ELECTRON-PROTON ELASTIC SCATTERING AT HIGH MOMENTUM TRANSFERS*

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We report preliminary values of electron-proton elastic scattering measurements at the Stanford Linear Accelerator Center (SLAC) carried out at squared four-momentum transfers q^2 from 0.7 to 25.0 $(\mathrm{GeV}/c)^2$. In the newly accessible region of q^2 above 10 $(\mathrm{GeV}/c)^2$, the principal feature is the continued decrease of the magnetic form factor approximately as $1/q^4$.

The primary electron beam passed through a liquid-hydrogen target and the scattered electrons were momentum analyzed with a magnetic spectrometer utilizing a complex counter system.[1,2] Primary energies from 4.0 to 17.8 GeV and scattering angles from 12.5 to 35.1 deg were used. The raw-data cross sections varied from about 10^{-31} cm^2/sr to about 2×10^{-39} cm^2/sr (0.3 count/h).

The incident electron beam had a momentum spread of 0.2 to 1.0% total $\Delta p/p$. Extensive magnetic measurements and precision surveying allowed the mean initial energy to be fixed to better than 0.5%, which is equivalent to an uncertainty of less than 4% in the cross section. The beam direction was defined to better than 0.1 mrad by alignment of the beam spot on two fluorescent screens. The electron beam had a pulse width of 1.5 μsec and a repetition rate up to 360 cps, and could deliver up to 20-μA average current at 18 GeV at the time of these measurements.

The beam current was measured using a toroid transformer and two thin-foil secondary-emission monitors. A Faraday cup with absolute gain known to within 0.2%[3] was used for regular calibration of the monitors and was removed from the beam during data runs. The beam monitors were stable to within 1 to 2%.

The targets were of the condensation type. From pressure measurements the inferred density was 0.07035 g/cm^3. Acoustic and differential temperature measurements indicated that no correction was necessary for bubbling in the liquid hydrogen.

The spectrometer, which can analyze particles of momentum up to about 8 GeV/c, has two magnets which bend the particles upward through 30 deg and three quadrupoles which provide two separated focal planes. A 55-element scintillation-counter hodoscope at the first focal plane measured the scattering angle θ to within ±0.15 mrad, for 16-mrad total acceptance. A 41-element hodoscope at the second focal plane defined the momentum p of the scattered particles to within ±0.05%, for 4.0% total momentum acceptance. Particle-identification information was obtained from pulse-height spectra of a lead-Lucite total-absorption shower counter whose efficiency was close to 100%. The acceptance solid angle is defined by the spectrometer vacuum tank and is approximately 0.75 msr. Ray tracing with an electron beam and calculations agreed well, and we feel that the error on the solid angle is less than 4%.

Data were recorded on-line using an SDS-9300 digital computer. An event, defined by a fast coincidence between two trigger counters, caused the computer to transfer information from the hodoscopes and pulse-height analyzers onto magnetic tape. As counting rate permitted, on-line analysis of the data was performed which allowed extensive monitoring of the data and equipment.

A number of corrections were made to the data in computing the experimental cross sections. The most important were the following: (1) Radiative losses arising from real and virtual bremsstrahlung during the scattering process.[4] We assumed exponentiation, and this correction varied from 1.23 to 1.40. (2) Radiative losses due to real bremsstrahlung in the target and thin windows.[5] This correction varied from 1.18 to 1.35. We used 59.4 g/cm^2 for the radiation length in liquid hydrogen. (3) Electronic and computer dead times. The beam intensity was adjusted to reduce these effects to less than a few percent, and the losses were accurately monitored. (4) Event decoding losses. About 8 % of the events had ambiguous signatures in the hodoscope arrays. Of these, 5 % were due to double tracks or inefficient counters and were recovered. At this time, 3 % still remain uncertain and these have been included as a systematic error. (5) Target-window subtractions. These were typically 1 to 5 %.

