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Spectrum of a radio pulse from an exploding black hole*

R. D. Blandford *W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA*

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Summary. Black holes of mass $\lesssim 10^{11}$ kg may be able to radiate all of their rest mass in an essentially instantaneous explosion by the Hawking process provided that the number of available modes increases sufficiently rapidly with temperature. Rees has pointed out that should this happen, electron–positron pairs may form a significant fraction of the explosion products and that if the hole is surrounded by a typical interstellar magnetic field, the kinetic energy of the pairs may be efficiently transformed into radio waves. In this paper the spectrum of the observed radiation is calculated and conditions necessary for this to be detectable are further discussed.

1 Introduction

In Rees (1977) it is pointed out that if a small black hole of mass $m \lesssim 10^{11}$ kg explodes impulsively as a consequence of radiating by the Hawking (1974) process, then up to ~ 50 per cent of the rest mass energy may eventually be transformed into electron–positron pairs of typical energy $\sim 100\,(m/10^{11}\,\text{kg})^{-1}$ MeV. These pairs may then behave electromagnetically like a relativistically expanding conductor capable of excluding surrounding magnetic field. In one interpretation, the field can be decomposed into virtual photons that are essentially reflected by the moving surface with energies boosted by a factor $\sim \gamma^2$ where γ is the Lorentz factor with which the surface moves. This means that if the initial flux density is B, the conductor expands to a radius $R_c \sim (3m/2\pi\epsilon_0\gamma^2B^2)^{1/3}$ before being decelerated provided that the inertia of the surrounding gas is ignorable. The characteristic frequency radiated is $\sim \gamma^2 c/R_c$. For a hole of $m \sim 10^8$ kg, exploding in a typical interstellar field of 0.5 nT, $\gamma \sim 10^5$, $R_c \sim 10^6$ km, and the total radiated energy is dominated by photons of frequency $\nu \sim 1$ GHz. This is potentially observable if the explosion occurs within the Galaxy.

In the following section we calculate the spectrum of electromagnetic radiation from a spherical conductor expanding into a uniform magnetic field. This is specialized in Section 3 to a finite uniform expansion and in Section 4 to the case of an impulsive ultrarelativistic

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explosion. The conditions that must be satisfied if this analysis is to be applicable to a black hole explosion are discussed further in Section 5, amplifying the treatment in Rees (1977).

2 Expansion of a spherical conductor in a uniform magnetic field

Consider a spherical conductor of infinite conductivity whose surface, of radius $R(t)$, expands into a previously uniform magnetic field of flux density B_0 parallel to the z-axis. We wish to calculate the Fourier components of the radiation field and hence obtain an expression for the observed spectral power. On kinematic grounds we expect the field to be concentrated within a shell of thickness $\sim R/2\gamma^2$ where $\gamma = (1-\dot{R}^2)^{-1/2}$ and $c=1$ in this and the following two sections. From flux conservation we see that the field strength in this shell is $B_\theta \sim -\gamma^2 B_0 \sin\theta$ where r, θ, ϕ are standard spherical polar coordinates. The corresponding surface current density can be obtained by transforming into the (primed) frame moving with the conductor, i.e.

$$J_\phi \sim \frac{J'_\phi}{\gamma} \sim \frac{\epsilon_0 B'_\theta}{\gamma} \sim \frac{\epsilon_0 B_\theta}{\gamma^2} \sim -\epsilon_0 B_0 \sin\theta.$$

The strength of this toroidal current will vary as the expansion proceeds and is always $\propto \sin\theta$. This suggests that the radiation field is purely $l=1$, $m=0$ magnetic dipole radiation, a guess which we now confirm.

The method used is a straightforward adaptation of that developed by Colgate & Noerdlinger (1971) in their treatment of electromagnetic pulse production in supernovae. We express the magnetic field $\mathbf{B}(\mathbf{r}, t)$ as a sum of the static field $\mathbf{B}_0$ and the magnetic dipole field $\mathbf{B}_1(\mathbf{r}, t)$ whose Fourier transform can be written

$$\tilde{\mathbf{B}}_1(r,\theta;\omega) = B_0 C(\omega)\exp(i\omega r)\left[2\hat{\mathbf{e}}_r\frac{\cos\theta}{r^2}\left(1+\frac{i}{\omega r}\right) - \hat{\mathbf{e}}_\theta\frac{i\omega\sin\theta}{r}\left(1+\frac{i}{\omega r}-\frac{1}{\omega^2 r^2}\right)\right] \tag{1}$$

