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Analysis of the pulsar P–$\dot{P}$ distribution

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Summary. A new technique for the systematic evaluation of models of pulsar period evolution on the basis of a complete observational sample is outlined and applied to the existing incomplete sample. Possible selection effects are discussed. It is concluded that the simple law for the rate of change of period P, $\dot{P} \propto P^{2-n}$ is incompatible with the assumption of stationarity and pulsar 'death' at large periods, if $n > 2$. Models with $n < 2$, or with $n \gtrsim 2.5$ and torque decay on a time-scale of 1 Myr are consistent with the data. Another possibility is that the beaming fraction decreases by a factor ~5 as a pulsar slows down. A new procedure for deriving a rigorous lower limit to the creation rate of pulsars within the sample is presented, and it is shown that most pulsars appear to be born with large values of $\dot{P}$.

1 Introduction

To date over 300 radio pulsars have been discovered; roughly one-third of these have measured period derivatives, $\dot{P}$. Although the sample with measured $\dot{P}$s certainly suffers from selection effects, the full-sky sample of 328 is believed to be flux-limited at ~10 mJy and fairly complete in period space (J. H. Taylor, private communication). The sample is unlikely to be substantially enlarged with existing instrumentation. Since $\dot{P}$s for the full sample will be measured in the next few years, it is now appropriate to develop tools for the extraction of information about pulsar evolution from the observed distribution function $f(P, \dot{P})$ of pulsars per unit period P and unit $\dot{P}$. Other variables available for use in a complete statistical analysis of the pulsar sample include: dispersion measure (DM), which in conjunction with pulsar positions can be used to estimate their distance d and height z above the galactic plane; proper motions, which give velocities perpendicular to the line-of-sight; flux measurements, which give estimates of the radio luminosity; and finally pulse widths, shapes and polarization which may allow us to infer some details of the pulsar magnetic field strength and orientation.

Many attempts have been made to determine the birth rate and evolutionary history of pulsars from their periods and $\dot{P}$s beginning with the investigation of Gunn & Ostriker (1970), who used the then existing sample of 41 objects (including only 14 values of $\dot{P}$). Gunn & Ostriker concluded that pulsar magnetic fields must decay on a time-scale of roughly 4 Myr, and estimated the galactic birthrate at one per 10–100 yr, with the major uncertainty existing in the beaming factor. The latter result has been largely confirmed by subsequent analyses of successively larger samples (e.g. Lyne, Ritchings & Smith 1975; Manchester & Taylor 1977). An independent determination of the life-span of pulsars is provided by the proper motions and the galactic scale-height, and leads to a mean age of ~5 Myr (Helfand & Tademaru 1977; Hanson 1979; Lyne 1979). This estimate is independent of the distance determination, although it is subject to some strong selection effects.

These results are perplexing for two reasons, discussed at greater length by Manchester & Taylor (1977). First, as their lifetimes are much less than the age of the Galaxy, pulsars ought to constitute a stationary population. Their derived birthrate appears at least to equal, and may exceed the estimated local supernova rate (*cf.* Milne 1979; Tammann 1977) and the birth rate of stars believed to be possible pulsar progenitors. This suggests that most pulsars are formed by a surprisingly inconspicuous mechanism (e.g. within dense molecular clouds) and involving stars not previously thought capable of evolving to neutron stars. The discrepancy is widened by the fact that only two out of more than 100 supernova remnants show convincing evidence of a central pulsar from either radio or X-ray searches (although there are roughly 10 *possible* associations (*cf.* Manchester & Taylor 1977; Tuohy & Garmire 1980)). The second difficulty is that, while the mean kinematic age of ~5 Myr is roughly the same as the *median* 'timing age' $P/\dot{P}$, many pulsars have much larger values of $P/\dot{P}$, and indeed 11 per cent of the present sample of 107 have timing ages $\gtrsim 10^8$ yr, implying that $P/\dot{P}$ is not always a good estimate of physical age. This has motivated the development of various models in which $\dot{P}$ decreases exponentially with time, following a rapid decline of the torque exerted on the star – e.g. surface magnetic field decay (first suggested in Ostriker & Gunn 1969) or field alignment with the spin-axis (*cf.* Jones 1976), or decoupling of the core from the crust, suddenly decreasing the moment of inertia on which the torque acts (Harding, Guyer & Greenstein 1978). Unfortunately, the last two models are based on some rather specialized assumptions about the physical conditions in neutron stars, while the first is hard to reconcile with the essentially infinite time for field decay in their superconducting interiors (though, see Flowers & Ruderman 1977).

This great profusion of models suggests the need for some systematic way of testing their predictions against the observed distributions, and for obtaining information about the birth and death functions of pulsars. In particular, it is important to do this in a manner independent of the probably biased proper motion measurements and the uncertain distance determinations. We outline one such method below, and illustrate its use by applying it to the existing sample of 107 pulsars with measured $\dot{P}$s given by Taylor (private communication).

