Supplemental Material
Floquet second-order topological insulators from nonsymmorphic space-time symmetries

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TWO DIMENSIONAL FLOQUET SOTI IN CLASS AIII UNDER A TWO-STEP DRIVE

In this section, we consider a two-step driven Floquet system first introduced in Ref. [1], in which the authors showed that the model can harbor anomalous Floquet edge modes protected by the combination of chiral symmetry and time-glide symmetry. We show that this model describes a Floquet SOTI with anomalous Floquet corner modes, when a pair of edges in the system are mapped onto each other via reflection about the time-glide plane.

Within one full period in time, this system is driven by two static Hamiltonians $H_\pm$ in the each half period, defined as

$$H_\pm(k_x, k_y) = \cos k_x \tau_x + \sin k_x \tau_y + J(\cos k_y \sigma_x \pm \sin k_y \tau_y \sigma_y), \tag{S1}$$

where the chiral and the time-glide symmetries are realized by $\mathcal{S} = \tau_z\sigma_z$ and $\mathcal{M} = \tau_x$.

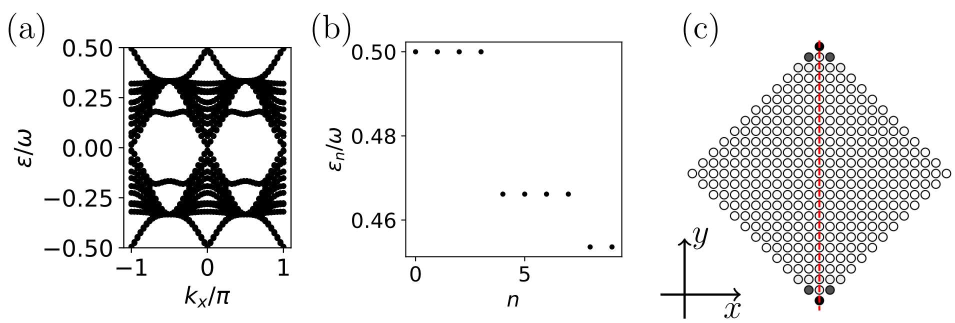

Figure S1. (a) Quasienergy spectrum of the time-glide symmetric Floquet system defined in Eq. (S1) as a function of momentum k_x, when periodic boundary condition along x, and open boundary condition along y are imposed. There are 13 sites along y in the calculation. (b) Quasienergy spectrum for the Floquet modes near $\omega/2$, when the system contains pairs of edges which are mapped onto each other via reflection about the time-glide plane, see the geometry in (c). (c) Support of the wave function for one of the Floquet mode at $\omega/2$. The parameters are $\omega = J = 3$.

In SFig. S1(a), we reproduce the quasienergy spectrum as the one in Ref. [1], when periodic boundary condition is imposed along x. We see that both at $k_x = 0$ and at $k_x = \pm\pi$, the system has gapless Floquet edges states at $\omega/2$. Hence, when we cut the system such that there are two pairs of edges that are mapped onto each other via the reflection about the time-glide plane, anomalous Floquet corner modes at $\omega/2$ appear, see (b). Moreover, since there are two gapless modes, there are four degenerate corner modes at $\omega/2$, twice as many as in the ones in the harmonically driven model introduced in the main text. In (c), we show the support of the wave function for one of the degenerate corner modes.

EMERGENT REFLECTION SYMMETRY IN THE FREQUENCY-DOMAIN FORMULATION

In this section, we show that the full enlarged Hamiltonian in the frequency domain aquires a reflection symmetry, whenever the original time-periodic Hamiltonian has a time-glide symmetry. Moreover, the reflection plane coincide with the time-glide plane.

Let us write down the enlarged Hamiltonian in the frequency domain explicitly as

$$\mathcal{H} = \begin{pmatrix} \ddots & & & & \\ & H_0+\omega & H_1 & H_2 & \\ & H_1^\dagger & H_0 & H_1 & \\ & H_2^\dagger & H_1^\dagger & H_0-\omega & \\ & & & & \ddots \end{pmatrix} \tag{S2}$$

with

$$H_n(\mathbf{k}) = \frac{1}{T}\int_0^T dt\, H(\mathbf{k},t)e^{-in\omega t}. \tag{S3}$$

Let us first summarize how time-reversal $\mathcal{T}$, particle-hole $\mathcal{C}$ and chiral $\mathcal{S}$ symmetries transform $H_n(\mathbf{k})$ [2]:

$$\mathcal{T}H_n(\mathbf{k})\mathcal{T}^{-1} = H_n^*(-\mathbf{k}) \tag{S4}$$

$$\mathcal{C}H_n(\mathbf{k})\mathcal{C}^{-1} = -H_{-n}^*(-\mathbf{k}) \tag{S5}$$

$$\mathcal{S}H_n(\mathbf{k})\mathcal{S}^{-1} = -H_{-n}(\mathbf{k}). \tag{S6}$$

Hence, one can define the effective time-reversal $\mathscr{T}$, particle-hole $\mathscr{C}$ and chiral $\mathscr{S}$ symmetries for the enlarged Hamiltonian $\mathcal{H}$ as

$$\mathscr{T} = \begin{pmatrix} \ddots & & & & \\ & \mathcal{T} & & & \\ & & \mathcal{T} & & \\ & & & \mathcal{T} & \\ & & & & \ddots \end{pmatrix}, \quad \mathscr{C} = \begin{pmatrix} & & & \cdots \\ & & \mathcal{C} & \\ & \mathcal{C} & & \\ \mathcal{C} & & & \\ \cdots & & & \end{pmatrix}, \quad \mathscr{S} = \begin{pmatrix} & & & \cdots \\ & & \mathcal{S} & \\ & \mathcal{S} & & \\ \mathcal{S} & & & \\ \cdots & & & \end{pmatrix}. \tag{S7}$$

Let us assume the time-dependent Hamiltonian $H(\mathbf{k},t)$ has an additional time-glide symmetry, namely

$$\mathcal{M}H(\mathbf{k},t)\mathcal{M} = H(-k_x,\mathbf{k}_\parallel,t+T/2). \tag{S8}$$

When acting on $H_n(\mathbf{k})$, the time-glide symmetry becomes

$$\mathcal{M}H_n(\mathbf{k})\mathcal{M} = (-1)^n H_n(-k_x,\mathbf{k}_\parallel). \tag{S9}$$

This enables us to define an effective reflection symmetry

$$\mathscr{R} = \begin{pmatrix} \ddots & & & & \\ & \mathcal{M} & & & \\ & & -\mathcal{M} & & \\ & & & \mathcal{M} & \\ & & & & \ddots \end{pmatrix}, \tag{S10}$$

which is block diagonal with blocks alternating between $\mathcal{M}$ and $-\mathcal{M}$. In this way, we map the original time-glide symmetric Floquet system into a reflection symmetric static system, without changing the AZ classes.

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[1] T. Morimoto, H. C. Po, and A. Vishwanath, Phys. Rev. B **95**, 195155 (2017).
[2] S. Yao, Z. Yan, and Z. Wang, Phys. Rev. B **96**, 195303 (2017).