THE ASTROPHYSICAL JOURNAL, **237**:99–104, 1980 April 1

FG SAGITTAE—1975 TO 1978

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Received 1979 April 2; accepted 1979 October 4

ABSTRACT

The spectrum of the extremely peculiar variable star, FG Sagittae, has been followed with high dispersion over the interval from 1975 to 1978, thus continuing the work of Herbig and Boyarchuk and of Langer, Kraft, and Anderson.

A new technique for crude abundance determinations is presented. Using the spectrum ratio method, we find that the steady increase in the *s*-process element abundances, which began in approximately 1964, has ceased. A cooling of only 500 K is permitted over the 3 years of our observations, at least on the specific dates we observed the star. The agreement between convective time scales derived from a simple application of the mixing length theory and the observed time scale for the *s*-process episode implies that this event occurred as a result of a convective zone penetrating for a short time into a region where *s*-process nucleosynthesis had already occurred.

Subject headings: nebulae: planetary — nucleosynthesis — stars: individual — stars: interiors — stars: variables

I. INTRODUCTION

The spectroscopic and photometric behavior of the extremely peculiar variable star FG Sagittae prior to 1973 has been discussed by Herbig and Boyarchuk (1968; henceforth HB) and by Langer, Kraft, and Anderson (1974; henceforth, LKA), while photometry from 1968 to 1977 has been presented by Stone (1979). To summarize briefly, the spectral type has grown progressively later, from B4 Ia in 1955 (Henize 1961) to F4 Ip in 1973 (LKA); the continuum temperature is cooling at a rate of approximately 300 K yr^{-1} (Stone 1979); the emission from an expanding gaseous envelope (prominent in the spectra of HB) had almost faded away in 1972 (LKA); and the star is at the center of a planetary nebula with an outer radius of 18″ which has been present since the earliest direct photographs in the beginning of this century. A further outstanding event in the history of FG Sge was the *s*-process episode discovered by LKA which began in about 1964. This was an enhancement of the abundances of *s*-process elements, particularly La, Ce, Ba, Y, and Zr, from approximately the solar abundance prior to 1964 to approximately 25 times the solar abundance in late 1972, while to within the observational errors, the abundance of the non–*s*-process elements such as Fe and Ti remained unchanged. The planetary nebula has normal He, O, Ne, and S abundances (Hawley and Miller 1978).

We have followed this peculiar object during the interval 1975–1978, obtaining a 4 m echelle spectrum of its once each spring. A list of the available spectra is given in Table 1. They are all 4.6 Å mm^{-1} at 5000 Å, taken with the Singer camera and cooled RCA image intensifier, and cover the region from 4900 to 7000 Å. Simultaneously exposed sensitometer plates permit a conversion to intensity using the KPNO PDS digital microphotometer. In § II we present a new technique for determining coudé stellar abundances in cases where identification of every feature and measurement of equivalent widths is not feasible.

A brief summary of the principal results, obtained by applying the ratio technique to this series of spectra as described in § III, follows: (1) The rare earth abundances have stopped increasing. The maximum

TABLE 1

ECHELLE OBSERVATIONS OF FG SAGITTAE

Spectrum	Date (UT)
SI 274..............	1975 June 6
SI 1236	1976 July 7
SI 1742	1977 August 20
SI 2175	1978 June 17

[1] Operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

[2] Operated jointly by the Carnegie Institution of Washington and the California Institute of Technology.

increase over the 3 yr period is less than 0.05 dex, for a cooling of 500 K over this time span. (2) The spectrum over the period from 1975 to 1978 indicates that the star has cooled by not more than 500 K over the 3 yr period, at least on the specific dates covered by our spectra. (3) The spectrum is entirely in absorption, including Hα. There is no evidence for expansion of the atmosphere. (4) The sharp cores seen in H and K by LKA are interstellar. From these observations, we derive the convective velocity in the interior and surface regions of FG Sge and compare it to the theoretically expected values, using the mixing length theory in § IV.

