

Redefine statistical significance

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Supplementary Information:

Supplementary Text

R code used to generate Figures 1 and 2

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Figure 1

All four curves in Figure 1 describe the relationship between (i) a P -value based on a two-sided normal test and (ii) a Bayes factor or a bound on a Bayes factor. The P -values are based on a two-sided test that the mean μ of an independent and identically distributed sample of normally distributed random variables is 0. The variance of the observations is known. Without loss of generality, we assume that the variance is 1 and the sample size is also 1. The curves in the figure differ according to the alternative hypotheses that they assume for calculating (ii).

Because these curves involve two-sided tests, all alternative hypotheses are restricted to be symmetric around 0. That is, the density assumed for the value of μ under the alternative hypothesis is always assumed to satisfy $f(\mu) = f(-\mu)$.

The curve labeled “Power” corresponds to defining the alternative hypothesis so that power is 75% in a two-sided 5% test. This is achieved by assuming that μ under the alternative hypothesis is equal to $\pm(z_{0.025} + z_{0.75}) = \pm 2.63$. That is, the alternative hypothesis places $\frac{1}{2}$ its prior mass on 2.63 and $\frac{1}{2}$ its mass on -2.63.

The curve labeled UMPBT corresponds to the uniformly most powerful Bayesian test (2) that corresponds to a classical, two-sided test of size $\alpha = 0.005$. The alternative hypothesis for this Bayesian test places $\frac{1}{2}$ mass at 2.81 and $\frac{1}{2}$ mass at -2.81. The null hypothesis for this test is rejected if the Bayes factor exceeds 25.7. Note that this curve is nearly identical to the “Power” curve if that curve had been defined using 80% power, rather than 75% power. The Power curve for 80% power would place $\frac{1}{2}$ its mass at ± 2.80 .

The Likelihood Ratio Bound curve represents an approximate upper bound on the Bayes factor obtained by defining the alternative hypothesis as putting $\frac{1}{2}$ its mass on $\pm \bar{x}$, where \bar{x} is the observed sample mean. Over the range of P -values displayed in the figure, this alternative hypothesis very closely approximates the maximum Bayes factor that can be attained from among the set of alternative hypotheses constrained to be of the form $0.5 \times [f(\mu) + f(-\mu)]$ for some density function f .

The Local- H_1 curve is described fully in the figure caption. A fuller explanation and discussion of this bound can be found in ref. 15.

Equation 2 and Figure 2

This equation defines the large-sample relationship between the false positive rate, power $1 - \beta$, type I error rate α , and the probability that the null hypothesis is true when a large number of independent experiments have been conducted. More specifically, suppose that n independent hypothesis tests are conducted, and suppose that in each test the probability that the null

hypothesis is true is ϕ . If the null hypothesis is true, assume that the probability that it is falsely rejected (i.e., a false positive occurs) is α . For the test $j = 1, \dots, n$, define the random variable $X_j = 1$ if the null hypothesis is true *and* the null hypothesis is rejected, and $X_j = 0$ if either the alternative hypothesis is true or the null hypothesis is not rejected. Note that the X_j are independent Bernoulli random variables with $\Pr(X_j = 1) = \alpha\phi$. Also for test j , define another random variable $Y_j = 1$ if the alternative hypothesis is true *and* the null hypothesis is rejected, and 0 otherwise. It follows that the Y_j are independent Bernoulli random variables with $\Pr(Y_j = 1) = (1 - \phi)(1 - \beta)$. Note that Y_j is independent of Y_k for $j \neq k$, but Y_j is not independent of X_j . For the n experiments, the false positive rate can then be written as:

$$FPR = \frac{\sum_{j=1}^n X_j}{\sum_{j=1}^n X_j + \sum_{j=1}^n Y_j} = \frac{\sum_{j=1}^n X_j/n}{\sum_{j=1}^n X_j/n + \sum_{j=1}^n Y_j/n}.$$

By the strong law of large numbers, $\sum_{j=1}^n X_j/n$ converges almost surely to $\alpha\phi$, and $\sum_{j=1}^n Y_j/n$ converges almost surely to $(1 - \phi)(1 - \beta)$. Application of the continuous mapping theorem yields

$$FPR \xrightarrow{\text{a.s.}} \frac{\alpha\phi}{\alpha\phi + (1 - \phi)(1 - \beta)}.$$

Figure 2 illustrates this relationship for various values of α and prior odds for the alternative, $\frac{1-\phi}{\phi}$.