(Jackson 1975). The corresponding electric field frequency component is

$$\tilde{\mathbf{E}}_1(r,\theta;\omega) = B_0 C(\omega)\exp(i\omega r)\,\hat{\mathbf{e}}_\phi\frac{i\omega\sin\theta}{r}\left(1+\frac{i}{\omega r}\right). \tag{2}$$

Here $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_\theta$, $\hat{\mathbf{e}}_\phi$ are unit vectors. The boundary conditions to be satisfied on $r=R(t)$, $t \geqslant 0$ are

$$B_r = 0 \tag{3}$$

$$E_\phi + \dot{R}B_\theta = 0 \tag{4}$$

with $\dot{R} = dR/dt$, corresponding to the usual junction conditions on the normal and tangential field components. Writing Fourier transforms in the form

$$\mathbf{B} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega\exp(-i\omega t)\,\tilde{\mathbf{B}},$$

we express condition (3) as

$$\frac{2}{\sqrt{2\pi}R^2}\int_{-\infty}^{\infty} d\omega\exp[i\omega(R-t)]\,C(\omega)\left(1+\frac{i}{\omega R}\right) = -1. \tag{5}$$

Differentiating this with respect to t, we obtain

$$\frac{i}{\sqrt{2\pi}R}\int_{-\infty}^{\infty} d\omega\exp[i\omega(R-t)]\,C(\omega)\left[\omega\left(1+\frac{i}{\omega R}\right) - \dot{R}\omega\left(1+\frac{i}{\omega R}-\frac{1}{\omega^2R^2}\right)\right] = \dot{R} \tag{6}$$

which is recognizable as condition (4). If we solve equation (5) for $C(\omega)$, then the fields (1), (2) will satisfy both boundary conditions, thus vindicating our guess.

If we introduce the quantity

$$\phi(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \exp\,[i\omega(R-t)]\, C(\omega), \tag{7}$$

then straightforward manipulation of equation (5) shows that $\phi(t)$ satisfies the differential equation

$$\dot{\phi}+\phi/R+{}^3\!/\!_2 R\dot{R}=0. \tag{8}$$

For a finite expansion

$$\phi(t)=-\,{}^3\!/\!_2\int_{-\infty}^{t} dt' R'\dot{R}' \exp\left(-\int_{t'}^{t} dt''/R''\right) \tag{9}$$

where $R'=R(t')$ etc. Next we take the Fourier transform of equation (7) and integrate by parts to obtain

$$C(\omega)=\frac{i}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt \exp\,[i\omega(t-R)]\, \dot{\phi}/\omega. \tag{10}$$

Finally from equation (1) we see that the spectral energy radiated per sterad $I_{\omega\Omega}$ is

$$\begin{aligned} I_{\omega\Omega} &= 2\epsilon_0 B_0^2\omega^2 \sin^2\theta\, |C(\omega)|^2 \\ &= \frac{\epsilon_0 B_0^2}{\pi}\sin^2\theta \left|\int_{-\infty}^{\infty} dt \exp\,[i\omega(t-R)]\, \dot{\phi}\right|^2. \end{aligned} \tag{11}$$

Integrating over frequency and solid angle we obtain for the total radiated energy

$$\begin{aligned} I &= \int d\Omega \int_0^{\infty} d\omega I_{\omega\Omega} \\ &= \frac{8\pi}{3}\epsilon_0 B_0^2 \int_{-\infty}^{\infty} dt \frac{\dot{\phi}^2}{1-\dot{R}}. \end{aligned} \tag{12}$$

We also need expressions for the electromagnetic field strength and surface current density at the surface of the conductor

$$\begin{aligned} B_\theta(R) &= -E_\phi(R)/\dot{R} \\ &= -\frac{B_0\sin\theta}{2\pi R\dot{R}}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} d\omega \exp\{i\omega[(t'-t)-(R'-R)]\}\, \phi'(1-\dot{R}')(i\omega-1/R). \end{aligned}$$

(We have substituted equation (10) into equation (2), taken the Fourier transform and reversed the order of integration.) Performing the integration and using the differential equation (8), we obtain finally

$$B_\theta(R)=-\frac{B_0\sin\theta}{(1-\dot{R})}\left(\frac{3}{2}+\frac{\phi}{R^2}\right). \tag{13}$$

In addition by transforming into the frame of the conductor and back again, we find that the surface current density is given by

$$J_\phi = (1 - \dot{R}^2)\,\epsilon_0 B_\theta = -\left(\frac{3}{2} + \frac{\phi}{R^2}\right)(1 + \dot{R})\,\epsilon_0 B_0 \sin\theta. \tag{14}$$

(This current is the source of the radiated field and it can be shown that a calculation of the spectral power starting from equation (14.67) of Jackson (1975) yields an expression identical to equation (11).)