II. THE RATIO TECHNIQUE

As the study of stellar abundances progresses from the simpler spectra characteristic of the early-type stars to the complex ones characteristic of the coolest stars, the techniques of line identification and measurement of equivalent widths (W_λ) for individual features become more and more inadequate. The crowding of lines, the difficulty of locating the continuum, and the presence of strong features absent in the well-studied earlier spectral types conspire to make a conventional abundance analysis almost impossible. In the case where a suitable comparison object is available, we describe a new technique, the ratio method, for determining crude abundances.

The basic premise of the ratio technique is that the spectrum (intensity versus wavelength, even if the wavelength scale is unknown) must exist in digital form. This can be obtained either by microphotometering a photographic plate and converting the resulting densities to intensities or by the direct use at the telescope of a linear digital detector. The same wavelength region must be covered in the comparison object and the unknown. Then the two linear arrays are shifted so that the spectral features overlay each other. (Such shifts arise both from radial velocity differences and from relative misalignment of the spectra on the detector or on the platen of the microphotometer.) The spectrum of the unknown is then divided point by point by the spectrum of the comparison object. It is this ratio spectrum, which we shall denote by D_λ, that can provide crude abundance information for the unknown relative to the comparison object. If the unknown is identical to the comparison spectrum, D_λ will be 1.0 (assuming the spectra have been normalized so that in each case 1.0 is the apparent continuum for all λ). Given information on the difference in the atmospheric parameters between the two stars (i.e., effective temperature and, to a lesser extent, surface gravity) and given that the lines are formed in absorption with a normal curve of growth having the usual form of a linear segment, a segment dominated by microturbulence, and a damping portion, it is feasible to extract crude relative abundance information from the values of D_λ, where D_λ departs from unity and the wavelengths where this occurs.

We demonstrate that the function D_λ can be so used, via some numerical tests, to indicate the expected noise levels resulting from the use of photographic spectra (echelle spectra on IIIa-J emulsion behind a Carnegie image tube). In Figure 1*a* we show approximately 25 Å in the spectrum of NGC 2420, star A, when the echelle blaze function has been removed and the continuum normalized to unity. Figure 1*b* shows the ratio spectrum, D_λ, over the same wavelength region for two echelle spectra of this star taken on the same night, but

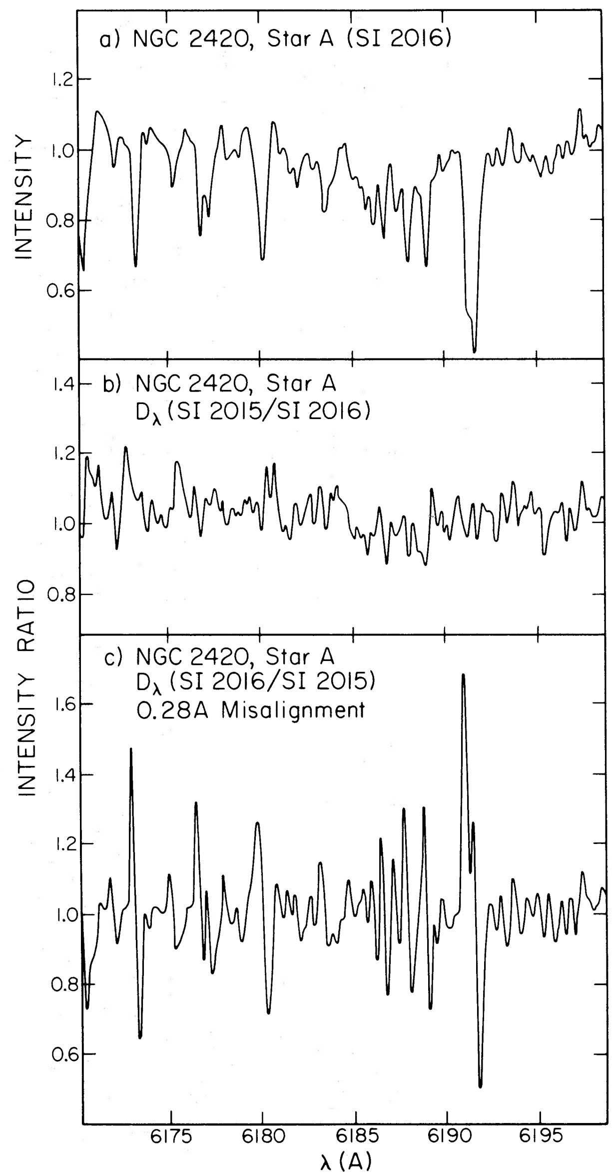

FIG. 1.—(*a*) A region from the spectrum of star A in NGC 2420, where intensity is plotted as a function of wavelength. The echelle blaze has been removed to make the continuum have an intensity of 1.0. (*b*) The function D_λ resulting from the division of two echelle spectra of star A in NGC 2420. (*c*) The function D_λ when one of the spectra of this star is misaligned by 0.28 with respect to the other one.

