

The Final Parsec Problem

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Abstract. The coalescence of massive black hole binaries is one of the main sources of low-frequency gravitational radiation that can be detected by LISA. When two galaxies containing massive black holes merge, a binary forms at the center of the new galaxy. We discuss the evolution of the binary after its separation decreases below one parsec. Whether or not stellar dynamical processes can drive the black holes to coalesce depends on the supply of stars that scatter against the binary. We discuss various mechanisms by which this supply can be replenished after the loss cone has been depleted.

INTRODUCTION

The prospect that low-frequency gravitational radiation will be detected by LISA has recently energized theoretical inquiries into the formation and the evolution of massive black hole binaries (MBHB). MBHB are sources of low-frequency gravitational radiation that will provide highest signal/noise for LISA, but the event rate for these sources is much less well known than for the other two principal astrophysical sources, compact binaries and extreme mass ratio inspiral (see contributions by G. Nelemans and S. Sigurdsson, this volume). Astronomical evidence for the existence of black holes with masses $M_{\text{bh}} \gtrsim 10^6 M_{\odot}$ in galaxy spheroids with central stellar velocity dispersions $\sigma \gtrsim 100 \text{ km s}^{-1}$ is increasingly compelling; the evidence for black holes with masses $100 - 10^6 M_{\odot}$ in low-dispersion spheroids is still equivocal. When two galaxies merge, a MBHB forms at the center of the new galaxy [1, 2]. There has been considerable interest in determining whether black hole coalescence occurs efficiently following galaxy mergers, since almost all predictions of MBH coalescence event rates equate the galaxy merger rate – derived from models of structure formation in which galaxies merge hierarchically [3, 4, 5, 6, 7] – with the MBH coalescence rate. Detailed estimates of the MBH coalescence rate are fundamentally limited by the resolution of electromagnetic telescopes. It is therefore expected that MBH coalescences as observed by LISA will facilitate the first definite conclusions regarding galaxy merger rates and MBH demographics at high redshifts [8].

The mutual gravitational capture of the black holes is facilitated by the dynamical drag imparted to the orbiting MBHs by the overlapping galaxies. The inner parts of elliptical galaxies and many spiral bulges are well described by power-law (“cuspy”) stellar density profiles of the form $\rho \sim r^{-\gamma}$ with $\gamma \approx 2$ [9]. High luminosity ellipticals often exhibit a shallowing of the density profile ($\gamma < 2$) near the very centers, where the MBHs are located. Numerical simulations of galaxy mergers have shown that the

black holes remain embedded in their donor cusps throughout the merger [10]. Since the dynamical drag is a function of the combined black hole and stellar cusp mass, which exceeds that of the black hole until the very final stages of the merger, the black hole inspiral and the MBHB formation take place on a dynamical time scale for black holes of comparable mass. In the unequal mass case, the infall time scale is lengthened proportionally to the black hole mass ratio M_1/M_2 ($M_1 \geq M_2$), but for $M_2 \gtrsim 10^6 M_\odot$ remains short compared to time scale for the subsequent evolution [35], which is the subject of this contribution.

GRAVITATIONAL SLINGSHOT AND BINARY DECAY

The galaxy merger delivers the black holes to within the distance $a_{\text{hard}} = G\mu/4\sigma^2$, where $\mu = M_1 M_2 / (M_1 + M_2)$ is the reduced mass. This distance is about 1 parsec for $M_1 = M_2 = 2 \times 10^7 M_\odot$. At that point, the binary continues to decay by scattering stars super-elastically [11]. Stars in the merged galaxy with orbits approaching the binary closely enough to be perturbed by the rotating quadrupole component of the binary's potential belong to the "loss cone." The loss cone is defined in analogy with a similar structure characterizing the distribution of the stellar-mass objects around solitary massive black holes [12, 13]. When a star inside the loss cone impinges on the MBHB, it exchanges kinetic energy with the binary and is shot out at an average velocity comparable to the binary's orbital velocity $\bar{v}_* \sim \sqrt{G(M_1 + M_2)/a} \gg \sigma$. This is a form of the gravitational slingshot mechanism commonly used to accelerate spacecraft in the solar system. As a result, the binary's binding energy increases and its semi-major axis a decreases.

