Comments

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Comment on "Growth of a Weak Magnetic Field in a Turbulent Conducting Fluid with Large Magnetic Prandtl Number"

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THE effect of a uniform straining motion $\mathbf{u} = (\alpha_1 x_1, \alpha_2 x_2, \alpha_3 x_3)$, where $\alpha_1 + \alpha_2 + \alpha_3 = 0$, and $\alpha_1 > \alpha_2 > \alpha_3$, on what may be called a "blob" of weak magnetic field, that is a magnetic field which is exponentially small outside a certain bounded region, has recently been examined by Pao.[1] The magnetic field $H_i(\mathbf{x}, t)$ is expressed [Eq. (3.7)] as a weighted integral over all space of its initial value, which is prescribed statistically in terms of its two-point correlation. Pao shows that, if $\alpha_2 = 0$, the value of the mean-square magnetic field at the origin ultimately tends to a constant. He then infers the effect of turbulence on a weak random magnetic field, and in particular concludes that, if the magnetic Prandtl number $\nu/\lambda > 100$, then $\overline{H^2}$ initially increases and then tends to a constant value. This inference and conclusion seem unjustified for the following reasons.

Firstly, the magnetic field distribution was not solenoidal. With a solenoidal initial field (i.e., one for which there is no net flux of $\mathbf{H}$ across any coordinate plane), it can be shown from Pao's Eq. (3.7) that ultimately $H_1(\mathbf{x}, t)$ varies with time like $e^{-\alpha_3 t}$, t^{-1}, $e^{\alpha_2 t}$, according as α_2 is greater, equal, or less than zero. The 2- and 3-components decrease even more rapidly. Moreover, confining the analysis to the case $\alpha_2 = 0$, where the decay with time is algebraic rather than exponential, is too restrictive in view of the fact that $\overline{\alpha_1 \alpha_2 \alpha_3}$ is negative in all known forms of turbulence. But in any event, the mean-square magnetic field at any point of a single blob eventually decays to zero although there will be, in general, initial amplification if λ is sufficiently small.

Secondly, and more importantly, the analysis of a single blob cannot in itself be conclusive, since the overlapping of neighboring blobs cannot be neglected. It is true that a random magnetic field can be thought of as initially decomposed into an array of nonoverlapping solenoidal blobs, each blob contained within a region of uniform straining motion:

$$\mathbf{H}(\mathbf{x}, 0) = \sum_r \mathbf{H}^{(r)}(\mathbf{x}, 0).$$

However, the volume occupied by the blob $\mathbf{H}^{(r)}$ ultimately increases exponentially [as $e^{(\alpha_1+\alpha_2)t}$ if $\alpha_2 > 0$, as $t^{\frac{1}{2}} e^{\alpha_1 t}$ if $\alpha_2 = 0$, and as $e^{\alpha_1 t}$ if $\alpha_2 < 0$]. Thus, more and more blobs must overlap as time passes and, after a long time, at any point a large number (proportional to $e^{(\alpha_1+\alpha_2)t}$ if $\alpha_2 > 0$, and $t^{\frac{1}{2}} e^{\alpha_1 t}$ if $\alpha_2 = 0$, and $e^{\alpha_1 t}$ if $\alpha_2 < 0$) of blobs which were initially distinct must overlap. If the magnetic fields in the blobs are uncorrelated, the mean-square magnetic field at any point behaves like

$$\overline{H^2} \sim \begin{cases} e^{(\alpha_1+\alpha_2)t} e^{-2\alpha_2 t} = e^{(\alpha_1-\alpha_2)t} & \text{if} \quad \alpha_2 > 0, \\ t^{\frac{1}{2}} e^{\alpha_1 t} t^{-2} = t^{-\frac{3}{2}} e^{\alpha_1 t} & \text{if} \quad \alpha_2 = 0, \\ e^{\alpha_1 t} e^{2\alpha_3 t} = e^{(\alpha_3-\alpha_2)t} & \text{if} \quad \alpha_2 < 0. \end{cases} \tag{1}$$

Thus in this case for any strain field (except the axisymmetric fields having $\alpha_1 = \alpha_2$ or $\alpha_2 = \alpha_3$, $\overline{H^2}$ increases exponentially. This result was originally proved conclusively by Pearson[2] who considered the action of irrotational uniform strain on a weak random vorticity field.

Pearson's mathematics may equally be applied to a magnetic field and the result (1) more rigorously obtained. The exponential increase is associated with the fact that the total magnetic energy of a single blob increases like $e^{(\alpha_1-\alpha_2)t}$, $t^{-\frac{3}{2}} e^{\alpha_1 t}$, $e^{(\alpha_3-\alpha_2)t}$, according as $\alpha_2 >, =,$ or <0, even though the magnetic field at any point of the single blob decreases.

If the blobs are positively correlated, then the increase in mean-square magnetic field will be even more rapid than (1). If they are sufficiently negatively correlated, then it is possible that $\overline{H^2} \to 0$ as $t \to \infty$. It has been advocated elsewhere[3] that the result (1) for a weak random field in a uniform straining motion does not hold for homogeneous turbulence. In any case, it seems certain that results are needed for blobs in nonuniform straining motions before conclusions may be drawn for the magnetic field in a turbulent conducting fluid.

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[2] J. R. A. Pearson, J. Fluid Mech. **5**, 274 (1959).
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