The conservation of energy during this expansion is discussed in the Appendix.

3 Finite uniform expansion

The formulae derived above enable us to calculate the radiated power for arbitrary $R(t)$. The simplest example is provided by a finite uniform expansion commencing at $t = 0$.

$$\begin{aligned} R &= vt \qquad 0 \leqslant t \leqslant \tau \\ &= v\tau \qquad \tau \leqslant t. \end{aligned}$$

From equation (9) we obtain

$$\begin{aligned} \dot{\phi} &= -\frac{3v^3 t}{(1 + 2v)}, \quad 0 \leqslant t \leqslant \tau \\ &= \frac{3}{2}\left(\frac{v^2}{1 + 2v}\right)\tau \exp\left[-\left(\frac{t - \tau}{v\tau}\right)\right], \quad \tau \leqslant t. \end{aligned}$$

The spectral power can then be obtained from equation (11) in closed form. Of immediate interest is the ultrarelativistic limit

$$I_{\omega\Omega} = \frac{\epsilon_0 B_0^2 \sin^2\theta\,\tau^4}{\pi}\,\frac{2[1 - \cos(\omega/\omega_c)] - 2(\omega/\omega_c)\sin(\omega/\omega_c) + (\omega/\omega_c)^2}{(\omega/\omega_c)^4} \tag{15}$$

with

$$\omega_c = [(1 - v)\,\tau]^{-1} \approx 2\gamma_f^2 \tau^{-1},$$

where

$$\gamma_f = (1 - v^2)^{-1/2}.$$

The limiting forms of equation (15) are

$$\begin{aligned} I_{\omega\Omega} &= \frac{\epsilon_0}{4\pi} B_0^2 \sin^2\theta\,\tau^4 + 0(\omega^2), \quad \tau^{-1} \ll \omega \ll \omega_c \\ &= \frac{4\epsilon_0 \gamma_f^4 B_0^2 \sin^2\theta\,\tau^2}{\pi\omega^2} + 0(\omega^{-4}), \quad \omega \gg \omega_c \end{aligned} \tag{16}$$

and the total energy radiated per sterad is

$$I_\Omega = \frac{2\epsilon_0}{3}\gamma_f^2 B_0^2 \sin^2\theta\,\tau^3. \tag{17}$$

4 Ultrarelativistic adiabatic impulsive expansion

The finite uniform expansion treated above is not particularly realistic physically as it involves an infinite deceleration when $t = \tau$. A more realistic and dynamically self-consistent assumption is to conserve the total (kinetic plus radiated) energy per sterad, $E/4\pi$. To lowest order, the equation for the Lorentz factor of the surface of the conductor, $\gamma(t)$ is

$$\frac{E}{4\pi\gamma_f}\frac{d\gamma}{dt} = -\frac{1}{2}J_\phi B_\theta t^2, \quad \gamma \gg 1$$

where $\gamma_f = \gamma(0)$ and we have assumed that the particles remain non-relativistic in the co-moving frame. Substituting from equations (9), (13), (14) this becomes

$$d\gamma/dt = -[(3\gamma^2 t^2)/(\gamma_f \tau^3)]$$

where

$$\tau = [3E/(8\pi\epsilon_0\gamma_f^2 B_0^2 \sin^2\theta)]^{1/3}.$$

So,

$$\gamma = \gamma_f[1 + (t/\tau)^3]^{-1}. \tag{18}$$

Note that $\gamma(t)$ depends also on θ.

