Supporting Information for "The role of the Southern Ocean in abrupt transitions and hysteresis in glacial ocean circulation"

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Contents of this file

1. Perturbation test for bi-stability of hysteresis loop
2. Figures S1 to S5

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Perturbation test for bi-stability of hysteresis loop

To test the stability of the hysteresis loop, we have done a series of perturbation experiments to z_2 in both the Atlantic and Pacific. We have chosen to perturb the z position instead of the y position of interface 2 because the timescale for the diffusive adjustment of z is much longer than the eddy-induced relaxation of y. This longer adjustment timescale means that the perturbation has a larger affect. For a NADW flux of 8.75 Sv (in the middle of the hysteresis loop for $\phi = 1$), we displace the z position of interface 2 downward from its steady state on the upper (down-going) limb of the loop by 100–500 m and allow it to relax back towards equilibrium. We run the simulation for 50, 200, and 500 years so we can ascertain the direction that the model is relaxing toward (since none of them fully reach steady state). We find that there is indeed a bifurcation in the middle of the hysteresis loop—displacements of 100–300 m relax back towards the original state, while 400 and 500 m perturbations relax toward the other limb of the loop (Figure S1). If we look at the same perturbation experiment in z space instead of y space (Figure S2), we see that the bifurcation in z_2 occurs once the initial z position enters the κ transition region (where the κ transition region is defined by the parameter d, see Appendix Table A1).

