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Supplementary information

Observation of the exceptional-point-enhanced Sagnac effect

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Supplement: Observation of the exceptional-point enhanced Sagnac effect

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I. ORIGIN OF THE DISSIPATIVE COUPLING

In a standing-wave mode basis, the optical loss induced by the fiber taper or any other spatially localized absorption or dissipative scattering element will be different for each mode and can be captured by the following contribution to the Hamiltonian:

$$H_{\text{taper}} = \begin{pmatrix} -i\gamma_1 & 0 \\ 0 & -i\gamma_2 \end{pmatrix}. \tag{S1}$$

Changing to a traveling wave basis (CW and CCW) by using the relation $|\Phi_\pm\rangle = (|\text{CW}\rangle \pm |\text{CCW}\rangle/\sqrt{2}$ gives the following Hamiltonian in the new basis,

$$H_{\text{taper}} = \begin{pmatrix} -i\gamma_{\text{common}} & 0 \\ 0 & -i\gamma_{\text{common}} \end{pmatrix} + \begin{pmatrix} 0 & i\kappa \\ i\kappa & 0 \end{pmatrix} \tag{S2}$$

where $\gamma_{\text{common}} = (\gamma_1 + \gamma_2)/2$ and $\kappa = (\gamma_2 - \gamma_1)/2$. The first term is the common loss (out-coupling loss of the taper) while the second term is the dissipative backscattering in Eq. (1). Based on empirical evidence, we believe that most of the dissipative backscattering originates from taper scattering. Specifically, changes in the contact position of the taper on the resonator are observed to vary κ. As an aside, conservative backscattering is not considered in this analysis.

II. GAIN CLAMPING CONDITION (NON-ZERO κ)

Here, it is shown that the gain clamping condition for zero κ also holds for non-zero κ to a good approximation in the unlocked regime. The Hamiltonian (Eq.1) depends on the eigenvalue ω_s through the Brillouin gain factor $g_j = g_0/[1+2i(\omega_{p_j} - \Omega_{\text{phonon}} - \omega_s)/\Gamma]$. To account for this dependence in the threshold gain clamping condition, first separate the Brillouin gain factor into real and imaginary parts as follows:

$$\text{Re}(g_j) = \frac{g_0}{1 + 4\frac{(\omega_{p_j} - \Omega_{\text{phonon}} - \omega_s)^2}{\Gamma^2}} \tag{S3}$$

$$\text{Im}(g_j) = -\frac{2\left(\omega_{p_j} - \Omega_{\text{phonon}} - \omega_s\right)}{\Gamma}\text{Re}(g_j) \tag{S4}$$

Then, note that when mode pulling is small compared to the cavity linewidth (which is the case in this work), ω_s can be replaced by the cavity mode frequency (ω_0) in the denominator of Eq. (S3) thereby leaving the eigenvalue dependence only in the dispersive term (numerator). Furthermore, by defining normalized quantities:

$$I_j \equiv \frac{\text{Re}(g_j)|A_j|^2}{\gamma/2}, \quad k \equiv \frac{\kappa}{\gamma/2}, \quad n_{pj} \equiv \frac{\omega_{p_j} - \Omega_{\text{phonon}}}{\gamma/2}, \quad x \equiv \frac{\omega_s}{\gamma/2}, \quad x_0 \equiv \frac{\omega_0}{\gamma/2}, \quad r \equiv \frac{\gamma}{\Gamma}, \tag{S5}$$

the Hamiltonian (Eq. (1)) reduces to:

$$\tilde{H}_0 \equiv \frac{H_0}{\gamma/2} = x_0\mathbb{1} + \begin{pmatrix} i\left(I_1 - 1\right) + rI_1(n_{p1} - x) & ik \\ ik & i\left(I_2 - 1\right) + rI_2(n_{p2} - x) \end{pmatrix} \tag{S6}$$

The eigenvalues $x_\pm$ can be solved from $\det(\tilde{H}_0 - x\mathbb{1}) = Ax^2 + Bx + C = 0$ where

$$A = (1 + I_1 r)(1 + I_2 r) \tag{S7}$$

$$B = 2i - 2x_0 - (I_1 + I_2)(i - ir + x_0 r) - r(I_1 n_{p1} + I_2 n_{p2}) + I_1 I_2[2i + (n_{p1} + n_{p2})r] \tag{S8}$$

$$C = k^2 + (-i + x_0)^2 + (-i + x_0)[I_1(i + n_{p1}r) + I_2(i + n_{p2}r)] + I_1 I_2(i + n_{p1}r)(i + n_{p2}r) \tag{S9}$$

Because the two eigenvalues $x_\pm = (-B \pm \sqrt{B^2 - 4AC})/2$ must be real (i.e., above laser threshold operation), the following equations can be derived from $\mathrm{Im}(x_\pm) = 0$

$$\mathrm{Im}(B^2 - 4AC) = 2r(r+1)(I_1 - I_2)[I_1 I_2 r(n_{p1} - n_{p2}) + I_1(np_1 - x_0) + I_2(x_0 - n_{p2})] = 0 \tag{S10}$$
$$\mathrm{Im}(B) = 2rI_1I_2 + (I_1 + I_2)(1 - r) - 2 = 0 \tag{S11}$$

It follows from Eq. (S10) that $I_1 = I_2$. Inserting this result into Eq. (S11) gives $I_1 = I_2 = 1$ yielding $|A_j|^2 = \gamma/(2\mathrm{Re}(g_j))$, which (subject to the weak mode pulling approximation noted above) is also the $\kappa = 0$ gain clamping condition used to simplify the Hamiltonian to the form given in Eq. (2). Numerical solution of the eigenvalue equation confirms this result for the unlocked regime. On the other hand, numerical solution also shows that in the locked regime only one eigenvalue can be real for any combination of pumping powers (i.e., only one mode lases in the locked regime). Moreover, a low and high loss eigenvalue exist so that one mode has a lower threshold pumping power. An equal pump power solution ($I_1 = I_2$) is still possible for laser action, but this condition is no longer unique.