developed and microphotometered[3] separately. Note that the typical range of noise is $\pm 10\%$. The removal of the echelle blaze function is not perfect in these two spectra, as can be seen from the tendency for D_λ to be slightly larger than 1.0 at the short-wavelength side of the interval. However, the general noise level indicates that variations of weak spectral features with $W_\lambda < 50$ mÅ, which typically have central residual intensities of less than 15% at a resolution characteristic of these spectra, will be lost in the noise. Figure 1*c* shows the characteristic oscillating D_λ resulting from a 0.28 Å deliberate misalignment of one of the spectra of NGC 2420, star A, with respect to the other one.

We now demonstrate that variations in strong features can be seen using the D_λ technique. In Figure 2*a* is shown approximately 25 Å of the spectrum of ROA 58 in ω Cen. Figure 2*b* displays D_λ for the spectrum of ROA 219 in ω Cen compared to that of ROA 58. As shown by Persson *et al.* (1980), these two stars have approximately the same atmospheric characteristics, but ROA 219 has a significantly higher metal abundance. It is immediately apparent that the magnitude of the function D_λ at the position of the lines that vary in strength is significantly greater than 1 and is only slightly smaller than the ratios of W_λ measured directly from the spectra; furthermore D_λ becomes significantly larger than the mean value only at positions corresponding to strong lines. The slight overall departure from unity towards $D_\lambda = 1.1$ could be due to the presence in ROA 219 of many weak features which are undetectably weak in ROA 58 or to poor removal of the echelle blaze. For even larger changes in line strength than those of Figure 2, we expect D_λ, as eventually the line wings grow, to stay large over a wider wavelength range.

[3] All the echelle spectra described in this paper were obtained with the same spectrograph settings and microphotometered by the same person (B. Carder of KPNO) in a consistent manner.

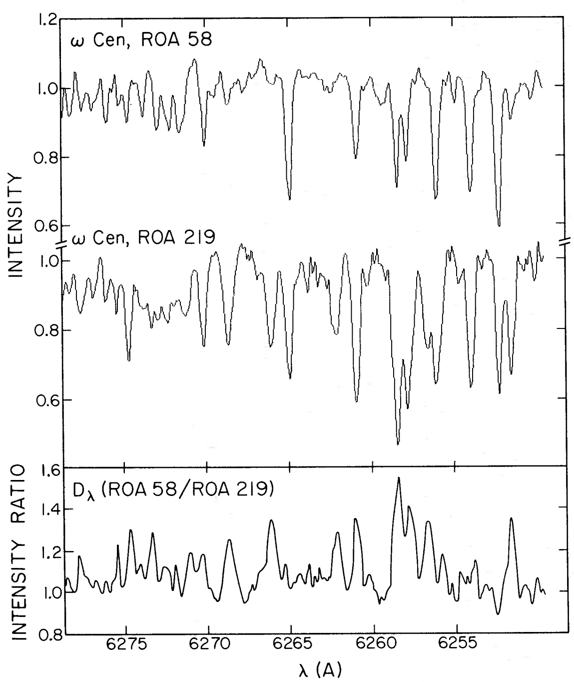

FIG. 2.—(*a*) A region from the spectra of stars ROA 58 and ROA 219 in ω Cen, where intensity is plotted as a function of wavelength. The echelle blaze has been removed to make the continuum have an intensity of 1.0. (*b*) The function D_λ resulting from dividing the normalized spectrum of ROA 58 by that of ROA 219.

Through these numerical tests we have established the general level of noise to be expected in the function D_λ and shown that we can easily detect changes in W_λ of 50% or larger for strong lines. The variations of weak lines ($W_\lambda \leq 50$ mÅ) cannot be studied, as they are lost in the noise.