In a crude approximation, the factor by which the binary separation decays can be related to the total mass in stars M_{scat} that are scattered against the binary between times t_1 and t_2 via [14]:

$$a(t_2) \sim a(t_1) \exp \left\{ - \frac{M_{\text{scat}}(t_1, t_2)}{\mathcal{J} (M_1 + M_2)} \right\}, \quad (1)$$

where \mathcal{J} is the mass ejection coefficient which in fact depends weakly on a/a_{hard} and the black hole masses.

Approximation (1) is widely used in the semi-analytic modeling of the MBHB population dynamics within hierarchical structure formation scenarios. Value of the mass ejection coefficient has been estimated from three-body scattering experiments [14]:

$$\mathcal{J} \approx 0.1 \ln \left(\frac{4a_{\text{hard}}}{a} \right) \approx 0.1 - 1. \quad (2)$$

This agrees with the value $\mathcal{J} \approx 0.5$ measured in N -body simulations of equal-mass MBH mergers [10]. If the galaxy is nearly spherical and the ejected stars return to interact with the binary more than once, the efficiency of mass ejection depends on the potential well depth $\Delta\Phi$ separating the energies at which stars enter and exit the loss cone [15]:

$$\mathcal{J} \approx \frac{1}{(\Delta\Phi/2\sigma^2)(a/a_{\text{hard}})}. \quad (3)$$

To shrink the binary by one e -folding, a stellar mass of $\mathcal{J} M_{\text{bh}}$ must be transported from an energy marginally bound to the black hole, to the galactic escape velocity.

COALESCENCE

If the semi-major axis decreases until it becomes less than [16]:

$$a_{\text{gr}} = \left\{ \frac{64 G^3 M_1 M_2 (M_1 + M_2) F(e)}{5 c^5} t_{\text{gr}} \right\}^{1/4}, \quad (4)$$

the emission of gravity waves leads to the coalescence of the black holes on a time scale t_{gr} . Here, $F(e)$ is an eccentricity-dependent factor equal to unity for a circular binary. N -body simulations of MBHB formation in galaxy mergers suggest that the eccentricities remain moderate, although our understanding of the MBHB eccentricity evolution is still incomplete. The factor by which the binary must shrink from its conception to coalescence is:

$$\frac{a_{\text{hard}}}{a_{\text{gr}}} \approx 34 \times e^{9.2/\alpha} \left(\frac{M_1 + M_2}{10^6 M_{\odot}} \right)^{1/4 - 2/\alpha} p^{3/4} (1+p)^{-3/2} \left(\frac{t_{\text{gr}}}{10^9 \text{ yrs}} \right)^{-1/4}, \quad (5)$$

where $p = M_2/M_1 \leq 1$ and $\alpha = d \log M_{\text{bh}} / d \log \sigma \approx 4 - 5$ is the logarithmic slope of the tight relation between the black hole mass and the central velocity dispersion of the galaxy [17, 18].

Therefore, to achieve coalescence in a Hubble time, the MBHB must shrink by 4 – 5 e -foldings, which requires the scattering of 10 – 20 $\times \mu$ worth of stars from the loss cone. After its initial emptying and the accompanying rapid shrinking of the binary by a factor of ~ 5 past a_{hard} (as observed in N -body simulations with $\rho \sim r^{-2}$ initial density profiles [10]), the mass of the loss cone $\lesssim \mu$ if the galaxy is approximately spherical. Once this mass is expended, the binary is still a factor $\gtrsim 10$ wider than the separation at which it would coalesce gravitationally in a Hubble time; the binary decay may therefore stall [2, 10, 19, 20, 21]. The problem is exacerbated if the initial cusp profile is shallower than r^{-2} . The apparent inability of stellar dynamical processes to drive the binaries to coalescence poses a potential problem for the detection of these sources by LISA. Circumstantial evidence, however, suggests that long-lived MBHBs may be rare: no smoking-gun detections of MBHBs have been reported (with the possible exception of OJ 287; see [22]), although a number of AGN have time-dependent features that have been provisionally attributed to MBHBs (Komossa, this volume). The theoretical difficulty of shrinking a MBHB by a factor of ~ 100 after its formation at a separation of ~ 1 pc is called the “final parsec problem.”