III. CHARACTERIZATION OF EIGENMODES

The eigenmodes of Eq. (2) are:

$$|\Psi_+\rangle = \frac{1}{N}\begin{pmatrix} i \\ \left[\Delta\omega_p/\Delta\omega_c + \sqrt{(\Delta\omega_p/\Delta\omega_c)^2 - 1}\right]\end{pmatrix} \tag{S12}$$
$$|\Psi_-\rangle = \frac{1}{N}\begin{pmatrix} -\left[\Delta\omega_p/\Delta\omega_c + \sqrt{(\Delta\omega_p/\Delta\omega_c)^2 - 1}\right] \\ i\end{pmatrix} \tag{S13}$$

where N is the normalization. These lasing eigenmodes are valid in the uncoupled regime of operation ($|\Delta\omega_p| > \Delta\omega_c$). As $\Delta\omega_p$ evolves from large and positive detuning towards $\Delta\omega_c$, $|\Psi_+\rangle$ and $|\Psi_-\rangle$) (corresponding to the $\pm$ roots of Eq. (3)) evolve from purely CCW and CW waves through admixtures of CCW and CW and then ultimately coalesce at $\Delta\omega_p = \Delta\omega_c$. On the other hand, As $\Delta\omega_p$ evolves from large and negative detuning towards $-\Delta\omega_c$, $|\Psi_+\rangle$ and $|\Psi_-\rangle$) evolve from purely CW and CCW waves through admixtures of CW and CCW and then ultimately coalesce at $\Delta\omega_p = -\Delta\omega_c$ (with opposite phase).

To make the data plot within the inset of Fig. 2d the laser output in the CCW direction (combination of two laser Stokes waves) was monitored. The ratio of powers of the components was determined by heterodyning the combined fields with a CCW pump field and then measuring the respective Pump-SBL$_{1,2}$ beat components on an electrical spectrum analyzer. The ratio of the powers in these beat frequency components is the ratio of the powers in the CCW Stokes' waves components:

$$\frac{I_{s2}}{I_{s1}} = \left| |\Delta\omega_p/\Delta\omega_c| + \sqrt{(\Delta\omega_p/\Delta\omega_c)^2 - 1}\right|^2 \tag{S14}$$

which follows from Eqs. (S12, S13).

It is also interesting to note that in the locked regime ($|\Delta\omega_p|/\Delta\omega_c < 1$) numerical solution shows that eigenvectors having equal admixture of CW and CCW waves occur when $I_1 = I_2$, but at distinctly different threshold power levels (i.e., the two states have different loss rates). Moreover, this pumping combination is not unique so lasing solutions featuring an unbalanced admixture of CW and CCW states are also possible. The (locked regime) equatorial trajectories shown in Fig. 1 represent the low and high loss $I_1 = I_2$ trajectories (i.e., equal CW and CCW admixture).

As an aside, the measurements in Fig. 2a and Fig. 2b feature additional lines in the spectra that are believed to originate from nonlinear mixing in the Brillouin interaction (a third order nonlinear interaction). This four-wave-mixing process becomes more significant near the EP where the CW and CCW modes strongly interact with each other. It is observed to impact the intensity of the beating lines but leaves their frequencies intact. As a result, data for the eigenmode amplitude components deviate from the theoretical value while the pump-SBL and dual-SBL frequency data fit well with the theory (see Fig. 2c and d).

IV. KERR-INDUCED SHIFT

The Kerr effect shifts the resonance frequency by adding the following term into the Hamiltonian:

$$H_{\mathrm{Kerr}} = \begin{pmatrix} -\eta\left(\left|\alpha_1\right|^2 + 2\left|\alpha_2\right|^2\right) & 0 \\ 0 & -\eta\left(2\left|\alpha_1\right|^2 + \left|\alpha_2\right|^2\right) \end{pmatrix} \tag{S15}$$

where $\eta = n_2 \hbar \omega^2 c / V n_0^2$ is the single photon induced nonlinear angular frequency shift. The corrected beating frequency (without rotation) reads:

$$\Delta\omega_s = \frac{1}{1+\gamma/\Gamma}\sqrt{\left[\frac{\gamma}{\Gamma}\Delta\omega_p + \eta\left(\left|\alpha_2\right|^2 - \left|\alpha_1\right|^2\right)\right]^2 - 4\kappa^2} \tag{S16}$$

The correction from the Kerr effect is therefore equivalent to shifting $\Delta\omega_p$ by angular frequency $\eta\Gamma(|\alpha_2|^2 - |\alpha_1|^2)/\gamma$. In the experiment, this Kerr shift was minimized by centering the locking zone at zero pump detuning by adjusting the two pump powers. After that, the pump powers were locked so that the two SBL powers are balanced. The subsequent pump detuning changes required to make the measurement affected the SBL power, but only negligibly. Specifically, the Kerr shift is around 10s of Hz after a pump detuning change by 200kHz. This is negligible in comparison to the Stokes frequency separation changes measured in Fig. 2c and Fig. 2d. Moreover, the dithering measurement in Fig. 3 was insensitive to these constant Kerr-induced shifts since it measured the amplitude of a sinusoidal rotation.