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by

Wilhelm H. Kegel

Technical Report No. 26

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A MODEL FOR PULSE STIMULATED EMISSION FROM A PLASMA

Wilhelm H. Kegel

Recently the observation of echoes radiated from a plasma was reported[1]. One type of experiment was to excite the plasma at the electron gyrofrequency by two short pulses with a time separation τ. The echo radiation was then observed at a time τ after the second pulse. This effect is analogous to the well known spin echo[2].

The aim of the present paper is to show theoretically for a similar experiment with a homogeneous magnetic field that there is another effect giving rise to a whole set of radiation peaks after the second pulse.

We consider a magneto plasma which is so dilute that the single particle approach is valid. For simplicity the plasma dimensions are assumed to be small compared to the wave length of the cyclotron radiation. The radiation of this plasma at the electron gyrofrequency depends on the relative phase of the particles and is, if we ask for the radiation in the direction of B, proportional to the quantity

$$\Phi(t) = \left(\sum_{i=1}^{N} r_i\right)^{-2} \left\{\left(\sum_{i=1}^{N} r_i \cos(\omega_c^i t + \alpha_i)\right)^2 + \left(\sum_{i=1}^{N} r_i \sin(\omega_c^i t + \alpha_i)\right)^2\right\} \quad (1)$$

where N is the number of particles considered, r_i is the Larmor radius of the i-th particle, ω_c^i is its gyrofrequency, and α_i its phase at time $t = 0$. In order to have an effect like the echo[1], the quantity Φ must depend on time and have a sharp maximum at the time the echo occurs. This means that the phase correlations of the particles have to be time dependent. In the approximation used here this requires the introduction of individual gyrofrequencies for the different particles. One way for this to occur is through the inhomogeneities of the magnetic field by analogy with the theory of the spin echo[2]. Another possibility, which will be followed in the present paper, is to assume the magnetic field to be homogeneous and to treat the gyrofrequency as a function of the energy according to the relativistic mass effect.

We consider the plasma to be excited at the gyrofrequency by two pulses which are so short that for the acceleration of the electrons the differences of their gyrofrequencies are unimportant. We are interested in the radiation after the second pulse and treat the problem by means of the following model: Before the first pulse all particles are assumed to have the same energy and, by that, the same Larmor radius r_o, but different equally distributed phases. Then we have in the r-φ-diagram (r = Larmor radius, φ = phase) the picture given in Fig. 1a. It is convenient to consider this diagram in a coordinate system rotating with an average gyrofrequency $\bar{\omega}_c$ (defined by equations (3) and (8)). Then φ is the phase difference with respect to one specified particle. Directly after the first pulse we have the r-φ-diagram given in Fig. 1b, the particles equally distributed on the small dashed circle. We now assume that the energy the particles received from the pulse is large compared to the initial (thermal) energy, i.e., $R \gg r_o$. Then all particles in Fig. 1b have almost the same phase and we may approximate the distribution on the circle given as dashed curve, by a uniform distribution on the diameter given as a solid line. This means, the only effect of the initial energy we keep is that the particles have different energies after the first pulse according to their phase at the onset of the pulse.

As time proceeds, phase differences arise between the particles according to the differences in their gyrofrequencies. The gyrofrequency in our model depends on the energy, i.e., on r, and is in general given by[3]

$$\omega_c^i(r_i) = \frac{e\,c\,H}{E^i(r_i)} \tag{2}$$

where $E^i(r_i)$ is the relativistic energy. With the assumption $v^i \ll c$ follows from (2) for the differences in ω_c:

$$\Delta\,\omega_c^i = \frac{\bar{\omega}_c^3\,R^2}{2c^2}\,(1 - \frac{r_i^2}{R^2}) \tag{3}$$

if we attribute $\Delta\,\omega_c^i = 0$ to particles with $r_i = R$. Correspondingly, the relative phase at a time τ after the first pulse is

$$\varphi(r,\tau) = A\,\tau\,(1 - r^2/R^2) \tag{4}$$

with $A = \bar{\omega}_c^3\,R^2/2c^2$.