R code used to generate Figure 1:

```
type1=.005
type1Power=0.05
type2=0.25
p=1-c(9000:9990)/10000
xbar = qnorm(1-p/2)

# alternative based on 80% POWER IN 5% TEST
muPower = qnorm(1-type2)+qnorm(1-type1Power/2)
bfPow = 0.5*(dnorm(xbar,muPower,1)+dnorm(xbar,-muPower,1))/dnorm(xbar,0,1)

muUMPBT = qnorm(0.9975)
bfUMPBT = 0.5*(dnorm(xbar,muUMPBT,1)+dnorm(xbar,-
muUMPBT,1))/dnorm(xbar,0,1)

# two-sided "LR" bound
bfLR = 0.5/exp(-0.5*xbar^2)

bfLocal = -1/(2.71*p*log(p))

#coordinates for dashed lines
data = data.frame(p,bfLocal,bfLR,bfPow,bfUMPBT)
U_005 = max(data$bfLR[data$p=="0.005"])
L_005 = min(data$bfLocal[data$p=="0.005"])
U_05 = max(data$bfLR[data$p=="0.05"])
L_05 = min(data$bfUMPBT[data$p=="0.05"])

# Local bound; no need for two-sided adjustment

#plot margins
par(mai=c(0.8,0.8,.1,0.4))
par(mgp=c(2,1,0))

matplot(p,cbind(bfLR,-1/(2.71*p*log(p))),type='n',log='xy',
        xlab=expression(paste(italic(P) ,"-value")),
        ylab="Bayes Factor",
        ylim = c(0.3,100),
        bty="n",xaxt="n",yaxt="n")
lines(p,bfPow,col="red",lwd=2.5)
lines(p,bfLR,col="black",lwd=2.5)
lines(p,bfUMPBT,col="blue",lwd=2.5)
lines(p,bfLocal,col="green",lwd=2.5)
legend(0.015,100,c(expression(paste("Power")), "Likelihood Ratio
Bound", "UMPBT",expression(paste("Local-",italic(H)[1], "
Bound"))),lty=c(1,1,1,1),
      lwd=c(2.5,2.5,2.5,2.5),col=c("red","black","blue","green"), cex =
0.8)
#text(0.062,65, "\u03B1", font =3, cex = 0.9)

#customizing axes
#x axis
axis(side=1,at=c(-2,0.001,0.0025,0.005,0.010,0.025,0.050,0.100,0.14),
```

```

        labels =
c("", "0.0010", "0.0025", "0.0050", "0.0100", "0.0250", "0.0500", "0.1000", ""), lwd=1,
tck = -0.01, padj = -1.1, cex.axis = .8)
#y axis on the left - main
axis(side=2, at=c(-0.2, 0.3, 0.5, 1, 2, 5, 10, 20, 50, 100), labels =
c("", "0.3", "0.5", "1.0", "2.0", "5.0", "10.0", "20.0", "50.0", "100.0"), lwd=1, las
= 1,
        tck = -0.01, hadj = 0.6, cex.axis = .8)
#y axis on the left - secondary (red labels)
axis(side=2, at=c(L_005, U_005), labels = c(13.9, 25.7), lwd=1, las= 1,
        tck = -0.01, hadj = 0.6, cex.axis = .6, col.axis="red")
#y axis on the right - main
axis(side=4, at=c(-0.2, 0.3, 0.5, 1, 2, 5, 10, 20, 50, 100), labels =
c("", "0.3", "0.5", "1.0", "2.0", "5.0", "10.0", "20.0", "50.0", "100.0"), lwd=1, las
= 1,
        tck = -0.01, hadj = 0.4, cex.axis = .8)
#y axis on the right - secondary (red labels)
axis(side=4, at=c(L_05, U_05), labels = c(2.4, 3.4), lwd=1, las= 1,
        tck = -0.01, hadj = 0.4, cex.axis = .6, col.axis="red")

###dashed lines
segments(x0 = 0.000011, y0= U_005, x1 = 0.005, y1 = U_005, col = "gray40",
lty = 2)
segments(x0 = 0.000011, y0= L_005, x1 = 0.005, y1 = L_005, col = "gray40",
lty = 2)
segments(x0 = 0.005, y0= 0.00000001, x1 = 0.005, y1 = U_005, col =
"gray40", lty = 2)

segments(x0 = 0.05, y0= U_05, x1 = 0.14, y1 = U_05, col = "gray40", lty =
2)
segments(x0 = 0.05, y0= L_05, x1 = 0.14, y1 = L_05, col = "gray40", lty =
2)
segments(x0 = 0.05, y0= 0.00000001, x1 = 0.05, y1 = U_05, col = "gray40",
lty = 2)

