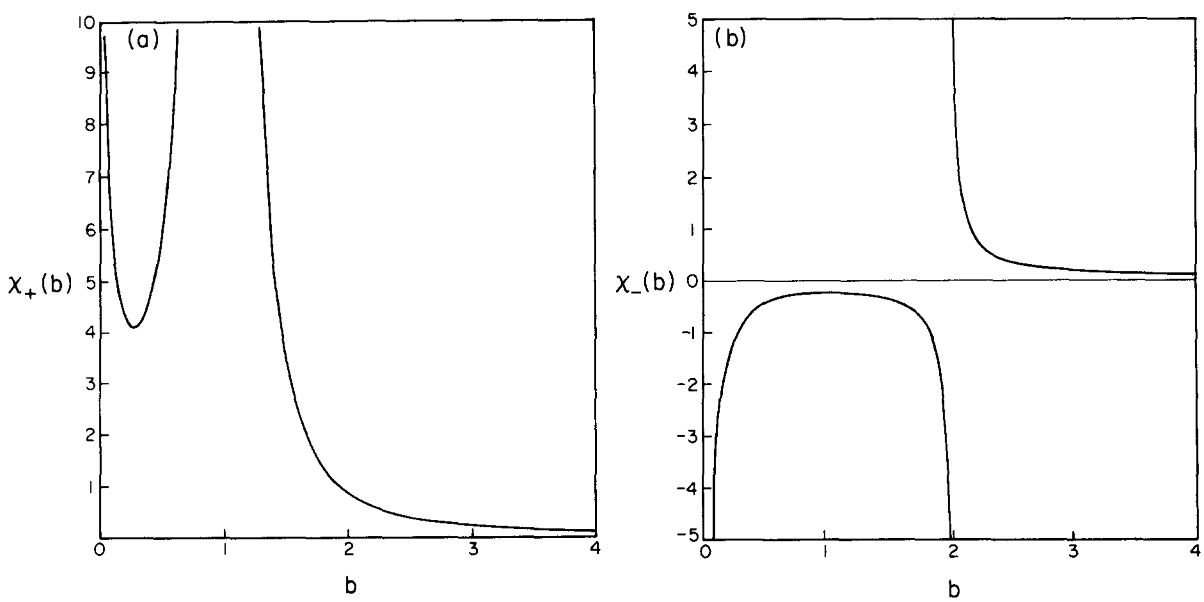

FIG. 2. Plot of $\chi_{\pm}(b)$.

Recall that the ballistic term represents the transient response of a plasma electron to the leading edge of the incident wave front. Referring to Fig. 3, it is apparent that the sideband lifetime is limited because electrons streaming along the magnetic field leave the scattering volume in time $t_0 \sim L/v_\parallel$, carrying with them the transient information imparted by the wave front. Thus in order to recreate the ballistic term, the pump wave must be pulsed repetitively. If one wishes to follow the time evolution of the plasma (instead of obtaining a single "snapshot" at one time), the repetition rate should be of order $\sim(1/t_0)$ in order to continuously recreate the ballistic term. However, as discussed below, the high power required of the pump wave limits the number of pulses per unit time that can be achieved in practice.

Since the pump is pulsed, the Fourier decomposition of the pump will consist of a wide spectrum of frequency components, potentially masking the weaker sideband signal at $\omega \pm \omega_c$. Suppose, for example, that one cycle of the received signal $f(t)$ is represented by

$$f(t) = A_p e^{i\omega t} + a_s e^{i(\omega - \omega_c)t}, \quad -t_0/2 < t < t_0/2,$$

where A_p is the amplitude of the pump and a_s is the amplitude of the sideband at $\omega - \omega_c$ ($A_p \gg a_s$). If the signal is Fourier analyzed over an interval τ (where $\tau < t_0$), then

$$g(\Omega) = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) e^{-i\Omega t}\, dt,$$

or

$$g(\Omega) = A_p \operatorname{sinc}\left(\frac{(\omega - \Omega)\tau}{2}\right) + a_s \operatorname{sinc}\left(\frac{(\omega - \omega_c - \Omega)\tau}{2}\right).$$

The value of the transformed signal at $\Omega = \omega - \omega_c$ is

$$g(\omega - \omega_c) = A_p \operatorname{sinc}(\omega_c \tau/2) + a_s. \tag{21}$$

