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Roy W. Gould

Technical Report No. 29 December 1965

CALIFORNIA INSTITUTE OF TECHNOLOGY

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ECHO PHENOMENA *

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It has recently been shown experimentally [1] that a plasma excited by multiple pulses of radiation near the electron cyclotron frequency can exhibit echo phenomena, similar to that which occurs in a spin system [2]. We wish to point out two mechanisms by which echoes can arise in classical oscillator systems, and in particular in a system of gyrating charged particles, as in a plasma. Although a nonlinear treatment of the system is necessary to fully understand the echo phenomena, a linear treatment is nevertheless very illuminating for it shows how the echo may arise.

Consider the response of an ensemble of non-interacting charged particles to a pulsed electric field which rotates with angular velocity $\omega \approx \omega_c = qB/m$ about the axis of the static magnetic field. In a velocity space system (primed) which rotates with the electric field, E' is constant and the linearized equation of motion is

$$\dot{v}' + \omega_c' \times v' = (q/m)E' , \qquad (1)$$

where $\omega_c' = \omega_c - \omega$ is assumed small and can be neglected during the pulses. The first pulse, of duration $t_1 (\omega_c' t \ll 1)$, causes all particles to acquire a velocity $v_1' = qE_1' t_1/m$. (See fig. 1 for behaviour in v' space.)

Following the first pulse, the velocity vector of each particle rotates with angular velocity ω_c' which, due to inhomogeneities in the magnetic field, may be different for different particles. After a time $\tau \gg 1/\langle \omega_c'^2 \rangle^{\frac{1}{2}}$, the tips of the velocity vectors become nearly equally distributed (solid circle in fig. 1b) and the macroscopic current has decayed substantially due to "phase mixing". Velocity vectors at B, for example, will have reached B by having rotated through angles $\frac{1}{4}\pi \pm 2\pi n$, $n = 0, 1, 2, \ldots$. A second radio frequency pulse changes v' of each particle by $v_2 = qE_2' t_2/m$. (For simplicity only we consider two identical pulses $v_2' = v_1'$.) Fig. 1c shows the

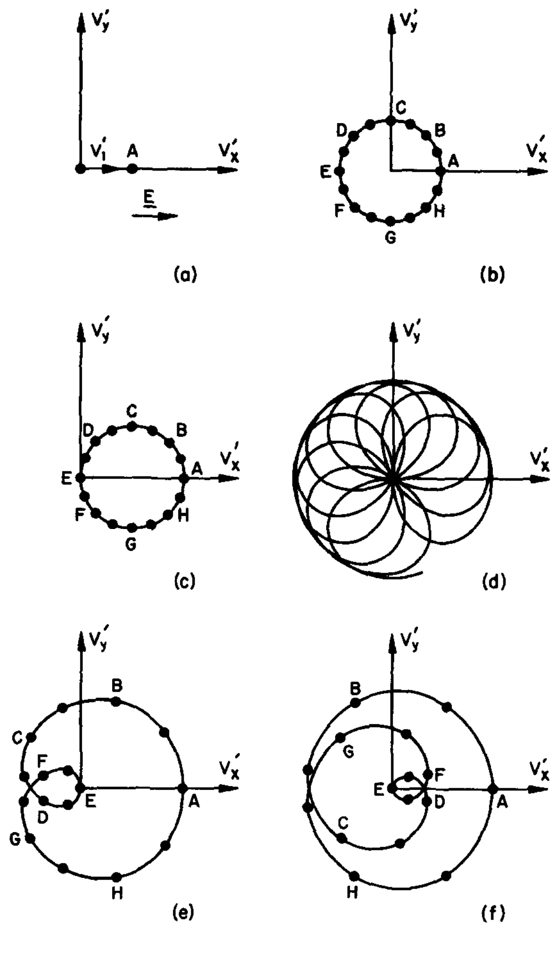

Fig. 1. Distribution of particle velocities in a rotating velocity space a) after the first pulse: b) before the second pulse; c) after the second pulse; d) prior to echo; e) at time of first echo; f) at time of second echo.