In Section 2, we describe the simplest, model-independent way of obtaining information on the source function of pulsar birth and death, and explain why the 'dynamical' treatment outlined in Section 3 is necessary. In Section 4 we examine the constraints put on existing models, and possible selection effects which could be invoked to save the models. In Section 5 we analyse the lower bound on the galactic creation rate. Section 6 reiterates our conclusions.

2 A 'kinematic' approach to the source function

We have plotted the 107 pulsars on the conventional P–$\dot{P}$ diagram in Fig. 1. Each pulsar moves through the diagram as time advances, having become accessible to observation at some point in the P–$\dot{P}$ plane. It will then disappear from the sample at some other point.

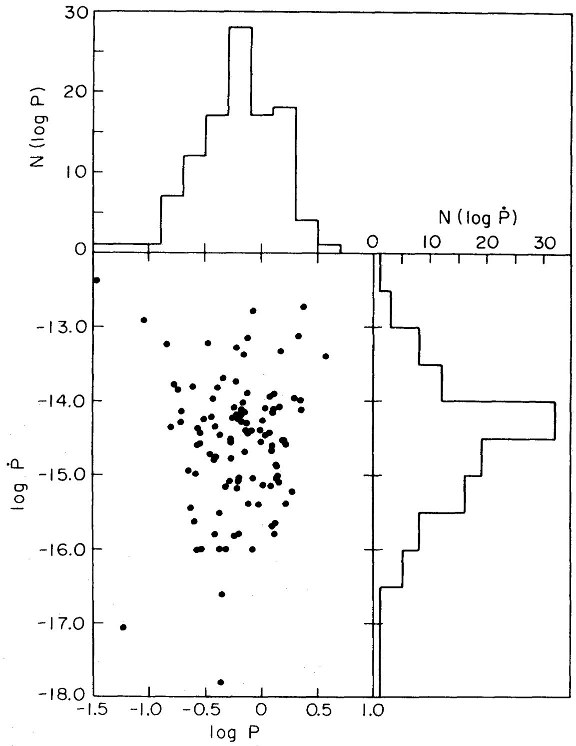

Figure 1. Pulsar period derivatives $\dot{P}$ are plotted versus period P for the 107 pulsars with measured $\dot{P}$. Above is a histogram of the number of pulsars in logarithmic intervals of period. On the right is a similar histogram for $\dot{P}$. Note the slow fall-off in density at the large $\dot{P}$ end which leads to the problems of divergent flux integrals. Here, and in all the figures, periods are measured in seconds, period derivatives are dimensionless.

Observations indicate that this motion is 'smooth', the net effect of glitches on the period being insignificant compared to that of the continuous $\dot{P}$. The time-scale for an appreciable movement in the diagram is just the timing age $P/\dot{P}$. As the timing ages are all much less than the age of the galaxy, we assume that the distribution of pulsars in the P–$\dot{P}$ diagram has reached a steady state. Furthermore, we see pulsars (median distance 3 kpc) from a considerable fraction of the Galaxy, so any local deviation from equilibrium (in the vicinity of a spiral arm, for example) should be unimportant.

We therefore treat the motion in the P–$\dot{P}$ plane of the ensemble of observable pulsars as a stationary 'fluid' flow. Independent of any model of pulsar period evolution the distribution function $f(P, \dot{P})$ must satisfy the equation of continuity

$$\frac{\partial}{\partial P}(f\dot{P}) + \frac{\partial}{\partial \dot{P}}(f\ddot{P}) = S(P, \dot{P}), \qquad (1)$$

where $S(P, \dot{P})$ is the source function of *observable* pulsars – i.e. the number becoming newly observable minus the number becoming unobservable in a unit cell per unit time. With only 107 pulsars (or even with the anticipated sample of 300) covering five orders of magnitude in $\dot{P}$ and two in period, direct computation of S from the observed f is clearly impossible. The data can be smoothed by an integration over $\dot{P}$ to yield the source function in P alone

$$\frac{d}{dP}(N(P)\langle\dot{P}\rangle) = S(P), \qquad (2)$$

where

$$N(P)=\int_{-\infty}^{\infty} f d\dot{P},$$

$$\langle\dot{P}\rangle=\frac{\int f\dot{P}\,d\dot{P}}{N(P)},$$

$$S(P)=\int S(P,\dot{P})\,d\dot{P}.$$

We might also consider the introduction of other coordinate pairs with more physical significance than P and $\dot{P}$ – say the 'age' $P/\dot{P}$ and $P\dot{P}$ (proportional to the square of the surface field strength in the rotating dipole model) for which exactly analogous formulae hold.