In the worst case, where $\sin(\omega_c\tau/2) = 1$, the sideband signal begins to dominate the pump signal when $a_s > 2A_p/\omega_c\tau$. Since $P_{\text{scatt}}/P_i = (a_s/A_p)^2$, the sidebands begin to dominate when

$$P_{\text{scatt}}/P_i > (2/\omega_c \tau)^2. \tag{22}$$

Combining Eqs. (22) and (20) yields the condition

$$P_i > 8\pi\left(\frac{c}{r_e} m v_\parallel^2\right)\left(\frac{1}{\chi}\frac{R}{\Delta x}\right)^2 \frac{(c/\omega_p)^4}{AL^2}, \tag{23}$$

where Δx, A, and L refer to the dimensions of the scattering volume (see Fig. 3). For an order-of-magnitude estimate of the required power, consistent with typical fusion parameters, suppose that $A \sim 100$ cm^2, $L \sim 10$ cm, $R/\Delta x \sim 200$ ($\Delta x \sim 0.5$ cm, $R \sim 100$ cm), $\chi \sim 0.1$ ($\omega/\omega_c \sim 3$), $\omega_p \sim 5.7 \times 10^{11}$ sec^{-1} (density $\sim 10^{14}$ cm^{-3}, $\omega_p/2\pi \sim 90$ GHz), $v_\parallel \sim 0.14c$ (electron temperature ~ 10 keV), which yields $P_i \gtrsim 13$ MW or $P_i\tau \gtrsim 31$ mJ/pulse ($\tau = L/v_\parallel \sim 2.4$ nsec).

An advantage is gained by varying the interval τ and averaging over many cycles. Suppose that the duration of interval number k is in the range $\tau_0 - \Delta\tau/2 < \tau_k < \tau_0 + \Delta\tau/2$. If an averaging is performed over N cycles, then Eq. (21) becomes

$$\langle g \rangle = \frac{A_p}{N} \sum_{k=1}^{N} \operatorname{sinc}\left(\frac{\omega_c \tau_k}{2}\right) + a_s.$$

For large values of N ($N \gtrsim \omega_c \Delta\tau$), the discrete sum can be replaced by an integral:

$$\langle g \rangle \approx \frac{A_p}{\Delta\tau} \int_{\tau_0 - \Delta\tau/2}^{\tau_0 + \Delta\tau/2} \operatorname{sinc}\left(\frac{\omega_c \tau}{2}\right) d\tau + a_s.$$

Note that the averaging must be performed before the signal is rectified at the detector. Such an averaging scheme is possible in principle but may be technically difficult to achieve. Clearly the reduction in required power is made at the expense of degrading the temporal resolution; density fluctuations which occur on a time scale short in comparison to $Nt_0 \sim (\omega_c \Delta\tau) t_0$ cannot be resolved. If $\Delta\tau/\tau_0 \ll 1$ is assumed, the above integral can be evaluated approximately and one obtains the condition

$$P_{\text{scatt}}/P_i \gtrsim [8/(\omega_c \Delta\tau)(\omega_c \tau_0)]^2. \tag{24}$$

Comparing Eqs. (22) and (24) shows that this averaging scheme reduces the required power by a factor of roughly $(4/\omega_c \Delta\tau)^2$. For $\omega_c \tau_0 \sim 2100$ ($B = 50$ kG) and $\Delta\tau/\tau_0 \sim \frac{1}{3}$, the above power estimate is reduced by a factor of 3.1×10^4 to give $P_i \gtrsim 420$ W or $P_i \tau_0 \gtrsim 1$ μJ/pulse.

Finally, recall that in our calculations, the pulsed wave source was represented in an idealized fashion by way of the Heaviside function, $E_y \sim H(t - x/c)\cos[\omega(t - x/c)]$, which corresponds to a switching rise time of zero. If this idealization is relaxed, it is found that the magnitude of the ballistic term is still given accurately by our analysis as long as the rise time is short in comparison to one period of the driving waveform.

IV. SUMMARY

A diagnostic method has been proposed which relies on a monochromatic, high-frequency ($\omega \gtrsim \omega_{ce}$), pulsed wave source to determine the density profile across a magnetic field gradient. A nonlinear mixing occurs between the pulsed pump wave and the electron cyclotron motion resulting in radiation at the sideband frequency $\omega \pm \omega_{ce}$, scattered coherently in the forward direction. The intensity of the radiation is proportional to the electron density at the point of scattering, so that a measurement of the sideband frequency spectrum gives the density profile along the path traversed by the pump wave. Although the diagnostic has many attractive features, the feasibility is limited by the intense power and the rapid rise time required of the pulsed wave source. For parameters relevant to fusion plasmas, and for the particular scattering geometry suggested here, the technique requires, roughly, 30 mJ pulses of 2 nsec duration. The required power can be reduced by a factor of roughly 10^4 by applying signal averaging techniques before detection. The rise time of the pulsed wave source must be short in comparison to one period of the incident wave.

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