Directly after the second pulse, which acts on the electrons at time τ after the first pulse, we have the r-φ-diagram given in Fig. 1c. The coordinates r and φ just before the second pulse are transformed into r^* and φ^* given by:

$$r^{*2}/R^2 = r^2/R^2 + D^2 + 2(r/R)\,D\,\cos(\varphi(r,\tau) - \varphi_o) \tag{5}$$

$$\varphi^*(t=0) = \varphi_o + \beta \tag{6}$$

$$\sin\beta = (r/r^*)\,\sin(\varphi - \varphi_o) \tag{7}$$

$$\Delta^*\,\omega_c = A(B - r^{*2}/R^2) \tag{8}$$

$$\varphi^*(t) = \varphi^*(t=0) + \Delta^* \omega_c t \tag{9}$$

where φ_o is the phase difference of the electric field of the second pulse with respect to a particle with $r = R$, B is an arbitrary constant which defines the particle with respect to which $\Delta^*\omega_c$ is measured, D gives the strength of the second pulse relative to the first ($D = 1$ for equal pulses), and t is now the time measured from the second pulse. We now can calculate $\Phi(t)$ for the time after the second pulse. The sums in (1) can be substituted by integrals over r from $R - r_o$ to $R + r_o$, if we express $\varphi^*(t)$ and r^* as functions of r.

In order to simplify these integrals to get qualitative results, we make use of the assumption $r_o \ll R$ and approximate r^2/R^2 by $1 + 2u$. Then we have for Φ :

$$\Phi(t) = \left(\int dr\ r^*\right)^{-2} \left\{ \left(\int_{-r_o/R}^{+r_o/R} du \left\{ D \cos[f(u)] + (1+u) \cos[f_1(u)] \right\} \right)^2 + \left(\int_{-r_o/R}^{+r_o/R} du \left\{ D \sin[f(u)] + (1+u) \sin[f_1(u)] \right\} \right)^2 \right\} \tag{10}$$

with

$$f(u) = \varphi_o + At\,[B - 1 - D^2 - 2u - 2D(1+u) \cos(-2A\tau u - \varphi_o)]$$

$$f_1(u) = -2A\tau u - \varphi_o + f(u) \tag{11}$$

$f(u)$ is a rapidly varying function except at the points

$$u_n = \frac{2n\pi + \varphi_o}{2A\tau} \qquad \text{and} \qquad u_m = \frac{(2m+1)\pi + \varphi_o}{2A\tau} \tag{12}$$

Since $f(u)$ appears as the argument of a sine or cosine, the main contribution to the integrals in (10) comes from these points. $\Phi(t)$ is a maximum if t is chosen so that $f(u_n) - f(u_{n-1}) = 2i\pi$ or $f(u_m) - f(u_{n-1}) = 2i\pi$ or $f(u_m) - f(u_{m-1}) = 2j\pi$, n, m, i, j being integers. So the maxima occur at times

$$t_i = i \frac{\tau}{1+D} \qquad \text{and} \qquad t_j = j \frac{\tau}{1-D} \tag{13}$$

More detailed information about the function $\Phi(t)$ has been obtained by numerical computations using the "exact" equations (1) and (4) - (9). Figure 2 gives as an example the result obtained for the parameters $A\tau = 50$, $r_o/R = .1$, $\varphi_o = \pi$ and $D = 1$. We see the sharp maxima at times given by (13), the peak intensities being proportional to $1/t$.

The parameters in the example given in Figure 2 are chosen such that the conditions are similar to those in the experiment of Hill and Kaplan[1]. An $A\tau = 50$ implies $\bar{\omega}_c\tau = 100\ c^2/\bar{v}^2$, where $\bar{v}$ is the average velocity of the electrons after the first pulse. If we choose, as in Reference 1, $\bar{\omega}_c = 5 \cdot 10^{10} \sec^{-1}$ and $\tau = 2 \cdot 10^{-7}$sec, we find $\bar{v}/c = .1$, which can be achieved for a plasma cross-section of 2.3 cm^2 (corresponding to an X-band waveguide) by a 10 W pulse in $6 \cdot 10^{-8}$sec. The essential difference with the experiment discussed in Reference 1 is the assumption of a very homogeneous magnetic field, or more precisely, that $\Delta_{inh}\omega_c \cdot \tau < 1$. With the parameters assumed above, this requires the inhomogeneities to be less than 10^{-4}. On the other hand, the echo effect[1] which gives rise to a radiation maximum at $t = \tau$ depends essentially on the existence of field inhomogeneities[1].