LOSS-CONE DIFFUSION

The final parsec problem is most severe in nearly spherical galaxies where the mass inside the loss cone is the smallest. The loss cone boundary is defined by the minimum

angular momentum $J_{\text{loss}} \sim \sqrt{G(M_1 + M_2)a}$ that a star can have and still avoid being perturbed by the MBHB. Consider the state of the loss cone after all the stars inside have been scattered *once*. Since the depletion of the orbital population inside the loss cone leads to a stalling in the decay rate, continued decay hinges on the rate at which stars diffuse into the loss cone via collisional relaxation. The relaxation can be modeled by means of the orbit-averaged Fokker-Planck equation [23] describing the evolution of the phase space density $f(E, J, t)$ subject to the boundary condition $f(E, J_{\text{loss}}, t) = 0$; i.e., stars are assumed to be removed from the system by the gravitational slingshot once they straddle the loss cone boundary. The loss cone flux is proportional to the gradient of the phase space distribution at its boundary, $\mathcal{F} \propto \partial f / \partial J|_{J_{\text{loss}}}$.

Although it is tempting to seek a steady-state solution $f_{\text{equi}}(E, J) \propto \log(J/J_{\text{loss}})$ [24, 25, 21], in reality the center of the galaxy is not likely to be collisionally relaxed [26], and thus the distribution of stars near the loss cone is never in a steady state on time scales of order the relaxation time. Indeed, the final stages of a galaxy merger when the MBHB forms proceed in a time much shorter than the galaxy crossing time. Therefore the distribution function $f(E, J)$ immediately following the formation of a hard binary can be far from that describing a steady-state flux of stars into the loss cone. Sudden draining of the loss cone during formation of the hard binary produces steep phase space gradients that are closer to the step function:

$$f(E, J) \approx \begin{cases} \bar{f}(E), & J > J_{\text{loss}} \\ 0, & J < J_{\text{loss}} \end{cases}. \quad (6)$$

Since the collisional transport rate in phase space is proportional to the gradient of f with respect to J , steep gradients imply an enhanced flux into the loss cone. The depletion of stars *outside* the loss cone affects the density profile of the galaxy and is identified with the cusp destruction. The broken power-law profiles of high-luminosity elliptical galaxies can be interpreted as fossil evidence for this process [27, 28].

The time evolution of the stellar distribution near the loss cone is an initial value problem equivalent to the diffusion of heat in cylindrical coordinates [12]. Ignoring the diffusion in E , the Fokker-Planck equation for diffusion in J reads ($J \geq J_{\text{loss}}$):

$$\frac{\partial f(E, J, t)}{\partial t} = \frac{\lambda(E)}{J} \frac{\partial}{\partial J} \left[J \frac{\partial f(E, J, t)}{\partial J} \right], \quad (7)$$

where $\lambda(E)$ is related to the orbit-averaged diffusion coefficient. Since the boundary angular momentum J_{loss} decreases with time, equation (7) can be solved iteratively by discretizing the decrements in J_{loss} and interpolating f between these via the Fourier-Bessel synthesis [15]. Solutions obtained this way can be compared to the collisionally relaxed, steady state ($\partial f / \partial t = 0$) solution normalized to the isotropic distribution (Figure 1a). In an example scaled to the galaxy M32 with a $3 \times 10^6 M_{\odot}$ black hole, the binary in the exact solution has decayed only $\sim 30\%$ more than that in the steady-state solution. The difference between the two, however, is much more substantial early on (Figure 1b), which is of crucial importance if episodic, violent dynamical perturbations such as the infall of satellite galaxies or giant molecular clouds rejuvenate the loss cone [29] by restoring the steep phase-space gradients instrumental for the enhanced diffusion (Figure 1c). For example, if the episodic replenishment in a galaxy like M32 occurs every

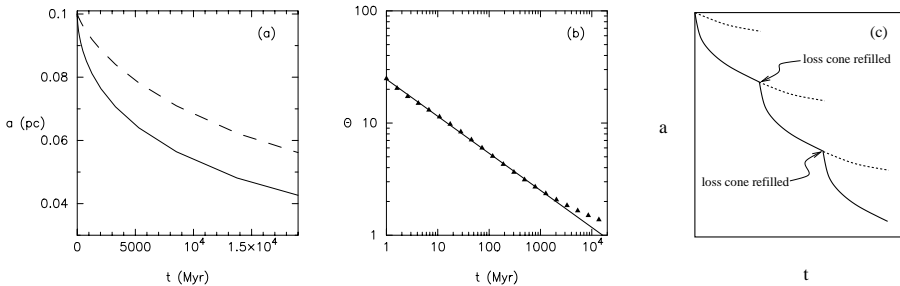


FIGURE 1. (a) Evolution of the semi-major axis a in a merger of equal galaxies with $\sim 10^6 M_\odot$ black holes starting from the initial separation of $a(0) = 0.1$ pc (solid line). The evolution is always more rapid than that predicted assuming that the galaxy is collisionally relaxed (dashed line). (b) Enhancement of the decayed compared with that of the collisionally-relaxed, steady-state solution, expressed in terms of $\Theta = \Delta a / \Delta a_{\text{equi}}$ (triangles) and a power-law fit (solid line). (c) Schematic illustration of the evolution of the semi-major axis in the presence of episodic refilling of the loss cone. (From [15].)