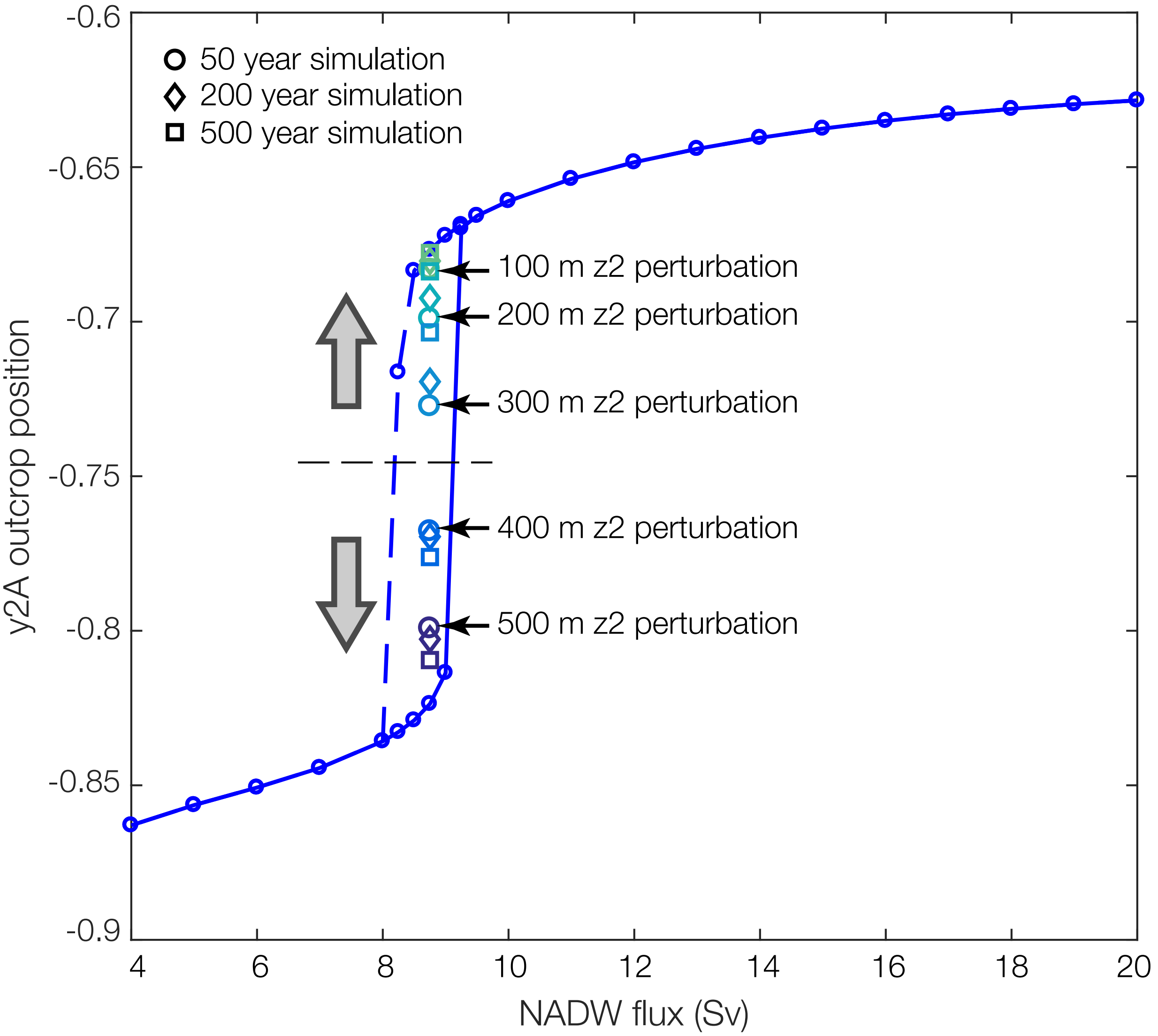

Figure S1. Perturbation experiments to test the stability of the hysteresis loop. Starting with the steady-state interface positions for a NADW flux of 8.75 Sv on the upper limb of the hysteresis loop, z_2^A and z_2^P were displaced and then allowed to relax for 50, 200, and 500 years (y_2^A for each of these simulations are plotted in circles, diamonds and squares, respectively).

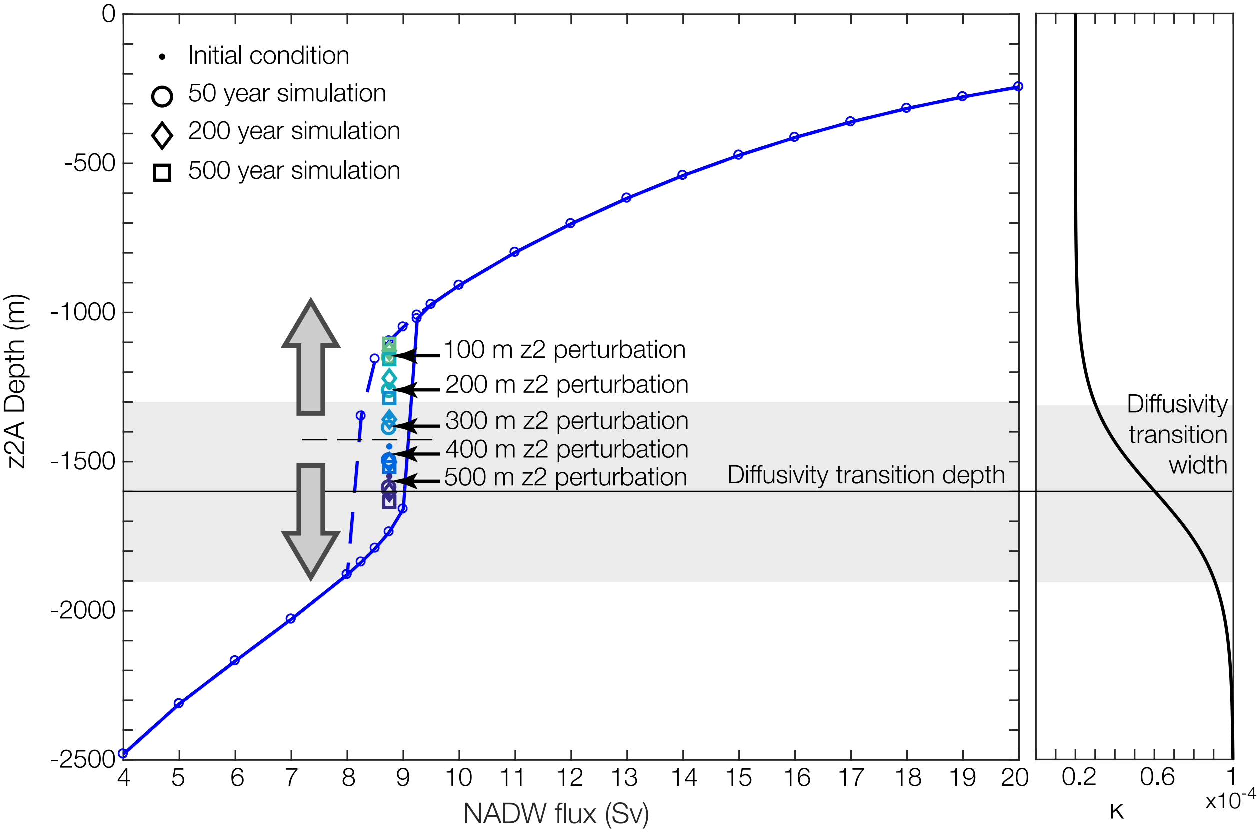

Figure S2. Perturbation experiments in z space. Same experiment as shown in Figure S1. Vertical diffusivity profile is also shown with diffusivity transition depth (1600 m) and diffusivity transition width ($d = 300$ m) marked.