When plasma dimensions no longer small compared to the wavelength are considered, one sees that the k-vectors of the exciting pulses have to be perpendicular to $\underset{\sim}{B}$. If $\underset{\sim}{k}$ and $\underset{\sim}{B}$ are parallel, particles excited at different phases can interchange their places by moving along the lines of force, giving rise thereby to statistical phase differences and spoiling the correlations which have been generated.

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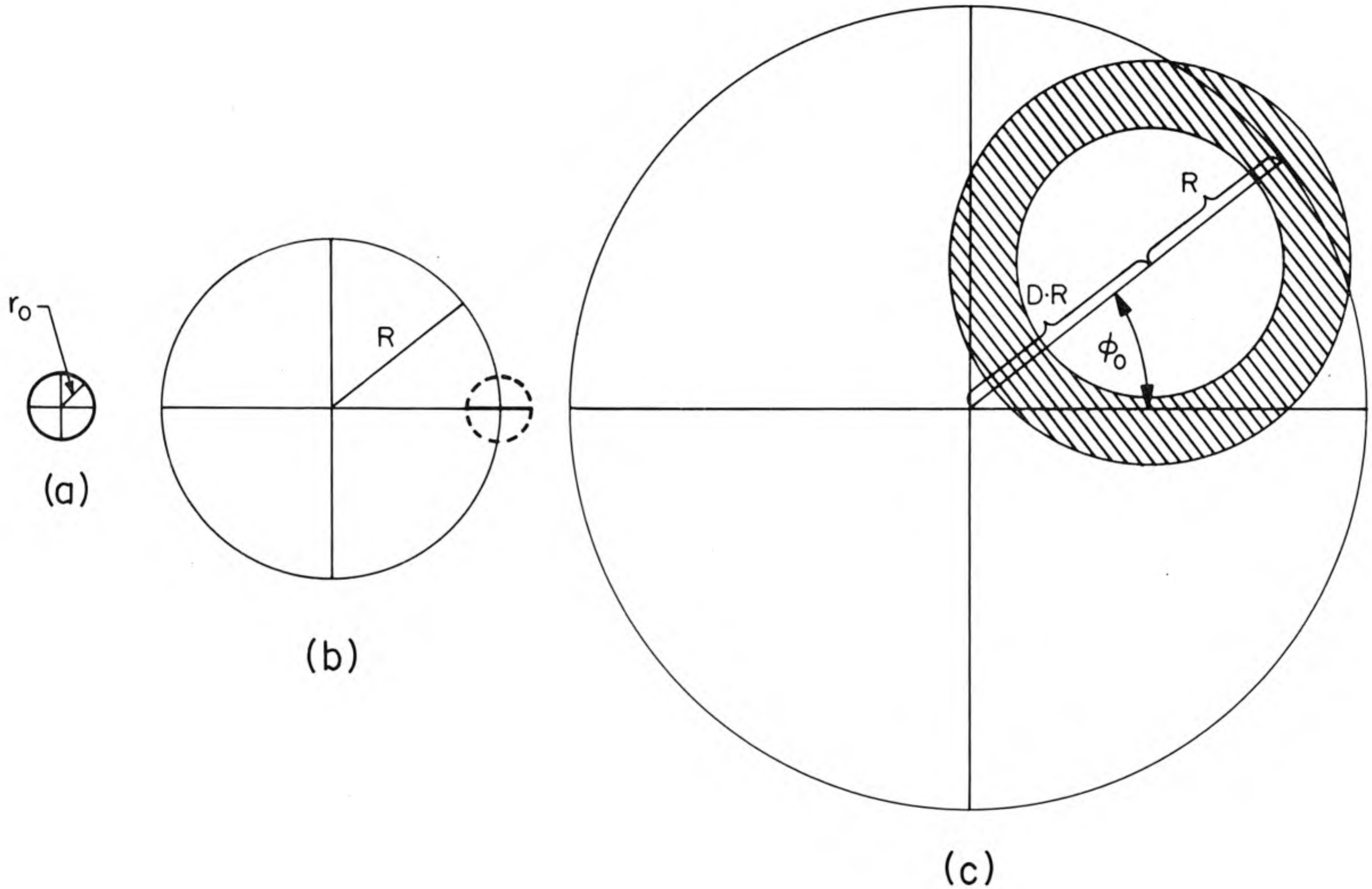

Figure 1. r-φ-diagram: a) before the first pulse, b) at the end of the first pulse, c) at the end of the second pulse.