10, 100, or 1,000 Myrs, the average MBHB decay rate will be $\Theta \approx 10, 5,$ or 3 times higher than what the equilibrium theory would have implied [15].

Brownian motion of the MBHB in the neighborhood of the geometric center of the galaxy can to some extent mimic the effects of collisional relaxation and drive stars into the loss cone. The Brownian motion results from the equipartition of kinetic energy between the MBHB and the stars in the galaxy, $\langle v_{\text{brown}}^2 \rangle \sim (m_*/M_{\text{bh}})\sigma^2$, where m_* is the average stellar mass [30, 31, 32]. As the binary wanders in space, it can sweep up stars that would have remained just outside the loss cone for a static binary. The time scale on which the loss cone refills in this fashion is:

$$t_{\text{brown}} \sim 400 \text{ Myr} \times \frac{a}{a_{\text{hard}}} \left(\frac{m_*}{M_\odot} \right)^{-1} \left(\frac{M_1 + M_2}{10^6 M_\odot} \right)^{2-3/\alpha} e^{14/\alpha} K(E), \quad (8)$$

where $K(E)$ is a function of the orbital energy such that $K(2\sigma^2) \sim 1$. The amplitude of the Brownian motion is only modestly enhanced by “super-elastic scattering” by the binary [30]. In galaxies $t_{\text{brown}} \gtrsim 1$ Gyr and thus the Brownian motion probably fails to significantly enhance the flux into the loss cone, in spite of earlier suggestions to the contrary [33]. In N -body simulations, however, the Brownian motion *saturates* the loss cone flux which is one of the many pitfalls that plague the numerical modeling of MBHBs.

SPHERICAL AND ASPHERICAL GALAXIES

Mode of interaction between the stars in a galaxy and a MBHB is also influenced by the geometry of the galactic potential, which could either be nearly spherical or substantially aspherical (axisymmetric, triaxial, or irregular).

In spherical galaxies all stars that are candidates for slingshot ejection by the MBHB encounter the binary within one crossing time from the moment the binary forms. The

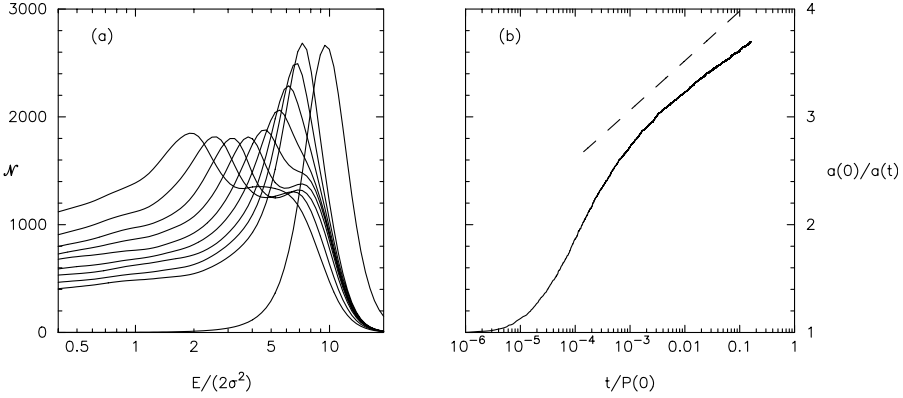


FIGURE 2. (a) Snapshots of the binding energy distribution $\mathcal{N}(E, t)$ of the stars residing inside the loss cone at $t = 0$. From right to left the data were taken at exponentially-increasing time intervals. As the binary separation decreases and the velocity of the slingshot ejection increases, stars inside the loss cone are heated through repeated scattering and shift to smaller E ; a significant fraction of them remain inside the loss cone at all times. (b) Evolution of the semi-major axis exhibiting the logarithmic behavior motivated in the text (solid line). The slope of $da/d \ln t$ is close to that given by the theoretical prediction (10); $P(0)$ is the dynamical time of the potential well in which the galaxy is embedded (dashed line). (From [15].)