Regrettably, these 'kinematic' approaches are doomed to failure. The reason can be seen immediately from Fig. 1 and the histogram in $\dot{P}$. The distribution of pulsars is roughly logarithmically uniform over five orders of magnitude in $\dot{P}$, and at large $\dot{P}$, $f(P,\dot{P}) \underset{\sim}{\propto} \dot{P}^{-1/2}$, so when we set about calculating $\langle\dot{P}\rangle \sim \int f(P,\dot{P})\dot{P}\,d\dot{P}$, the integral diverges – i.e. the flux $N(P)\langle\dot{P}\rangle$ is dominated by the few 'fast' pulsars with large $\dot{P}$. In fact, the 10 pulsars with largest $\dot{P}$ contribute 75 per cent of $\langle\dot{P}\rangle$. (This conclusion is not affected by the presence of unmeasured values of $\dot{P}$, since these are certain to be much smaller than average.) To compute $S(P)$ we would thus have to differentiate a histogram with effectively only one pulsar per bin! The failure of this technique is of interest none the less, since it is perhaps surprising that the source function in P should be so dominated by the youngest pulsars. The use of other simple coordinates like those mentioned above does not result in any improvement, since the uniform distribution over logarithmic intervals remains. Furthermore, the method is no longer model independent, for to calculate $\dot{x}$ where $x=x(P,\dot{P})$ is one of our new coordinates, we must know how to relate $\ddot{P}$ to the observable P and $\dot{P}$s, which requires an evolutionary model. If we are driven to this, however, we may as well seek another approach which will treat the data in a more equitable fashion.

3 A 'Dynamical' approach

We assume that the future history in the P–$\dot{P}$ plane is determined uniquely by a knowledge of P and $\dot{P}$ at some instant – i.e. $\ddot{P}$ is a known and well-behaved function of P and $\dot{P}$ over some domain in the P–$\dot{P}$ plane. This is a fairly strong assumption and precludes the possibility that other factors (e.g. observer orientation, surface ageing effects) seriously influence the detectability of pulsars. However, nearly all of the evolutionary models proposed to date satisfy this condition. Provided that $\dot{P}$ remains positive, it is straightforward to show that the 'force' law $\ddot{P}=f(P,\dot{P})$ can be integrated to furnish a constant of the motion $v(P,\dot{P})$ and a time $t(P,\dot{P})$. For if we choose some fiducial period P_0 (e.g. zero), then the differential equation can be solved to yield $\dot{P}_0$ as a function of P and $\dot{P}$. Any function of the fixed period P_0 and $\dot{P}_0$ (which varies from pulsar to pulsar) constitutes a suitable constant $v(P,\dot{P})$. The actual choice of this function and P_0 is determined by inspection on the grounds of simplicity. We will give some examples below. A second integration of the differential equation with perhaps a different fiducial period P_0' yields $P(t,P_0',\dot{P}_0')$ which can in principle be inverted to give $t=t(P,\dot{P})-t(P_0',\dot{P}_0')$ on substituting for $\dot{P}_0'(P,\dot{P})$ and $P_0'(P,\dot{P})$. Again simplicity usually dictates a suitable form for $t(P,\dot{P})$ Note that t is not necessarily to be interpreted as the time that has elapsed since the birth of the pulsar. We then define a 'position' coordinate $x(P,\dot{P})=vt$.

Since the continuity equation is invariant under coordinate transformation, we can choose

x and v as our new coordinates to obtain

$$v \frac{\partial \bar{f}}{\partial x} = \bar{S}(x, v), \tag{3}$$

where $\bar{f}$ and $\bar{S}$ are f and S as defined previously and multiplied by the Jacobian

$$J = \left| \frac{\partial (x, v)}{\partial (P, \dot{P})} \right|. \tag{4}$$

In particular, if $\bar{S}(x, v)$ is zero in some domain of the x–v plane, then equation (3) can be integrated to give $\bar{f} = \bar{f}(v)$ – i.e. in the absence of birth and death, the density of pulsars remains constant as they move along the direction of increasing x with uniform velocity v. In theory, we could assume $\bar{S} = 0$, and use the requirement of continuity and the observed distribution function to relate x and v. In practice, the density of observed pulsars in phase space is far too low to permit such a model-independent determination of the law of period evolution, and the best than can be done is to use equation (3) to test a given evolutionary model. Two simple examples follow:

(i) $\ddot{P} = (2 - n)\dot{P}^2/P$

This is the simplest type of model generally considered. The case $n = 3$ corresponds to magnetic dipole radiation and a constant moment of inertia. The presence of higher multipoles will tend to increase n (the braking index), and inertial effects will reduce it from this value (e.g. Roberts & Sturrock 1972). The only pulsar for which n has been measured reliably is the Crab pulsar which gives $n = 2.5$ (Groth 1975). It is unlikely that additional values will be measured in the foreseeable future due to the presence of timing noise (Taylor, private communication). Suitable coordinates are