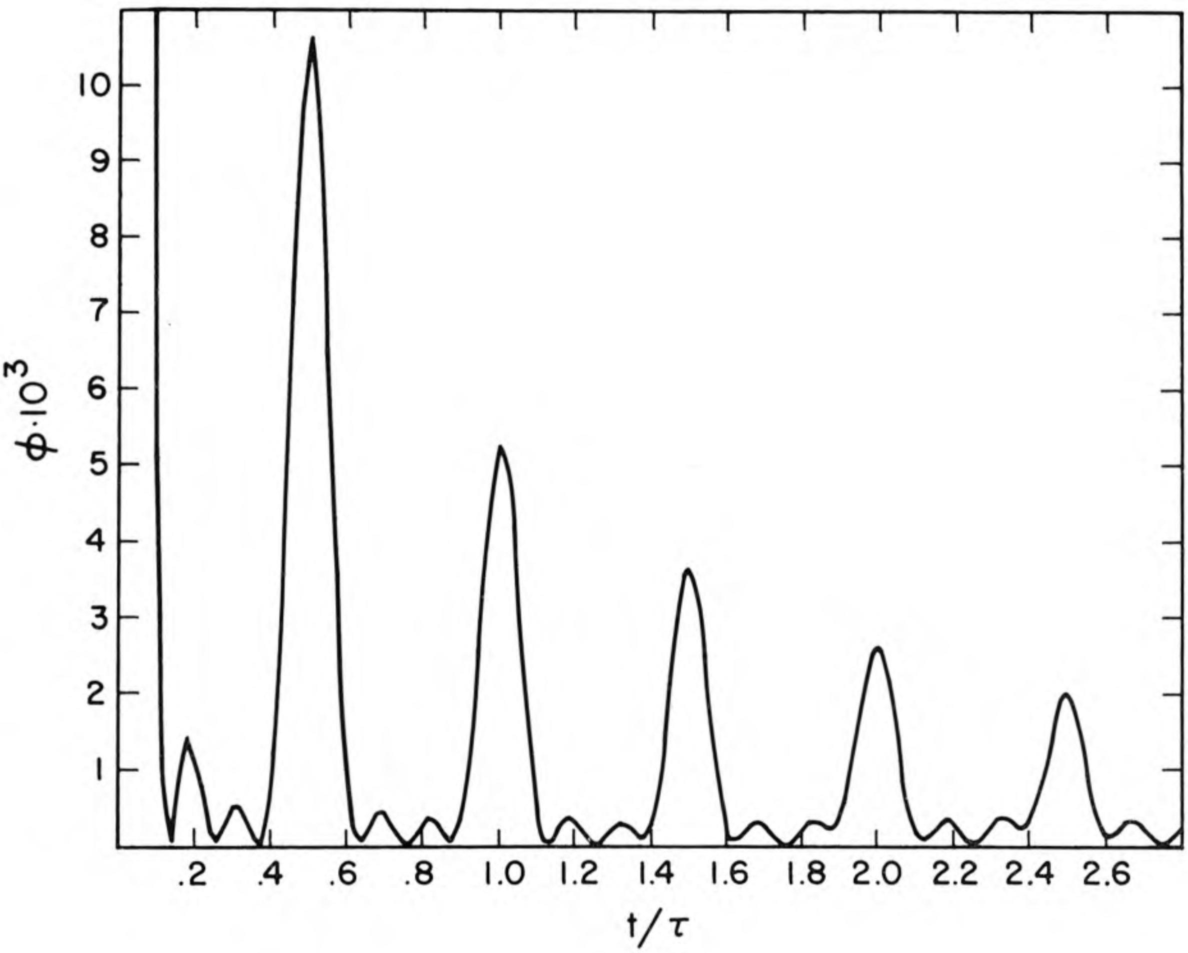

Figure 2. The plasma radiation after the second pulse as a function of time.

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A plasma in a homogeneous magnetic field, excited by two short pulses at the gyrofrequency is considered. It is shown that the relativistic mass effect leads to radiation maxima after the second pulse at times that are multiples of $\tau/1 \pm D$, where τ is the time separation of the exciting pulses and D is their relative strength.

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