To extract values of G_M from our data, we have used the Rosenbluth equation and the relation $G_E = G_M/\mu$ which is consistent[6] with the determination of G_E at lower q^2. Even if G_E were equal to zero, our values of G_M would change less than 4 % for q^2 greater than 5 (GeV/c)^2. Table I lists the values of cross sections and G_M/μ obtained in this preliminary analysis. The quoted errors arise only from counting statistics, fluctuations in beam monitoring (±1 to 2 %), and relative uncertainties in the radiative corrections (±1.5 %). At some values of q^2 several runs have been combined. Systematic errors are not included and are estimated to be less than 6 % overall.

Figure 1 shows the SLAC cross-section data and the data of earlier observers[7] for q^2 greater than 0.7 (GeV/c)^2. We have plotted the measured cross sections divided by the cross sections calculated from the Rosenbluth formula and assuming the dipole relation $G_E = G_M/\mu = (1+q^2/0.71)^{-2}$. Any other approximate fit could have been used to display differences among the various measurements. Considering the estimated systematic errors, which are not shown, the SLAC results appear to be in agreement with previous experiments.

Figure 2 compares the SLAC measurements with several expressions for G_M/μ, all divided by $(1+q^2/0.71)^{-2}$. This empirical dipole relation is obtained from a fit to the lower-q^2 data. Spectral functions based on narrow vector-meson resonances have been studied[8] which

Table I. Preliminary SLAC electron-proton elastic-scattering data. Only random errors from counting statistics, monitor fluctuations, and relative uncertainties in radiative corrections are shown. The G_M/μ values are derived from the Rosenbluth equation assuming $G_E = G_M/\mu$.

q^2 (GeV/c)^2	INCIDENT ELECTRON ENERGY (GeV)	ELECTRON SCATTERING ANGLE (deg)	$\frac{d\sigma}{d\Omega}$ (cm^2/sr)	$\frac{G_M}{\mu}$
0.690	4.004	12.50	$(.3097 \pm .0036)\times 10^{-30}$	$.2640 \pm .0015$
1.091	3.998	16.25	$(.4897 \pm .0086)\times 10^{-31}$	$.1630 \pm .0014$
1.538	4.003	20.00	$(.997 \pm .016)\times 10^{-32}$	$.1053 \pm .0008$
2.505	7.905	12.61	$(.5102 \pm .0040)\times 10^{-32}$	$(.5245 \pm .0021)\times 10^{-1}$
2.609	7.909	12.92	$(.4255 \pm .0062)\times 10^{-32}$	$(.4998 \pm .0036)\times 10^{-1}$
2.533	6.035	17.21	$(.2296 \pm .0051)\times 10^{-32}$	$(.5098 \pm .0056)\times 10^{-1}$
2.476	5.294	19.75	$(.1801 \pm .0042)\times 10^{-32}$	$(.5285 \pm .0062)\times 10^{-1}$
2.444	4.004	27.50	$(.833 \pm .027)\times 10^{-33}$	$(.5398 \pm .0086)\times 10^{-1}$
3.270	4.002	35.00	$(.123 \pm .010)\times 10^{-33}$	$(.325 \pm .013)\times 10^{-1}$
3.770	10.000	12.47	$(.970 \pm .017)\times 10^{-33}$	$(.2644 \pm .0023)\times 10^{-1}$
5.082	10.702	14.00	$(.1913 \pm .0048)\times 10^{-33}$	$(.1538 \pm .0019)\times 10^{-1}$
5.000	10.001	15.00	$(.1772 \pm .0077)\times 10^{-33}$	$(.1600 \pm .0035)\times 10^{-1}$
6.276	11.349	15.10	$(.571 \pm .023)\times 10^{-34}$	$(.1022 \pm .0021)\times 10^{-1}$
7.512	12.000	16.08	$(.195 \pm .010)\times 10^{-34}$	$(.711 \pm .018)\times 10^{-2}$
8.766	12.700	16.86	$(.707 \pm .053)\times 10^{-35}$	$(.495 \pm .019)\times 10^{-2}$
9.992	13.320	17.62	$(.392 \pm .023)\times 10^{-35}$	$(.421 \pm .012)\times 10^{-2}$
12.49	14.657	18.78	$(.840 \pm .087)\times 10^{-36}$	$(.244 \pm .013)\times 10^{-2}$
15.02	16.008	19.72	$(.328 \pm .038)\times 10^{-36}$	$(.184 \pm .011)\times 10^{-2}$
20.03	17.806	22.90	$(.45 \pm .13)\times 10^{-37}$	$(.103 \pm .015)\times 10^{-2}$
20.00	17.314	24.04	$(.284 \pm .066)\times 10^{-37}$	$(.88 \pm .10)\times 10^{-3}$
25.03	17.314	35.09	$(.42 \pm .20)\times 10^{-38}$	$(.73 \pm .17)\times 10^{-3}$