$$v = \dot{P}P^{n-2}$$

$$x = P^{n-1}/(n-1). \tag{5}$$

(ii) $\ddot{P} = (2 - n)\dot{P}^2/P - \dot{P}/\tau$

This model allows for an exponential decay of the magnetic field

$$\dot{P}P^{n-2} \propto \exp(-t/\tau). \tag{6}$$

The magnetic field alignment model of Jones (1976) gives the same equation, provided his parameter $\alpha \ll 1$. The decay time τ must be the same for all pulsars, for if it varies from pulsar to pulsar, a third-order differential equation is necessary, we need to know $\ddot{P}_0$ to specify the motion uniquely, and our approach cannot be used. Integration yields the coordinates

$$\begin{aligned} v &= \dot{P}P^{n-2} \, [1 + P/(n-1)\dot{P}\tau], \\ x &= \dot{P}P^{n-2}\tau \, [1 + P/(n-1)\dot{P}\tau] \ln [1 + P/(n-1)\dot{P}\tau], \end{aligned} \tag{7}$$

cf. Fujimura & Kennel (1979). For $\tau \gg P/\dot{P}$ these coordinates reduce to those of case (i).

4 Evolutionary models in the light of the existing sample

In the standard electromagnetic pulsar model (example (i) with $n = 3$), the relevant coordinates are $v = P\dot{P}$, $x = ½ P^2$. The existing sample of 107 pulsars is plotted in these coordinates in Fig. 3. If this law provided a good model of pulsar evolution (we know that it does not), and birth and death were negligible within some area of the x–v plane (i.e. $\bar{S} = 0$), then

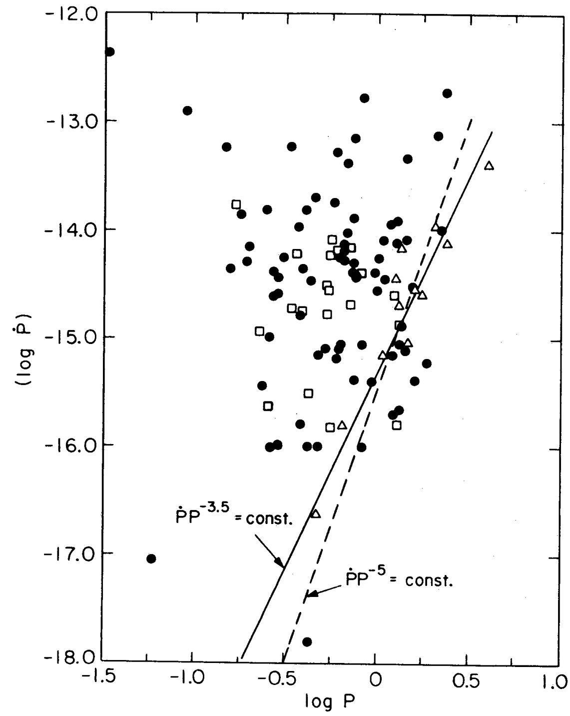

Figure 2. As in Fig. 1, but with Ritchings' (1976) observations of nulling shown to illustrate the correlation with $\dot{P}P^{-k}$, $2.5 \lesssim k \lesssim 5$. In the figure, and in Figs 3–5, a filled circle indicates a pulsar not examined for nulling. The squares mark those pulsars which spend less than 5 per cent of their time in a null state. The triangles indicate pulsars which Ritchings observed to null more than 5 per cent of the time.

pulsars should be uniformly distributed throughout this area apart from statistical fluctuations. In fact there is both theoretical and observational support for the hypothesis that emission ceases when $\dot{P}P^{-k}$ (where k is an exponent variously estimated to be in the range 2–5) falls below a critical value. In electrodynamical terms this implies that either the potential difference across the open field lines or vacuum electric field component resolved normal to the surface is inadequate to maintain a continuous supply of plasma (e.g. Ruderman & Sutherland 1975, who predict that $k = 2.25$). Furthermore, Ritchings' (1976) observations of pulse nulling provide additional evidence that death occurs close to this cut-off. As he first pointed out, the pulsars with observable nulling are strikingly clumped at the right-hand side of the distribution in the P–$\dot{P}$ plane along a line $\dot{P}P^{-k}$ = constant, where $2.5 \lesssim k \lesssim 5$ (replotted in Fig. 2). The obvious interpretation is that, as pulsars near the cut-off, they spend an increasingly large fraction of their time in a null state, finally disappearing when this fraction increases to unity.

We temporarily adopt this hypothesis (*cf.* Fujimura & Kennel 1979), and assume that 'death' occurs only to the right of the cut-off line $\dot{P}P^{-k} = K$. If 'birth' occurs only at very small x, then $\bar{S} = 0$, and $\bar{f}(x, v)$ is a function of v alone to the left of the cut-off line. If birth is present over the observed range of x, then $\bar{S} > 0$ and, for fixed v, $\bar{f}$ must be an increasing function of x. A quantitative test of this prediction that makes use of most of the sample is suggested by analogy with Schmidt's (1968) cosmological $\langle V/V_{\rm max}\rangle$ test. For a given cut-off line located approximately by the nulling pulsars, we define $x_{\rm max}(v)$. We then form the average $\langle x/x_{\rm max}\rangle$ for all pulsars with $x < x_{\rm max}(v)$. In a steady state with $\bar{S} = 0$, $\langle x/x_{\rm max}\rangle = 0.5$. The presence of birth requires $\langle x/x_{\rm max}\rangle > 0.5$.