Expanding the argument of the exponential in (11), we have

$$t - R \simeq \frac{t}{2\gamma_f^2}\left[1 + \frac{1}{2}\left(\frac{t}{\tau}\right)^3 + \frac{1}{7}\left(\frac{t}{\tau}\right)^6\right]; \quad t \ll \gamma_f^{1/3}\tau$$

and so

$$I_{\omega\Omega} = \frac{\epsilon_0 B_0^2 \sin^2\theta}{\pi}\tau^4 |F(\omega/\omega_c)|^2 \tag{19}$$

where

$$F(\xi) = \int_0^\infty dx\, x \exp[i\xi(x + x^4/2 + x^7/7)]$$

and

$$\omega_c = 2\gamma_f^2/\tau = [(64\pi\epsilon_0\gamma_f^8 B_0^2 \sin^2\theta)/3E]^{1/3} \tag{20}$$

$|F(\omega/\omega_c)|^2$ is plotted in the figure. Limiting forms are

$$|F(\xi)|^2 = 0.615\,\xi^{-4/7} - 0.514\,\xi^{-1/7} + 0.027\,\xi^{2/7} + 0.037\,\xi^{5/7} + 0(\xi^{8/7}), \quad \xi \lesssim 0.1$$
$$\sim \xi^{-4}(1 - 75,600\,\xi^{-6} + \ldots) \quad \xi \gtrsim 10.$$

The low-frequency part of the spectrum is produced at times when the sphere's surface is decelerating. (The spectral index of 4/7 can be understood by observing that for $t \gg \tau$, the characteristic frequency radiated is $\omega \sim \gamma^2 t^{-1} \propto t^{-7}$ and the energy radiated at this frequency, viewed as scattered virtual photons $\propto \gamma^2 B^2 t^3 \propto \omega^{3/7}$. Hence the energy radiated in a given bandwidth $\propto \omega^{-4/7}$.) The explanation for the difference between the high-frequency spectral index in this case (4) and that in the previous section (2) can be traced to the absence or presence of a sudden deceleration.

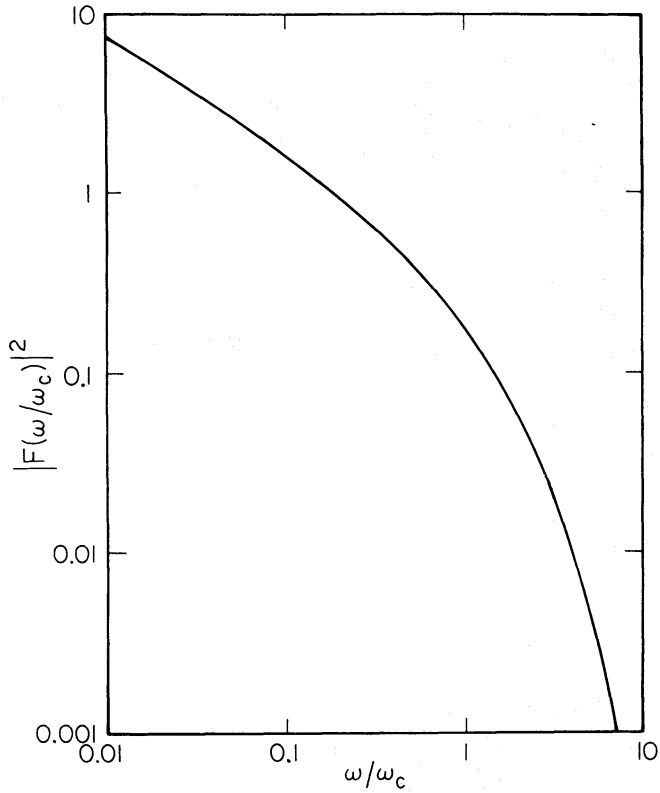

Figure 1. Observed energy spectrum from an ultrarelativistic adiabatic impulsive explosion. $|F(\omega/\omega_c)|^2$ and ω_c are defined in equations (19) and (20).

5 Radiation from exploding black holes

We now apply the above results to the problem of an exploding black hole as discussed by Rees (1977). We shall not discuss the likelihood that a low mass hole evaporate with an impulsive release of energy save to re-emphasize that a necessary condition for this to occur is that a very large (i.e. $\gg 10^6$) number of particle modes be available for the hole to radiate at some critical mass where the hole's temperature exceeds the pion rest mass.

An explosion can be characterized by an isotropic release of energy, E, in the form of electron–positron pairs moving with a broad distribution of radial Lorentz factors centred on γ_f and an explosion time δ (measured in the frame of the external medium). (Production of proton–antiproton pairs is also possible under some circumstances.) By the time that there is a significant electromagnetic interaction with the surrounding magnetic field, electron–positron annihilation will be ignorable although it may be of crucial importance in determining the energy yield E at earlier times.