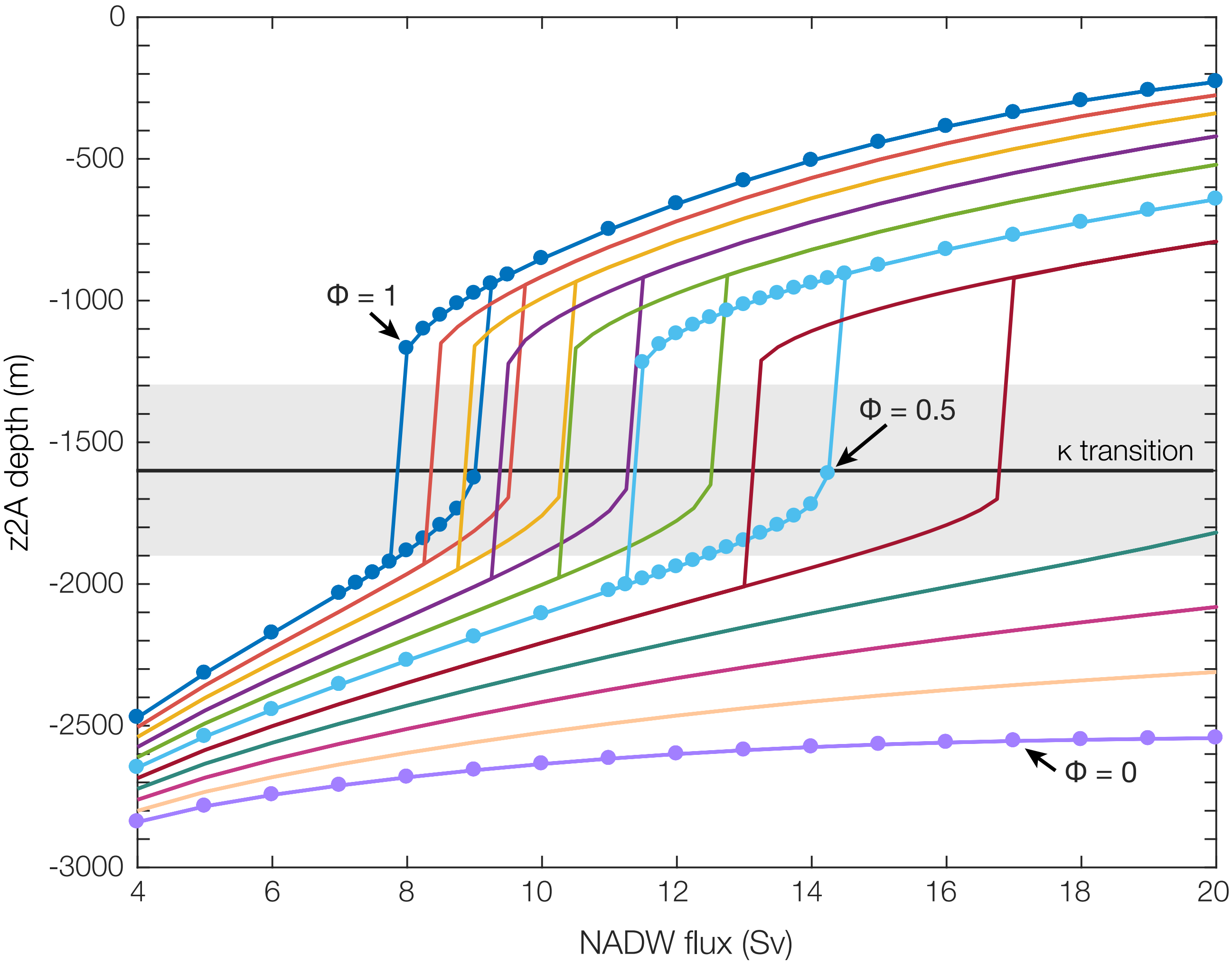

Figure S3. Hysteresis loops for all values of ϕ plotted as z_2^A. The depth of the kink in the vertical diffusivity profile is at 1600 m (thick horizontal line) and the width of the transition is ±300 m. For all of these experiments, y_{si} is at -0.5ℓ.