In Table 1, we present the computed values of $\langle x/x_{\rm max}\rangle$ for the models presented in Section 3 for differing values of n and τ. We have also varied the exponent k that fixes the

Table 1. Some examples of how variation of the model parameters affects $\langle x/x_{max}\rangle$. Here n = braking index; T = torque decay time; k = exponent of cut-off line $\dot{P}P^{-k} = K$ = const.; N = number of pulsars included (i.e. to the left of cut-off line). The standard deviation of the mean is $(12N)^{-1/2} = 0.035$ for $N = 70$.

n	T (yr)	k	N	$\langle x/x_{max}\rangle$
3.0	∞	5	70	0.343
2.5				0.397
2.0				0.484
2.5	10^5	3	70	0.571
	10^6			0.460
	10^7			0.416
	10^8			0.413
2.5	2×10^6	5	70	0.381
		4		0.415
		3		0.437
		2		0.409
		1		0.444
3.0	∞	3	98	0.254
			81	0.279
			70	0.374
			52	0.409

cut-off line and chosen values of the constant K such that roughly two-thirds of the sample is retained in the average. This ensures that all nulling pulsars in Ritchings' (1976) list are excluded. The expected error is $(12N)^{-1/2} \sim 0.035$, where N, the number of the sample is ~ 70. Errors in the determination of $\dot{P}$ contribute to the total error in the present sample but should become negligible after a few years observation. In Figs 3–5 we show examples of the distribution of pulsars in the models.

Taken at face value, these results indicate that, unless the braking index $n \gtrsim 2$ or torque decay occurs with a time constant $\lesssim 1$ Myr with $n = 2.5$–3, $\langle x/x_{max}\rangle$ is significantly less than 0.5 and the dynamical model is invalid. (The latter conclusion was reached independently by Fujimura & Kennel (1979) using a fundamentally similar but formally different and more specialized technique.)

It appears that our results are not strongly influenced by observational selection. We have tested the sample to see whether or not there is an inverse correlation between luminosity L and x which would over-represent pulsars at small x and thus lower the value of $\langle x/x_{max}\rangle$ below 0.5. For example, Ruderman & Sutherland (1975) give $L \sim 10^{30}\,(P/1\mathrm{s})^{-15/7}$ erg s^{-1} as an upper bound to the radio luminosities (for $n = 3$). Manchester & Taylor (1977) quote a luminosity function of the form

$$N(L)\,dL \propto L^{-(r+1)}H(L - L_{min})\,dL \tag{8}$$

where $r \cong 1.1$, H is the step function and $L_{min} \sim 10^{26}$ erg s^{-1}. For a two-dimensional galactic distribution, the observed luminosity function in a flux-limited sample is thus

$$N_{obs}(L)\,dL \propto L^{-r}H(L - L_{min})\,dL. \tag{9}$$

We can parametrize the effects of a correlation between L and $\zeta = x/x_{max}$ by expressing the exponent r as a linear function of x, $r = q + p\zeta$ (e.g. for the Ruderman–Sutherland luminosity law and model (i) with $n = 3$, $p = 15/14$). The mean value of ζ in the sample is then