```

R code used to generate Figure 2:

```
pow1=c(5:999)/1000 # power range for 0.005 tests
pow2=c(50:999)/1000 # power range for 0.05 tests
alpha=0.005 # test size
pi0=5/6 # prior probability
N=10^6 # doesn't matter

#graph margins
par(mai=c(0.8,0.8,0.1,0.1))
par(mgp=c(2,1,0))

plot(pow1,alpha*N*pi0/(alpha*N*pi0+pow1*(1-pi0)*N),type='n',ylim = c(0,1),
xlim = c(0,1.5),
      xlab='Power',
      ylab='False positive rate', bty="n", xaxt="n", yaxt="n")
#grid lines
segments(x0 = -0.058, y0 = 0, x1 = 1, y1 = 0,lty=1,col = "gray92")
segments(x0 = -0.058, y0 = 0.2, x1 = 1, y1 = 0.2,lty=1,col = "gray92")
segments(x0 = -0.058, y0 = 0.4, x1 = 1, y1 = 0.4,lty=1,col = "gray92")
segments(x0 = -0.058, y0 = 0.6, x1 = 1, y1 = 0.6,lty=1,col = "gray92")
segments(x0 = -0.058, y0 = 0.8, x1 = 1, y1 = 0.8,lty=1,col = "gray92")
segments(x0 = -0.058, y0 = 1, x1 = 1, y1 = 1,lty=1,col = "gray92")

lines(pow1,alpha*N*pi0/(alpha*N*pi0+pow1*(1-
pi0)*N),lty=1,col="blue",lwd=2)
odd_1_5_1 = alpha*N*pi0/(alpha*N*pi0+pow1[995]*(1-pi0)*N)
alpha=0.05
pi0=5/6
lines(pow2,alpha*N*pi0/(alpha*N*pi0+pow2*(1-
pi0)*N),lty=2,col="blue",lwd=2)
odd_1_5_2 = alpha*N*pi0/(alpha*N*pi0+pow2[950]*(1-pi0)*N)

alpha=0.05
pi0=10/11
lines(pow2,alpha*N*pi0/(alpha*N*pi0+pow2*(1-pi0)*N),lty=2,col="red",lwd=2)
odd_1_10_2 = alpha*N*pi0/(alpha*N*pi0+pow2[950]*(1-pi0)*N)
alpha=0.005
pi0=10/11
lines(pow1,alpha*N*pi0/(alpha*N*pi0+pow1*(1-pi0)*N),lty=1,col="red",lwd=2)
odd_1_10_1 = alpha*N*pi0/(alpha*N*pi0+pow1[995]*(1-pi0)*N)

alpha=0.05
pi0=40/41
lines(pow2,alpha*N*pi0/(alpha*N*pi0+pow2*(1-
pi0)*N),lty=2,col="green",lwd=2)
odd_1_40_2 = alpha*N*pi0/(alpha*N*pi0+pow2[950]*(1-pi0)*N)
alpha=0.005
pi0=40/41
```

```

lines(pow1,alpha*N*pi0/(alpha*N*pi0+pow1*(1-
pi0)*N),lty=1,col="green",lwd=2)
odd_1_40_1 = alpha*N*pi0/(alpha*N*pi0+pow1[995]*(1-pi0)*N)

#customizing axes
axis(side=2,at=c(-0.5,0,0.2,0.4,0.6,0.8,1.0),labels =
c("", "0.0", "0.2", "0.4", "0.6", "0.8", "1.0"),
      lwd=1,las= 1,tck = -0.01, hadj = 0.4, cex.axis = .8)
axis(side=1,at=c(-0.5,0,0.2,0.4,0.6,0.8,1.0),labels =
c("", "0.0", "0.2", "0.4", "0.6", "0.8", "1.0"),
      lwd=1,las= 1, tck = -0.01, padj = -1.1, cex.axis = .8)

legend(1.05,1,c("Prior odds = 1:40","Prior odds = 1:10","Prior odds =
1:5"),pch=c(15,15,15),
      col=c("green","red","blue"), cex = 1)

##### Use these commands to add brackets in Figure 2

library(pBrackets)

#add text and brackets
text(1.11,(odd_1_5_2+odd_1_40_2)/2, expression(paste(italic(P)," < 0.05
threshold")), cex = 0.9,adj=0)
text(1.11,(odd_1_5_1+odd_1_40_1)/2, expression(paste(italic(P)," < 0.005
threshold")), cex = 0.9,adj=0)
brackets(1.03, odd_1_40_1, 1.03, odd_1_5_1, h = NULL, ticks = 0.5,
curvature = 0.7, type = 1,
        col = 1, lwd = 1, lty = 1, xpd = FALSE)
brackets(1.03, odd_1_40_2, 1.03, odd_1_5_2, h = NULL, ticks = 0.5,
curvature = 0.7, type = 1,
        col = 1, lwd = 1, lty = 1, xpd = FALSE)

```