III. APPLICATION TO FG SAGITTAE

a) *Abundances and* $T_{\rm eff}$

The spectrum of FG Sagittae is extremely complex, with a myriad of lines, even in the region $\lambda > 5000$ Å where we observed. As the line density is as high as in an early M star of solar abundance, rather than attempt to identify the many absorption features, many of which are undoubtedly singly ionized rare earths, we have utilized the ratio technique described in § II to extract constraints on the behavior of $T_{\rm eff}$ and the abundances with time. We have taken the digital intensity spectra for each echelle order and carefully fitted a wavelength scale using a few lines of known λ in each order. The blaze function of the echelle was also removed by fitting a suitable function to continuum points between the lines. We have then divided corresponding orders from different years as functions of λ, using the spectrum from 1975 as a standard. If the absorption features do not change in strength from year to year, the division will yield a constant $D_\lambda(t_1/t_2)$, where t_1 and t_2 denote the years of the divisor and dividend spectrum, respectively. A strengthening of the absorption features in year t_1 as compared to t_2 will produce $D_\lambda(t_1/t_2)$, which decreases below its constant value at wavelengths corresponding to the absorption features. Figure 3 illustrates the 1975 spectrum transformed to intensity for $5850 \leq \lambda \leq 5940$ Å. The function D_λ(1975/1978) is also shown on a separate scale. The small general trend above 1.0 in D_λ(1975/1978) as λ increases is probably due to problems in correctly fitting the continuum location on the two echellograms. We have in fact considered only the function $D_\lambda(t_1/t_2)$, avoiding the difficult problem of finding identifications for the many features in the spectrum of FG Sge.

The principal result of this investigation is that, for the region from 5100 Å to 6600 Å, the function D_λ(1978/1975) implies that there are essentially no lines which increased or decreased in equivalent width (W_λ) by more than 40%, confirming the suggestion of Tenn and Carolin (1977). This result applies to partially resolved components of blends, as well as to

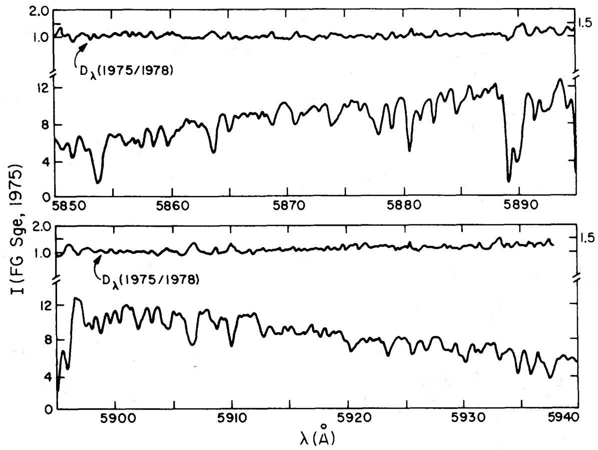

FIG. 3.—One echelle order of the 1975 spectrum of FG Sge transformed to intensity. The function $D_\lambda(1975/1978)$ is shown on a separate vertical scale. The small tendency toward values larger than 1.0 is probably due to problems in fitting the continuum.

individual absorption lines. From the numerical tests of § II, we assert that this statement is valid for lines stronger than 200 mÅ. We have checked this surprising conclusion by directly measuring from the intensity tracings W_λ for a limited number of relatively unblended lines in the 1975 and 1978 spectra, with