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new velocity distribution and there is again a macroscopic current, since each particle has $v'_x > 0$. Following the second pulse, each velocity vector rotates with its own angular velocity ω'_c producing, after a time ($< \tau$) the distribution of fig. 1d, in which the macroscopic current is essentially zero. The successive loops in (1d) are produced by particles which reached ABC... in (1b) by encircling the origin $0, 1, 2, \ldots, n$ times. It is readily shown that when $t = 2\tau$ the particles rebunch in velocity space as in fig. 1e; i.e., all particles which were at B, for example, in (1c) are again together at B in (1e), irrespective of the number of times they encircled the origin in reaching B. Despite the preferential bunching of particles in the region of $v'_x < 0$, their contribution to the macroscopic current is precisely cancelled by the contribution from the relatively fewer particles with $v'_x > 0$. However, certain types of nonlinearities may spoil this cancellation and lead to a macroscopic current and hence to an echo at $t = 2\tau$. After $t = 2\tau$ the velocities again disperse as in (1d) and rebunch periodically at $3\tau, 4\tau, \ldots$. Fig. 1f shows distribution at $t = 3\tau$. The macroscopic current is still zero at $t = n\tau$, unless some nonlinearity spoils the exact cancellation.

Two general types of nonlinearities should be considered: (A) those in which the effectiveness of the driving pulse is dependent upon the response already produced, (B) those which render the frequency of the oscillator amplitude-dependent (anharmonic oscillator).

A) Driving force effects. When charged particles in a static magnetic field are subjected to a spatially *inhomogeneous* electric field, the effectiveness of the electric field decreases with increasing orbit size. For a plane electromagnetic wave traveling with velocity c the effectiveness of the field is reduced by a factor $J_0(\omega\rho/c) = J_0(v/c)$, where ρ and v are the particles' gyro radius and velocity in the laboratory. Such an effect implies that the second pulse will shift particles HAB to the right by a slightly lesser amount than it shifts particles DEF to the right. The curve of fig. 1c is therefore elliptical and an exact cancellation no longer occurs at $t = 2\tau$. Thus there is an echo whose amplitude may be shown to be $v_1 v_2/8c^2$ times as large as the response to the second pulse. Subsequent echoes are proportional to higher powers of v^2/c^2 and are therefore much weaker. It may also be shown that, aside from the amplitude factor, the Fourier transform of the echo pulse is just the distribution function of gyrofrequencies, so that as in the spin case [2], a very inhomogeneous field leads to a short echo pulse and vice versa.

B) Anharmonic effects. The relativistic mass effect introduces a change in gyrofrequency $\Delta\omega_c/\omega_c = -v^2/2c^2$ which depends upon the particles' speed, or energy. As a result, velocity vectors along HAB acquire a slight additional rotation relative to those along DEF, which represent particles of lower energy. This causes velocity vectors along HAB to rotate toward negative (or positive) v'_y, thus spoiling the exact cancellation at $t = 2\tau$ and producing a macroscopic echo current. Similarly echoes at $t = 3\tau, 4\tau, \ldots$ are produced [3]. An energy-dependent gyrofrequency may arise in still other ways. For example, when the static magnetic field is spatially inhomogeneous, gradients give rise to guiding center drifts along surfaces of constant B. When the second spatial derivatives of B do not vanish, the effective gyrofrequency depends quadratically on orbit size, hence upon the energy. Whether this phenomenon is more important than the relativistic mass change depends upon the size of ρ^2/L^2 to v^2/c^2 (L = scale length for curvature of B-field).

In the plasma case discussed above, both types of nonlinearities A and B are dependent on v^2/c^2. However, when an anharmonic effect (B) is involved, the relevant quantity is actually $\omega_c\tau\, v^2/2c^2$, the anharmonic shift in orbit *phase* between pulses. Since, in general $\omega_c\tau \gg 1$, this effect can be important even for very small v^2/c^2.

Finally, we emphasize the generality of this result. Although the cyclotron echo has been used as an example, photon echoes have also been observed [4] and we expect a wide class of nonlinear oscillator systems to exhibit echoes, in particular *anharmonic* oscillator systems, provided that the oscillator lifetimes are not too much smaller than the interpulse spacing τ. To generalize the above discussion one can make use the substitution $(v_x, v_y) \rightarrow (q, p)$, the generalized coordinate and momentum of the oscillator. A single oscillator traverses a circular orbit in q-p plane (phase space) with angular velocity ω_c. Figs. 1a-f then describe the motion of an ensemble of oscillator in a (primed) phase space system which rotates with angular velocity ω (the driving frequency) with respect to the origin system. Observable quantities of interest might be $\langle p(t)\rangle$ or $\langle q(t)\rangle$.

It is a pleasure to acknowledge discussions with W. H. Kegel and P. E. M. Vandenplas.

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Due to small nonlinearities, an ensemble of oscillators may exhibit echoes when subjected to multiple pulses of radiation near their resonant frequency. Cyclotron echoes from gyrating charged particles is discussed as an example.

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