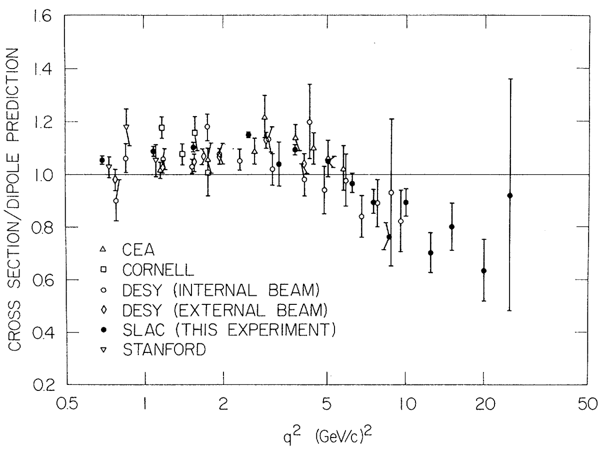

FIG. 1. Compilation of electron-proton elastic-scattering cross sections for q^2 greater than 0.7 $(\mathrm{GeV}/c)^2$. The cross sections are normalized to the Rosenbluth formula and the dipole relation. Systematic errors in the SLAC data are not shown, but would be up to 6%.

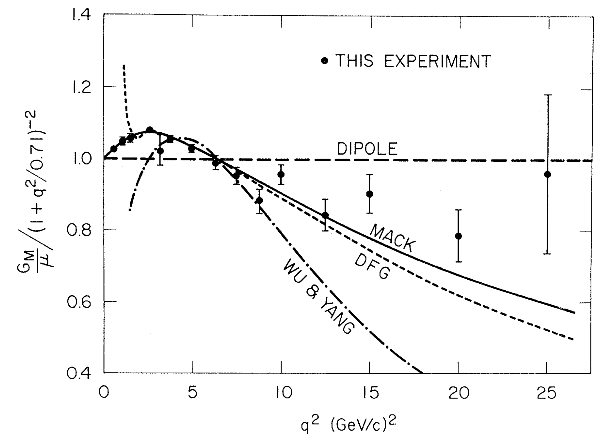

FIG. 2. Comparison of several theoretical expressions for G_M/μ and the SLAC data from Table I. Details of the fits are given in the text.

provide a satisfactory fit to the data up to $q^2 \sim 2$ $(\mathrm{GeV}/c)^2$, but they are not successful at high q^2.[9] A three-pole model using finite widths can be made[10] which will fit the proton data at all q^2. The dominant qualitative feature of the SLAC data is the clear absence of a $1/q^2$ behavior of the form factor for high momentum transfers and the appearance of an approximate $1/q^4$ variation shown by Fig. 2. The ideas of Massam and Zichichi,[11] Kroll, Lee, and Zumino,[12] and Schwinger[13] interpret one q^{-2}-dependent factor as arising from the propagator of the vector meson that is assumed to join the photon to the proton. Whether the remaining $1/q^2$ variation arises from a possible vector-meson–nucleon form factor is an open question.