TABLE 2

W_λ MEASUREMENTS OF SELECTED LINES

λ	PROBABLE IDENTIFICATION	W_λ (mÅ) SI 274 1975 June 24	SI 1236 1976 July 7	SI 2175 1978 June 17
5582.0	Ca I	265:	240:	425:
5889.9	Na I	830:	895:	940:
5895.9	Na I	575:	645:	660:
5889.9	Int. Na I	580:	575:	540:
5895.9	Int. Na I	565:	445:	545:
5711.1	Mg I	300	250	360
5586.8	Fe I	200:	165:	210:
5731.8	Fe I	175	195	285
5909.9	Fe I	185	245	325
6416.9	Fe II	525:	275:	350:
6222.6	Y I	485	595	555
4883.7	Y II	695:	660:	675:
4982.1	Y II	740:	825:	860:
5473.4	Y II	575:	590:	670:
5620.7	Y II	285	270	345
5728.9	Y II	425	465	540
5781.8	Y II	350	410	475
5092.8	Nd II	455:	480:	540:
5319.8	Nd II	285:	415:	390:
5451.1	Nd II	260	365	420
5322.8	Pr II	280	340	410
5330.6	Ce II	330	360	405:
5472.3	Ce II	350:	400:	425:
5350.2	Zr II	615	670	630
5853.7	Ba II	750	620	720
6390.5	La II	800	755	825
6774.3	La II	680	505	480

results completely consistent with the $D_\lambda(1978/1975)$ technique, as shown in Table 2. The line identifications suggested in Table 2 are probably correct for the strong rare earth lines, but are quite uncertain for the weaker lines of elements lighter than yttrium. The tabulated W_λ are uncertain by at least $\pm 15\%$. Cases of larger uncertainty because of blending are denoted by a colon.

One might expect that all lines stronger than 400 mÅ in the spectrum of FG Sge are on the damping part of the curve of growth, so that beyond this large W_λ, $W_\lambda^{ij} \propto \sqrt{(N_{ij})}$, where N_{ij} denotes the abundance of the ion i in the level j, and W_λ^{ij} is the equivalent width of the appropriate absorption line. However, curves of growth constructed from unpublished Fe II and Ti II W_λ measurements of LKA, kindly supplied by Dr. R. Kraft, show that at a line strength of $\log \lambda/W_\lambda = 4.0$, the best-fitting function appears to be $W^{ij} \propto (N_{ij})^\beta$, where $\beta = 0.30$ rather than 0.50. It may be necessary in this peculiar star to reach extremely large values of W_λ to get the strictly damping part of the curve of growth. To act conservatively, we adopt $\beta = 0.30$.

As the star is presumably cooling from a $T_{\rm eff}$ near 6500 K in 1975 to one closer to 5500 K in 1978 (Stone 1979), a simple ionization equilibrium calculation indicates that lines arising from the ground state of the singly ionized rare earths, Na I, and other neutral ions, will become stronger, since N_{i0} will increase as a result of the cooling. A perusal of any spectral atlas will demonstrate the strong increase in line strength to be expected under these conditions. We first ignore the supposed cooling and deduce limits for $\log |N_{ij}(1978)/N_{ij}(1975)|$ of 0.50 dex based on our observed limit of 40% change in the strong lines. If the star has not cooled at all over the interval from 1975 to 1978, the abundances of all the elements with strong lines in the wavelength interval covered by our spectra, which includes the rare earths, did not increase by more than 0.50 dex over this 3 yr time span. Any cooling, by the argument given above, can only decrease the deduced maximum abundance change of 0.50 dex for rare earths.

To evaluate more quantitatively the effect of the suggested (Stone 1979) 300 K cooling in $T_{\rm eff}$ yr^{-1}, we predicted equivalent widths for plane parallel, LTE, static models from the ATLAS code (Kurucz 1970), with $T_{\rm eff} = 6500$ K and 5500 K and surface gravity $\log g = 1.0$, for lines of various ions with lower excitation potentials of 0 and 2 eV in the appropriate wavelength range, so as to remove (to first order at least) the effects of the change in continuum opacity, with $T_{\rm eff}$ and the effect of the lower state not being the ground state. These models are poor descriptions of the atmosphere of FG Sge, as they predict for solar abundances of the non–s-process elements (believed to be valid for at least the even iron peak elements in FG Sge) values of W_λ smaller than those observed in the spectra. However, this is presumably due to the cooler outermost layers characteristic of extended atmospheres, while the qualitative magnitude of the trends with $T_{\rm eff}$ should be correct. The expected ratios of line

TABLE 3

PREDICTED DAMPING LINE RATIOS FOR CONSTANT ABUNDANCES AND 1000 K COOLING $T_{\rm eff}$

		$\frac{W_\lambda(T_{\rm eff} = 5500\ \rm K)}{W_\lambda(T_{\rm eff} = 6500\ \rm K)}$	
ION	χ (eV)	$\beta = 0.5$	$\beta = 0.3$
Na I	0	2.75	1.83
Ba II	0	2.75	1.83
	2	2.00	1.52
Nd II	0	2.30	1.65
	2	1.80	1.42

strengths for damped lines assuming constant abundances and a 1000 K cooling in $T_{\rm eff}$ over the 3 yr are given in Table 3 for representative lines.