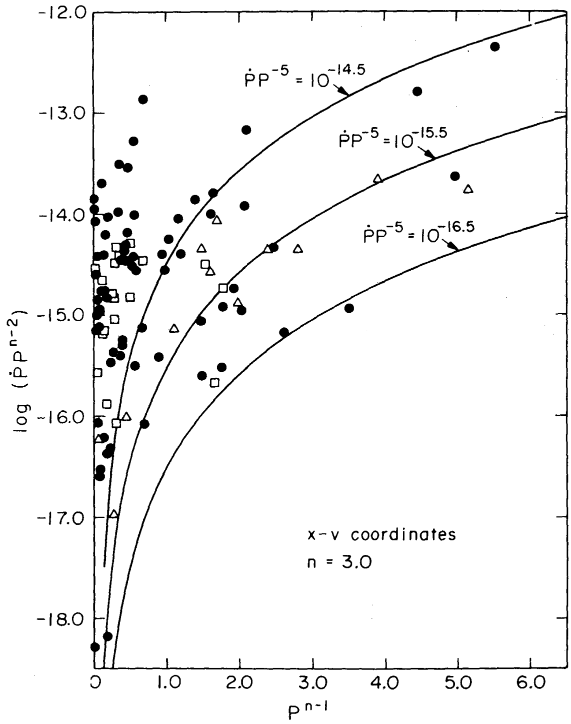

Fig. 3

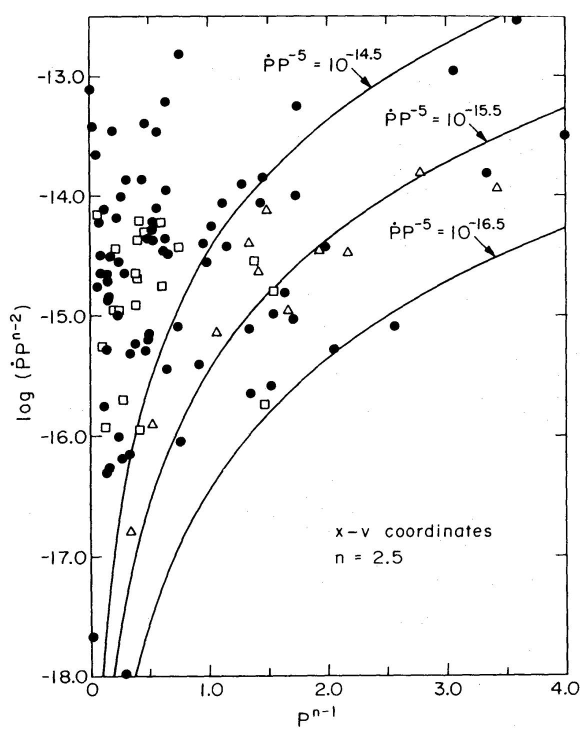

Fig. 4

Figures 3 and 4. x–v coordinates without torque decay. $v = \dot{P}P^{n-2}$ = const. Pulsars march from left to right along horizontal lines. In Fig. 3, $n = 3$. The significance of the strong concentration towards the left-hand side is discussed in Section 4. The superimposed lines are possible 'cut-off' lines also discussed in Section 4. The line $\dot{P}P^{-5} = 10^{-14.5}$ excludes all of Ritchings' possibly moribund pulsars and leads to $\langle x/x_{max}\rangle = 0.34$ (see Table 1). In Fig. 4, $n = 2.5$. The distribution is more uniform than for $n = 3$, but $\langle x/x_{max}\rangle = 0.40$ is still less than 0.5.

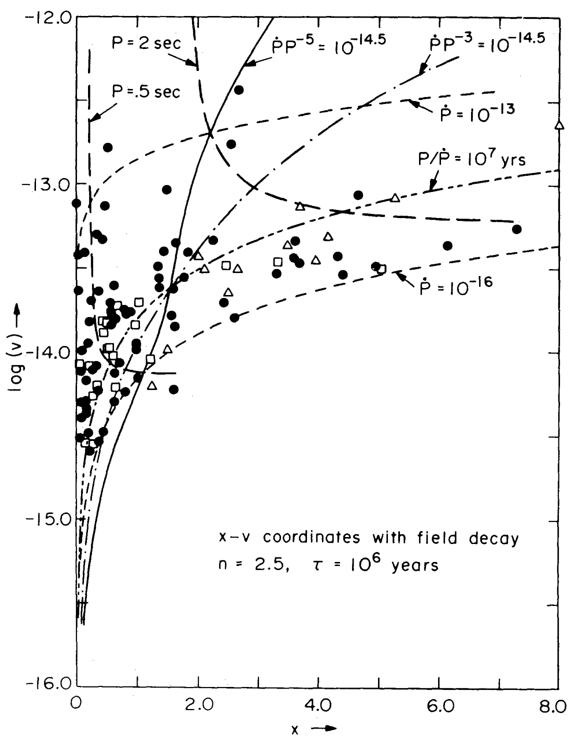

Figure 5. The x–v coordinate in the torque decay model (equation 7). The distribution is nearly uniform, and $\langle x/x_{\rm max}\rangle = 0.46$ with the cut-off line $\dot{P}P^{-3} = 10^{-14.5}$. To drive $\langle x/x_{\rm max}\rangle$ above 0.5 requires time constants for decay shorter than 0.5 Myr (see Table 1). Several cut-off lines, and lines of constant period and $\dot{P}$ are also shown.

calculated to be

$$\langle \xi \rangle = \langle x/x_{\rm max} \rangle = \frac{1-(1+p)\exp(-p)}{p(1-\exp(-p))} \tag{10}$$

a function tabulated in Table 2. If, for example, we wish to account for the computed value of $\langle x/x_{\rm max}\rangle$ with n in the range 2.5–3, then we need $p \gtrsim 1$, which in fact is expected on the Ruderman–Sutherland model. However, in the observed distribution of pulsars (plotted in Fig. 6), no such correlation is apparent and a value of $p > 1$ can be confidently rejected. The present sample may not be flux-limited, however, and again we must await the complete sample for a definite answer.