However, it is also emphasized by Fig. 2 that widely differing functional dependences approximately follow the trend of the data over three orders of magnitude. In particular, we show a simple example of a fractional exponential discussed by Drell, Finn, and Goldhaber[14] (curve DFG in Fig. 2), where $G_M/\mu = 27.8 \times \exp[-(q/0.040)^{1/2}]$. Also included is an expression derived by Mack[15] for G_M/μ that has the asymptotic form $\exp[-A \ln^2(aq^2)]$, where A and a are adjustable parameters.

Wu and Yang[16] suggested on theoretical grounds that there should be an asymptotic connection between p-p elastic scattering and the fourth power of the electromagnetic form factor. In their original fit an exponential decrease of $G_M(q^2)$ with q was suggested. In Fig. 2, we show the fit $G_M/\mu = 0.68\exp(-q/0.60)$, where 0.60 (GeV/c) is derived from p-p scattering and is not adjustable. More recently, Drell has pointed out[17] a correspondence between the e-p and p-p elastic-scattering data that suggests an asymptotic relationship. As the incident proton momentum increases, the shape of $(d\sigma_{pp}/dq^2)/(d\sigma_{pp}/dq^2)_{q^2=0}$ vs q^2 seems to be approaching $(G_M/\mu)^4$.

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ELECTROMAGNETIC MASS DIFFERENCE OF KAONS*

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Using the algebra of currents, modified Weinberg sum rules, and the tadpole model of Coleman, Glashow, and Schnitzer, we calculate the kaon electromagnetic mass difference in the soft-kaon limit to be -3.9 ± 0.6 MeV, in excellent agreement with experiment.

In the course of a recent calculation[1] of the electromagnetic mass difference of pions using chiral $SU(2) \otimes SU(2)$ current-algebra and soft-pion techniques, we introduced a modification of Weinberg's second sum rule[2] which rendered the result finite and in good agreement with experiment. In this note, we extend these considerations to the chiral $SU(3) \otimes SU(3)$ current algebra in order to calculate the second-order electromagnetic mass difference of kaons. We find that this method enables us to compute the "nontadpole" contribution[3,4] to the mass difference, and we find a considerably smaller value than that obtained by the authors of Ref. 4. When our result is combined with their phenomenological value for the tadpole contribution, the total mass difference thus calculated is in excellent agreement with experiment.

To order e^2, the kaon electromagnetic mass difference $\Delta(m_K^2) = m^2(K^+) - m^2(K^0)$ is given by

$$\Delta(m_K^{\ 2}) = -[2(2\pi)^4]^{-1} \int d^4q (q^2)^{-1} (g_{\lambda\beta} + a q_\lambda q_\beta / q^2) T^{\lambda\beta}(p,q), \tag{1}$$

where

$$T_{\lambda\beta}(p,q) = (2\pi)^3 e^2 \int dx\, e^{-iqx} [\langle K^+(p) | T(V_\lambda^{\ \mathrm{em}}(x) V_\beta^{\ \mathrm{em}}(0)) | K^+(p)\rangle - \langle K^0(p) | T(V_\lambda^{\ \mathrm{em}}(x) V_\beta^{\ \mathrm{em}}(0)) | K^0(p)\rangle] + \text{contact term}. \tag{2}$$

We now take the soft-kaon limit ($p_\lambda \to 0$) and use the chiral $SU(3) \otimes SU(3)$ current algebra and partial conservation of axial-vector currents in the form

$$\partial^\lambda A_\lambda^{\ (i)} = m_K^{\ 2} F_K \varphi_K^{\ (i)}, \quad i = 4, 5, 6, 7, \tag{3}$$

to obtain

$$T_{\lambda\beta}(0,q) = e^2 F_K^{\ -2} [\Delta_{\lambda\beta}^{\ V(3)}(q) + \Delta_{\lambda\beta}^{\ V(8)}(q) - 2\Delta_{\lambda\beta}^{\ A(5)}(q) + g_{\lambda\beta} C(q) + M_{\lambda\beta}(q)], \tag{4}$$