It is immediately apparent by comparing Table 3 with our maximum ratio of 1.40 for $W_\lambda(1978)/W_\lambda(1975)$ that $T_{\rm eff}$ could not have cooled by as much as 1000 K, thus avoiding the absurd conclusion that the abundances of such elements as Na and Fe actually decreased during this time span. Assuming specifically that the Na abundance did not decrease, the D lines imply that the maximum cooling permitted by our spectra was 300 K for $\beta = 0.5$ and 500 K for $\beta = 0.3$. Assuming a cooling of 500 K actually occurred, then the 0.50 dex maximum increase in the rare earth abundance over the time period deduced above is lowered to 0.05 dex. Extrapolating Stone's (1979) observed $\Delta T_{\rm eff}\ \rm yr^{-1}$ during the period 1968–1977 to the period 1975–1978 predicts a cooling of 930 K over the 3 yr. However, our total maximum cooling of 500 K is not completely inconsistent with Stone's photometry, as, especially in the last few years, there has been considerable scatter about the mean cooling curve by up to ± 500 K in $T_{\rm eff}$; and it is conceivable that the deviations in $T_{\rm eff}$ were, for the specific dates on which we had spectra, such as to minimize $T_{\rm eff}(1975)$ to $T_{\rm eff}(1978)$. To maintain closest consistency with Stone's results requires $\beta = 0.30$, in which case the maximum increase in the rare earth abundance is only 0.05 dex over the 3 yr time period.

We can attempt to derive $\Delta T_{\rm eff}$ directly from the spectra using the relationship for supergiants described by Barker *et al.* (1971) between $T_{\rm eff}$ and the width of Hα at a residual intensity of 0.90 [denoted $W_{0.9}$(Hα)]. However, the echelle blaze and weak lines in the wings of Hα affect the choice of continuum and produces an uncertainty in this result. Using a reasonable choice of continuum, we obtain $W_{0.9}$(Hα) $= 9.7$ Å in 1975 and 6.5 Å in 1978, to deduce $\Delta T_{\rm eff} \approx 400$ K, but by lowering the continuum to the limit of credibility, we can get $W_{0.9}$(Hα) $= 7.8$ Å in 1975 and 3.5 Å in 1978, which results in $\Delta T_{\rm eff} \approx 900$ K. Thus, this test is not as conclusive as might be desired in supporting our suggested smaller change in $T_{\rm eff}$ over the 3 yr time span.

We note that the resonance line of Li I has $W_\lambda \leq 150$ mÅ from our spectra. The coincidence of the ionization potentials of Li I and Na I allow us to set a crude upper limit on the Li abundance, assuming solar Na, which is N(Li) ≤ -8.0 dex.

b) Kinematical Considerations

At all times covered by our observations, the spectra of FG Sge showed diffuse broadened lines, typical of late type supergiants, with a full width at half maximum of approximately 0.3 Å when corrected for the intrinsic projected slit, corresponding to an excess broadening of approximately 15 km s^{-1}. Sharp, strong interstellar D lines were present at a radial velocity of -43 km s^{-1} with respect to the star, which corresponds to -2 km s^{-1} with respect to the Sun, using LKA's stellar V_r. These lines are interstellar, based on their radial velocities and profiles; they have remained constant in strength and radial velocity over the time span of these observations and confirm that the sharp absorption cores seen in H and K by LKA must also be of interstellar origin. There were also faint emission lines present, which could not arise from the night sky spectrum, as the exposure times were 10 minutes or less, except for SI 1236. Most likely, they arise in the known planetary nebula, as the spectra were widened by trailing along a decker which was either 7″ or 10″ long. These lines appear to have a radial velocity close to that of the star, as expected from the results of Flannery and Herbig (1973).