An alternative selection effect could arise if the beaming factor decreased with x, making fewer old pulsars accessible to observation. This might occur through a physical narrowing of the beam angle ϕ (emission from a fixed radius within the light-cylinder predicts $\phi \propto P^{-1/2}$; the Ruderman–Sutherland model $\phi \propto P^{-29/42}$) or through a progressive alignment of a narrow beam with the spin axis. We can estimate the change in the beaming factor necessary to produce the observed $\langle x/x_{\rm max}\rangle$ very simply. If $\phi \propto x^{-s}$ then $\langle x/x_{\rm max}\rangle = (1-s)/(2-s)$,

Table 2. Effect of a period–luminosity correlation on the observed pulsar distribution of $N(L) \propto L^{-(1+q+p\xi)}H(L-L_{\rm min})$, $\xi = x/x_{\rm max}$.

P	0.1	0.5	1	1.5	2
$\langle\xi\rangle$	0.49	0.46	0.42	0.38	0.34

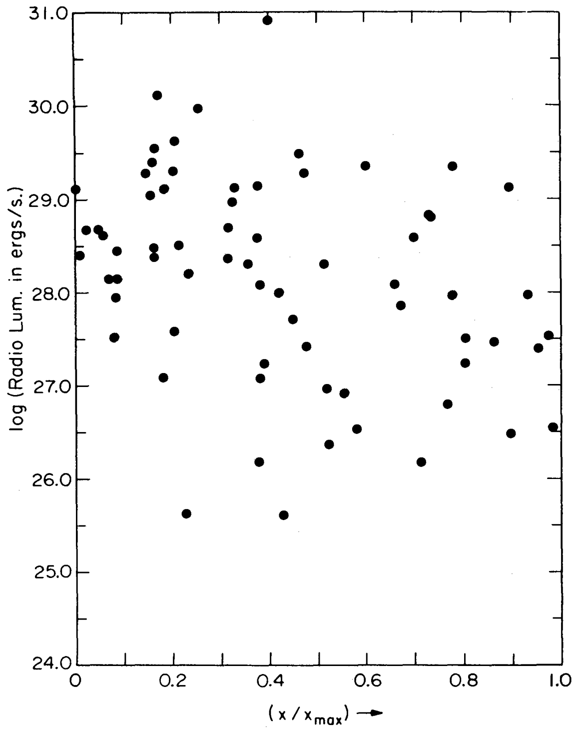

Figure 6. Radio luminosity L versus x/x_{max}. L is computed from the flux at 400 MHz and dispersion measure distances, and has been taken from Taylor & Manchester (1979). $x = 2/3P^{3/2}$, and x_{max} is determined by the crossing with a line of $\dot{P}P^{-3} = 10^{-14.5}$, assuming pulsar motion at constant $\dot{P}P^{1/2}$. No convincing correlation with luminosity is apparent. To reduce the value of $\langle x/x_{max}\rangle$ from 0.5 to its observed value of 0.41 through a correlation of x with L would require $n(x, L) \underset{\sim}{\propto} L^{-1.1(x/x_{max})}$. (See equation 10 and Table 2.)

as long as $s < 1$. Our result that $\langle x/x_{max}\rangle \leqslant 0.4$ would then imply that $s \geqslant 1/3$. For model (i) this requires that ϕ decline faster than $P^{s(1-n)}$ with increasing period. For $n = 2.5$, $\langle x/x_{max}\rangle = 0.4$ requires $\phi \propto P^{-1/2}$, a dependence consistent with theory. Typically we require the beaming factor to decrease by a factor of at least 3 and probably 5 as the pulse period increases from ~0.2 to ~2 s. A difficulty with this explanation of the low value of $\langle x/x_{max}\rangle$ is that the duty cycle does not appear to be significantly correlated with period (Manchester & Taylor 1977; *cf.* also Lyne & Smith 1979). (Note that it is important to define the duty cycle using the pulse width down to a given limiting flux rather than the equivalent width which is usually quoted.) However, if alignment occurs on the same time-scale as the pulsar slow-down and ϕ decreases so that $\phi \underset{\sim}{\propto} \sin\theta$ where θ is the angle between the beam and the spin axis, then the pulse width in longitude ($\propto \phi \operatorname{cosec}\theta$) will be constant while the beaming fraction ($\propto \phi \sin\theta$) will decrease as $\sin^2\theta$, provided the shape of the beam does not change as it shrinks. A typical decrease of θ from $\gtrsim 60°$ to $\lesssim 30°$ is then adequate to give a value of $\langle x/x_{max}\rangle$ consistent with model (i) and $n = 2.5$–3. Furthermore, in this model, $\sin\theta \propto P^{-\gamma}$ with $\frac{1}{4} \lesssim \gamma \lesssim \frac{1}{2}$. Dynamical models describing the alignment of pulsar magnetic moments have been published by several authors (Jones 1976, and references therein). There is, however, no simple argument that is free of detailed assumptions about the direction of the torque and the response of the neutron star crust that will yield this or any other law. One further consequence of alignment is that if the torque acting on the star is predominantly due to the magnetic dipole contribution ($\propto \sin^2\theta\, \Omega^3$) then the effective deceleration parameter will be increased to $n > 3$. A search for slow pulsars with large duty cycles, inevitable when $\theta \sim \phi$, would clearly be of interest (*cf.* Cordes & Dickey 1979).