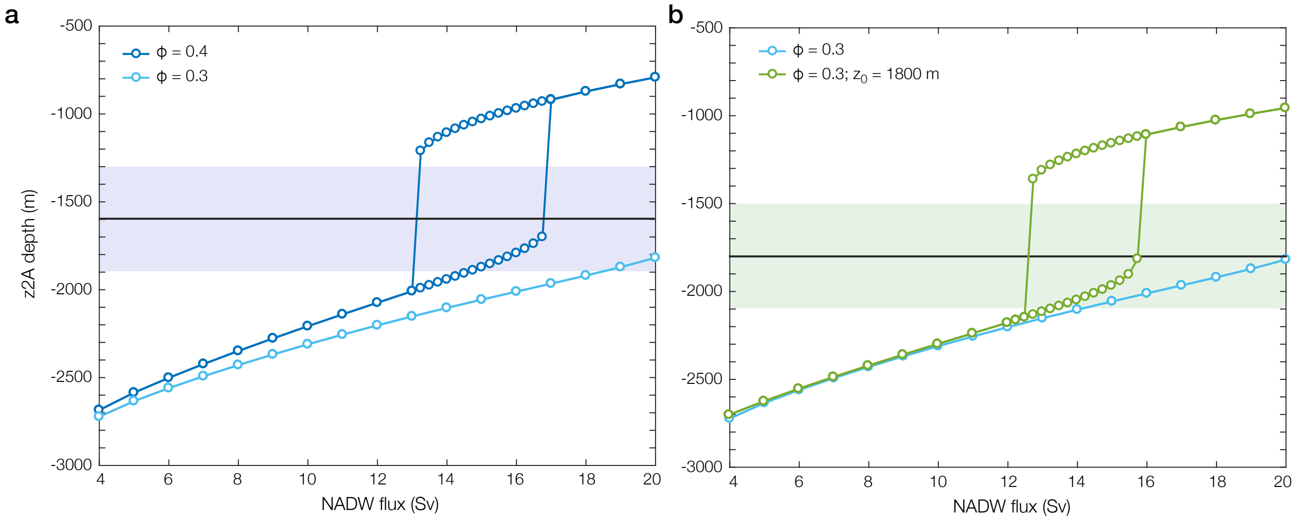

Figure S4. Comparison of hysteresis loops at the edge of they hysteresis stability region for different κ transition depths. A) Plots of z_2^A versus NADW flux for $\phi = 0.4$ (dark blue) and 0.3 (light blue) with a kink at 1600 m (horizontal line and shading). B) Comparison of loops for $\phi = 0.3$ with the kink at 1600 m (light blue) versus 1800 m (green with horizontal line and shading.)

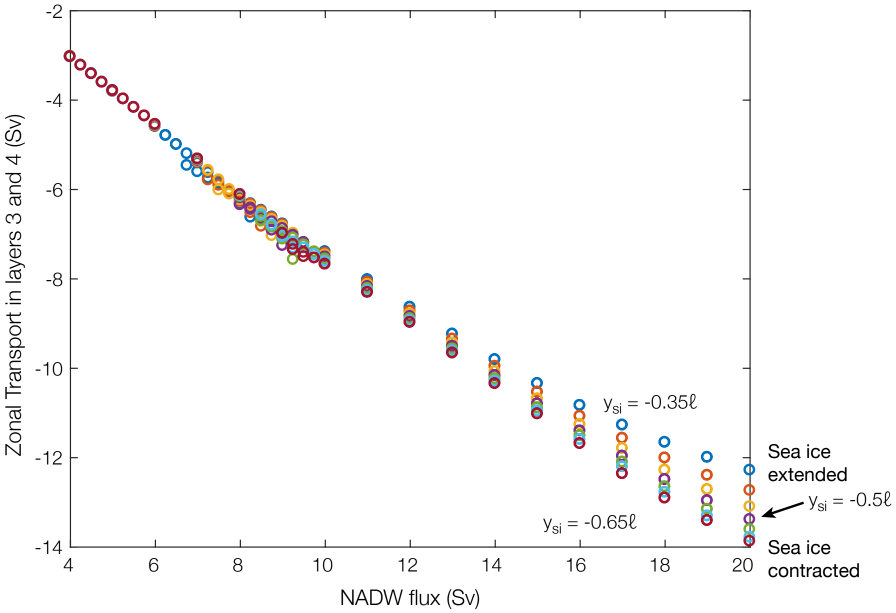

Figure S5. Assessment of circulation configuration for all y_{si} experiments. The relationship between NADW flux and circulation configuration is quite similar between all of the y_{si} experiments compared to the ϕ experiments (see Figure 11). The experiment with $y_{si} = -0.5$ is (labeled) is the same as the $\phi = 1$ experiment in Figure 11.