The stellar spectrum is entirely in absorption. Even at Hα, there is no definite evidence for emission. The radial velocity of Hα is not displaced from the mean stellar value. The other Balmer lines are not in the region covered by our spectra.

IV. IMPLICATIONS FOR CONVECTION

Based on our results and those of LKA, the *s*-process episode in FG Sge began in approximately 1964, prior to which time the abundances of the *s*-process elements were normal. The *s*-process elements increased in abundance until approximately 1972, when they reached a value of 25 times solar, then remained constant at that level until 1978. We may therefore ascribe the enhancement of these elements to an event which occurred instantaneously deep in the interior of FG Sge, while the 8 yr time scale over which *s*-process abundances increased in the atmosphere was due to the time during which convection brought the processed material from the interior to the surface. If we adopt $m_{\rm bol} = -4.5$, as suggested by LKA, and a $T_{\rm eff}(1978)$ equal to that of the Sun, we obtain $R_{\rm FG} = 5 \times 10^{12}$ cm. If we assume that the *s*-process even took place at a radius half that of the star, we may compute a mean convective velocity, $V_{\rm conv}$, where

$$V_{\rm conv} = \frac{\frac{1}{2}R_{\rm FG}}{8\ \rm yr} = 10^4\ \rm cm\ s^{-1}\ .$$

This velocity will apply only to the interior, of course. Assuming that the diffuse line profiles in the spectrum of FG Sge are due to macroturbulence rather than rotation, we may ascribe this turbulence to large-scale vertical motions of convective cells in the atmosphere. The resulting convective velocities for the atmosphere are about 15 km s^{-1}.

We can crudely compute the mean convective velocity in the interior and atmosphere of FG Sge from first principles using the mixing length theory. We follow the treatment of Cox and Guili (1968), assuming a mass of 1 $M_{\odot}$, and a mixing length equal to $R_{FG}/10$ for the interior and to the pressure scale height for the photosphere. The predicted mean convective velocity is then 10^4 cm s^{-1} for the interior and 100 km s^{-1} for the photosphere. However, $V_{conv} \leq V_{sound}$ in the atmosphere, so that $V_{conv} \lesssim 15$ km s^{-1} in the atmosphere, implying that the energy flux is not transported by convective motions there. Thus, the simple crude scaling of the mixing length theory yields for the mean convective velocity in the interior a value in excellent agreement[4] with that inferred from the observed time scale of the *s*-process event, while the predicted photospheric convective velocity is adequate to produce the observed diffuse line profiles. Therefore, the time scales of the *s*-process episode are consistent with a convective zone penetrating for a short time a region in which substantial *s*-process nuclear reactions have previously occurred.

An extensive discussion of convective mixing after He shell flashes is given by Christy-Sackmann and Despain (1974) and Sackmann (1979), while the possibility of *s*-process nucleosynthesis under such circumstances is explored by Iben (1975). We stress that the observations themselves imply an essentially instantaneous dredging up of *s*-process material into the H-rich envelope, with convection providing the observed, approximately 8 yr time scale for the *s*-process episode. It is still unclear why this mixing occurred 80 yr after the last He flash, when the star may be about to stop cooling and whether or not the mixing and the end of the cooling phase are in some way related.

[4] A more careful calculation, dividing the outer half of the star into five zones each of size $R_{FG}/10$ and evaluating the convective velocity in each zone, then considering the effective convective velocity as $\frac{5}{\sum (V^i_{conv})^{-1}}$ gives a larger effective convective velocity for the interior; but the effective convective velocity still agrees with the observed value to within a factor of 3.

V. THE FUTURE OF FG SAGITTAE

As of 1979 June the G band of CH was present, although not abnormally strong. No firm evidence for the presence of C_2 can be found. It will be extremely interesting to monitor this unusual object in the future to determine whether it will become a carbon star and whether it will continue cooling toward a white dwarf or loop back over toward the blue in an H-R diagram, as discussed by Paczyński (1971). The theoretical problem of why the *s*-process episode took place in FG Sge during the years 1964 to 1972 remains unresolved.

We thank Juliana Sackmann and Robert Kraft for helpful conversations. We grant absolution to the anonymous referee whose criticism forced us to work harder and greatly improved this paper.

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