Assuming that random motions in the comoving frame $\sim c$, the electrons will form a shell of thickness $= \max(c\delta, r/\gamma^2)$. A distant observer receiving photons from the expanding surface will only see a disk of radius $\sim r/\gamma$ and provided that $\gamma \gg 1$, $\dot{R}$ can be treated as effectively constant over the observed portion of the shell. This is important because as shown in the preceding section γ can vary across the surface at a fixed time t. To a high accuracy, the flux observed from such an anisotropic expansion will be similar to an isotropic expansion in which the radial Lorentz factor $\gamma(t)$ has the same variation as that part of the shell in the anisotropic expansion moving towards the observer.

If $E_{25} = E/10^{25}\,\mathrm{J}$, $\gamma_{f5} = \gamma_f/10^5$ and $b = B_0 \sin\theta/0.5\,\mathrm{nT}$, then from equation (20) we have for the critical frequency radiated

$$\nu_c = 1.1\gamma_{f5}^{8/3} b^{2/3} E_{25}^{-1/3}\,\mathrm{GHz}. \tag{21}$$

So if the simple dynamical model used in the preceding section is applicable, most of the energy will emerge at frequencies $\nu \sim \nu_c$ and for $\nu \ll \nu_c$, $\nu \gg \nu_c$ the appropriate energy spectral indices will be 0.57 and 4 respectively.

This, however, assumes that interaction with the surrounding plasma is ignorable and that the expanding shell behaves like a good conductor. If the explosion occurs in an H I cloud, there will be no interaction unless the surrounding medium can be ionized. Photoionization out to a radius $r = 10^6 r_6$ km requires that an energy $\sim 10^{23} r_6^2$ J be produced in the form of uv photons which in view of the nature of the explosion seems unlikely. Ionization by the radiated electric field requires that $\gamma_{f5} \gtrsim 20 b^{-1/2}$ and so for optimal conditions, $\gamma_{f5} \sim b \sim 1$, we expect that only the surrounding field will be swept up by the explosion.

If the surrounding material is ionized then it may share some of the explosion kinetic energy, thus reducing the efficiency of production of radio waves. If the electrons are treated as independent particles, it can be shown (e.g. Noerdlinger 1971) that the passage of the electromagnetic wave will accelerate them to an energy $\sim (eB_0 \sin\theta R)^2/2m_e$ assumed $\lesssim \gamma m_e c^2$ and so the *electrodynamic* power lost is

$$P_{ed} \simeq 9 \times 10^{20} n_{05} b^2 t^4\,\mathrm{W\,ster^{-1}}; \qquad t \lesssim 5\gamma_5^{1/2} b^{-1}\,\mathrm{s}$$

$$\simeq 2 \times 10^{22} n_{05} \gamma_5 t^2\,\mathrm{W\,ster^{-1}}, \qquad t \gtrsim 5\gamma_5^{1/2} b^{-1}\,\mathrm{s}$$

where $n_0 = 10^5 n_{05}\,\mathrm{m^{-3}}$ is the ambient electron density and t is measured in seconds. In contrast, the *electromagnetic* power is (using equation (18))

$$P_{em} \simeq 10^{23} \gamma_{f5}^2 b^2 t^2\,\mathrm{W\,ster^{-1}}$$

valid for

$$t \lesssim \tau = 2.8 E_{25}^{1/3} \gamma_{f5}^{-2/3} b^{-2/3}\,\mathrm{s}.$$

Provided that

$$\begin{aligned} \gamma_{f5} &\gtrsim 0.4 n_{05}^{3/10} E_{25}^{1/5} b^{-2/5}; \quad \gamma_{f5} \gtrsim 0.6 E_{25}^{2/7} b^{2/7} \\ &\gtrsim 0.2 n_{05} b^{-2}; \quad \gamma_{f5} \lesssim 0.6 E_{25}^{2/7} b^{2/7} \end{aligned} \tag{22}$$

only when $t \gtrsim \tau$ can P_{ed} exceed $P_{em} (\propto t^{-4})$. If this occurs at some time t_{ed} such that $\gamma_f^{1/3}\tau \gtrsim t_{ed} \gtrsim \tau$ the shell will quickly decelerate and the main spectral consequence will be a flattening at low frequencies to a spectral index ~ 0. If condition (22) is not satisfied the spectrum radiated will be approximately that derived in Section 3 with $\tau = t_{ed}$. In this discussion we have ignored the effect of the protons. What will probably happen is that the electrons will be accelerated impulsively by the shell subsequently sharing their radial momentum with the protons and giving the same total power loss from the expanding shell.