5 The galactic creation rate

In order to preserve stationarity, observable pulsars must be created at a rate at least equal to the integral of the flux across any line drawn in the x–v plane. If any pulsars 'die' to the left of our line, or are 'born' to the right, then the required creation may be considerably *higher*. The rate of crossing is thus *at least*

$$F = \int f_{\max}(v)\, v\, dv \tag{11}$$

where $f_{\max}(v)$ is the maximum over x of $f(x, v)$ at fixed v. For $n = 2.5$ and no field decay, $F = 10^{-12}$ pulsar s^{-1}, where 50 per cent of the flux is carried by the 10 pulsars with largest v (hence usually 'youngest' $P/\dot{P}$). For $n = 2.5$, and field decay with $T = 2$ Myr (Fig. 3), $F = 8 \times 10^{-13}\,s^{-1}$ and 30 per cent of the flux is carried by the 10 pulsars with largest v. The striking contribution to the flux by the 'fastest' pulsars is perhaps not unexpected in light of Section 2, and suggests that a few pulsars have lifetimes very much shorter than those of the majority. The obvious parallel with ordinary optical stars, where a flux-limited (say sixth magnitude) survey would produce an analogous result due to the over-representation of short-lived OB stars, naturally leads us to inquire whether the 'fast' pulsars are similarly over-represented (*cf.* Gunn & Ostriker 1970). This does not appear to be the case here, since the average distance of the 17 'fastest' pulsars is 3.7 kpc (compared with 3.9 kpc for the entire sample), and their median radio luminosity (compared as in Taylor & Manchester 1979) is 1.4×10^{28} erg s^{-1} (compared with $\approx 10^{28}$ erg s^{-1} for the entire sample).

With this in mind, the calculation of the galactic creation rate is simple. We assume that the sample of 107 pulsars has an x–v distribution representative of the galactic one, so that taking the number of potentially observable pulsars in the galaxy as 10^5 (Manchester & Taylor 1977) (or 4×10^5, Manchester 1979), the galactic creation rate is 10^3 (4×10^3) times the observed one, giving strong lower limits on the creation rate of 1/40 (1/10) yr^{-1} for $n = 2.5$ with field decay and 1/30 (1/8) yr^{-1} without. This result is not especially dependent upon the dynamical model, since model (i) is nearly always assumed to apply to those young pulsars which contribute most of the flux. The conventional technique of using a median age in the calculation of the birth rate is rather misleading, although it is consistent with this estimate. Note also that our argument uses only observable pulsars. The beaming fraction usually estimated from the observed pulse widths to be ~1/5 will increase this estimate to ~1/8 (1/2) yr^{-1}. Only if the galactic density of pulsars has been seriously over-estimated (through underestimated distances or strong selection effects) or if we drop the assumption of stationarity, does it seem possible to reduce this value to one compatible with the supernova rate.

There is the further difficulty mentioned in the introduction in that of the roughly 50 supernova remnants known within 5 kpc of the Sun (Milne 1979), only two are convincingly identified with pulsars. It may be that as well as switching off pulsars also have to switch on and in many cases the remnant has faded before the pulsar is observed.

6 Summary

Systematic procedures for the analysis of pulsar period data have been described and tested on the present incomplete sample. We expect that definitive statements will be possible only when observations from the full survey become available. Despite the limitations of the sample used, the following tentative conclusions were reached.

(1) Model-independent approaches to deriving the period evolution or birth-function are

unlikely to succeed because of the paucity of points in phase-space and the tremendous range of values covered.

(2) If we assume stationarity, a constant beaming factor and that pulsar death occurs only at large $\dot{P}P^{-k}$ for suitable k, then the simple magnetic dipole model of pulsar period evolution $\dot{P} \propto P^{-1}$ cannot account for the distribution of pulsars in P and $\dot{P}$.

(3) A simple deceleration law, $\dot{P} \propto P^{2-n}$, requires $n \lesssim 2$ at least for the shorter periods.

(4) Torque decay requires a time constant $\leqslant 1$ Myr if $n \gtrsim 2.5$.

(5) A beaming fraction that decreases from ~1/4 to ~1/20 as the pulsar ages is compatible with $n = 3$. A natural way for this to occur and satisfy the constraint of constant duty cycle is if the pulsar aligns so that $\phi \propto \sin\theta \overset{\propto}{\sim} P^{-1/2}$.

(6) A lower limit to the creation rate within the galactic volume surveyed by the present sample is fairly insensitive to the dynamical model assumed. The fraction of all pulsars *formed* with high magnetic field strengths is much larger than naive inspection of the fraction so *observed* would lead one to believe. Conversion to a galactic creation rate using the Manchester & Taylor (1977) estimate of the volume surveyed gives a value of one per 8 yr for a beaming fraction of 1/5, while using Manchester's (1979) estimate gives one every 2 yr.

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