The ambient electrons may also be coupled collectively to the expanding fireball. In the frame of the shell there will be a beam of electrons and protons with Lorentz factor γ which can be subject to the two-stream instability. If the proper plasma frequencies for the electron–positron plasma in the shell and the ambient electrons are ω_P and ω_{P0} respectively, then the growth rate for the instability in the frame of the shell is $\sim 0.7(\omega_P \omega_{P0}^2)^{1/3}$ (Bludman, Watson & Rosenbluth 1960) and so the beam will be unstable in the dynamical time $\sim t/\gamma$ if

$$E_{25}^{1/6} n_{05}^{1/3} \gamma_5^{-1} t^{1/2} \gtrsim 0.6 \tag{23}$$

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where we have assumed that the shell thickness $\sim ct/\gamma^2$. (If, instead, the thickness is $c\delta > ct/\gamma^2$, then the lhs of (23) must be multiplied by $(t/\delta\gamma^2)^{1/6}$ and equation (24) below modified accordingly.) Substituting for τ, we find that an additional condition for neglecting dynamical interaction with the ambient medium at least for frequencies $\nu \gtrsim \nu_c$ is

$$\gamma_5 \gtrsim 2E_{25}^{1/4} b^{-1/4} n_{05}^{1/4}. \tag{24}$$

So far we have assumed that the expanding shell behaves as a perfect conductor expelling the magnetic flux in accord with equation (4). In the frame of the expanding shell the ratio of the electron gyro frequency ω_G to the plasma frequency ω_P is

$$\omega_G/\omega_P \simeq (t/\tau)^{3/2} (\gamma/\gamma_f)^{1/2}$$

where we have again assumed $\delta \simeq t/\gamma^2$. From equation (18) we see that this ratio $\lesssim 1$ with approximate equality when $t \gtrsim \tau$. Under these conditions, the skin depth in the conductor will be $\sim c/\omega_P$ (Rosenbluth 1957) with the mean particle drift velocity in the conducting layer being $\sim (\omega_G/\omega_P)c$. Thus when $t \gtrsim \tau$ the plasma will probably be heated to a mildly relativistic temperature in order to suppress streaming instabilities. However, provided the mean particle random energy in the co-moving frame does not greatly exceed mc^2, the dynamical assumptions underlying Section 4 should remain valid. The condition that there be sufficient particles to supply the diamagnetic surface current is then that $c/\omega_P \lesssim ct/\gamma$ or

$$\gamma_{f5}\gamma_5 t E_{25}^{-1} \lesssim 10^5. \tag{25}$$

Imposing this condition at $t = \tau$ where it is most stringent gives

$$\gamma_{f5} \lesssim 3000 E_{25}^{1/2} b^{1/2} \tag{26}$$

essentially the same as given in Rees (1977).

If the shell thickness $\delta \gg t/\gamma^2$, its electrodynamic properties are more difficult to guess and certainly depend on its structure. If the field penetrates a substantial mass fraction into the shell, then the effective Lorentz factor will be $\sim (t/\delta)^{1/2}$ and the emitted spectrum will be consequently softened. On the other hand, if the field simply compresses the shell making the random particle energies relativistic, the random energy will be reconverted into bulk motion and the efficiency of conversion of explosion energy into radiation should remain high.

One further physical effect deserves mention – the possibility of a 'Razin-like' suppression of the emitted radiation resulting from a conduction current contributed by the swept-up plasma. We have seen that these particles will be accelerated to a radial Lorentz factor $\gamma_0 \sim 4000(bt)^2$ by the electromagnetic fields and they will contribute a dielectric constant at the characteristic radiated frequency $\omega \sim 2\pi\gamma^2/t$ satisfying

$$\epsilon - 1 \sim \gamma_0(\omega_{PO}^2/\omega^2) \quad 1 \lesssim \gamma_0 \lesssim \gamma.$$

Suppression will be avoided if

$$\gamma_5 \gtrsim 3 n_{05}^{1/2} b t^2; \quad t \lesssim 5\gamma_f^{1/2} b^{-1}.$$

At $t = \tau$ this becomes

$$\gamma_{f5} \gtrsim 4 b^{-1/7} n_{05}^{3/14} E_{25}^{2/7}; \quad \gamma_{f5} \gtrsim 0.6 E_{25}^{2/7} b^{2/7}. \tag{27}$$

A more detailed investigation of this effect is needed to determine whether or not the suppression will be partial or total.

The pulse may also be attenuated as a consequence of its propagation through the surrounding interstellar medium. The effective absorption will depend upon the balance between two effects; the electrodynamical acceleration of free electrons that results from the non-zero magnetic flux convected by the pulse (as discussed by Colgate 1975) and the reflection of low- (including zero) frequency Fourier components of the pulse below the effective plasma frequency, an effect that suppresses the electrodynamical absorption. As the electrons will generally be made relativistic by the pulse, non-linearity in the electrodynamics and instability resulting from electron–ion charge separation complicate any simple picture. This also requires further consideration.

Our conclusion is then similar to that of Rees (1977). Provided that the electrons and positrons can be produced sufficiently impulsively and survive annihilation, the most favourable conditions for detectability of black hole explosions occurring in the ionized interstellar medium with $n_{05} \sim b \sim 1$ arise if the explosions have $10^5 \lesssim \gamma_f \lesssim 10^7$ (applying conditions (22), (24), (26), (27) and assuming ~ 50 per cent yield in the form of pairs so that $E_{25} \sim \gamma_{f5}^{-1}$). Such an explosion at the Galactic Centre would yield a pulsed energy flux $\sim 10^{-26} \gamma_{f5}^{-16/7}$ J m^{-2} Hz^{-1} at $\nu \sim 1$ GHz. For any reasonable bandwidth the length of the pulse would be determined by dispersion (except at low frequencies where interstellar scattering would be important) lasting a time

$$\Delta t \sim 8 DM[\nu/(1\,\mathrm{GHz})]^{-3}[\Delta\nu/(1\,\mathrm{MHz})]\ \mu\mathrm{s}$$

where *DM* is the dispersion measure and $\Delta\nu$ the receiver bandwidth. Integration times $\sim \Delta t$ are clearly required for an optimal detection system.

In fact, pulses of approximately this magnitude have been seen by Hughes & Retallack (1973), and greater sensitivity is achievable with larger dishes. However, in order to exclude possible terrestrial sources of noise it is probably necessary to require the measurement of dispersion using multifrequency observations and simultaneous detection at different locations, as in Meikle *et al.* (1976). The predicted energy spectra discussed above and the expected 100 per cent linear polarization may help to distinguish such pulses from alternative astrophysical sources (such as giant pulses from faint pulsars, suggested by Hughes & Retallack (1973)).

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Appendix: conservation of energy in a spherical expansion

In Section 2 we calculated an expression (equation (12)) for the total radiated energy I. The source of this energy is the mechanical work

$$W=\int_{-\infty}^{\infty} dt \int d\Omega R^2 \dot{R} B_\theta J_\phi/2$$

done by the surface of the conductor. Substituting equations (12), (13) and (14), we obtain for the difference of these two quantities

$$W-I=\frac{4\pi\epsilon_0 B_0^2}{3}\int_{-\infty}^{\infty} dt \frac{[R^2\dot{R}(1+\dot{R})(3/2+\phi/R^2)^2-2\dot{\phi}^2]}{(1-\dot{R})}$$

$$=\frac{4\pi\epsilon_0 B_0^2}{3}\int_{-\infty}^{\infty} dt \left[\frac{9}{4}R^2\dot{R}-\frac{2\phi^2}{R^2}-3\phi\dot{R}-\frac{\phi^2\dot{R}}{R^2}\right]$$

where we have used the differential equation (8). Integrating the final term by parts then gives

$$W-I=\pi\epsilon_0 B_0^2 R^3$$

where R is to be evaluated long after the expansion has ceased. This term is readily identifiable as the potential energy U of the final state, i.e.

$$U=-\tfrac{1}{2}B_0\int_0^\pi R\, d\theta J_\phi \cdot \pi(R\sin\theta)^2$$

$$=\pi\epsilon_0 B_0^2 R^3.$$

This verifies energy conservation and demonstrates that in the ultrarelativistic limit all the dynamical energy is radiated as $U=0(\gamma^{-2})\,I$.