time-dependent loss cone solution derived in the previous section was based on the “sink” paradigm, in which a star is lost from the system as soon as it transgresses the loss cone boundary. This model is valid in the case of capture or tidal disruption of stars by a single black hole but is less relevant to MBHBs, since stars that interact with the binary simply receive kicks $(\Delta E, \Delta J)$ that transport them to another orbit without necessarily ejecting them from the galaxy. If the binary orbit decays on a time scale longer than the orbital period of an interacting star, stars inside the loss cone can remain inside the loss cone after ejection, encountering the binary again at their next pericenter passage. In principle a star can interact many times with the binary before being ejected from the galaxy or falling outside the loss cone; each interaction takes additional energy from the binary and hastens its decay.

We illustrate this “secondary slingshot” with a simple model in the spherical geometry. Consider a group of \mathcal{N} stars of mass m_* and energy per unit mass E that interact with the binary and receive a mean energy increment of $\langle \Delta E \rangle$. Averaged over a single orbital period $P(E)$, the binary hardens at a rate [15]:

$$\frac{d}{dt} \left(\frac{GM_1 M_2}{2a} \right) = m_* \frac{\langle \Delta E \rangle \mathcal{N}}{P(E)}. \quad (9)$$

In subsequent interactions, the number of stars that remain inside the loss cone scales as $J_{\text{loss}}^2 \propto a$, while the ejection energy is $\sim \Phi_{\text{eject}} + G(M_1 + M_2)/2a \sim a^{-1}$. Hence $\mathcal{N} \langle \Delta E \rangle \propto a^1 a^{-1} \propto a^0$. Assuming a singular isothermal sphere for the galaxy potential,

we derive the result [15]:

$$\begin{aligned} \frac{a(0)}{a(t)} &\sim 1 + \frac{4\sigma^2 a(0)}{G(M_1 + M_2)} \ln \left[1 + \frac{m_* \mathcal{N} \langle \Delta E \rangle}{2\mu\sigma^2} \frac{t}{P(E_0)} \right] \\ &\sim 1 + 0.25 \ln \left[\text{few} \times 10 \times \frac{t}{P(E_0)} \right]. \end{aligned} \quad (10)$$

Hence the binary's binding energy increases as the logarithm of the time. Figure 2 illustrates the evolution of a^{-1} in an N -body simulation where star-star interactions have been replaced by a smooth potential to inhibit relaxation. The observed rate of decay $da^{-1}/d\ln t \approx 1.7 \times 10^4$ is close to the prediction of equation (10), where we have $4\sigma^2/G(M_1 + M_2) = 2 \times 10^4$. Re-ejection at the rate of equation (10) would by itself induce changes in a^{-1} by factors of a few over a Hubble time, in addition to the shrinkage due to collisional loss cone refilling. For $P(E_0) = 10^{3.5,7}$ years, we obtain $a(0)/a(t_{\text{Hubble}}) \approx 5, 4,$ and $3,$ respectively.

In non-spherical galaxies where the total angular momentum J is not a conserved quantity, there exists a potentially larger population of orbits that can encounter the binary but only once per several orbital periods. The mechanisms of loss cone evolution and re-ejection discussed above in the spherical geometry would be modified somewhat in axisymmetric [21] or triaxial galaxies [36]. The triaxial case is potentially the most interesting: stellar bars are commonly observed in galactic nuclei, and torques from barlike potentials are often invoked to drive gas inflows during the quasar epoch [47]. Since orbital angular momentum is not conserved in the triaxial geometry, a large fraction of the stars in a triaxial bar can potentially interact with the central MBHB. These ‘‘centrophilic’’ orbits are typically chaotic due to scattering off the central mass; in spite of their unfavorable time-averaged shapes, chaotic orbits can make up 50% or more of the total mass of a triaxial nucleus [34].

Numerical integrations [36] reveal that the frequency of pericenter passages with $r_{\text{peri}} < a$ for a chaotic orbit of energy E is roughly linear in a , $\mathcal{N}(r_{\text{peri}} < a) \approx A(E)a$, up to a maximum pericenter distance of $r_{\text{peri,max}}(E)$. The total rate at which stars pass within a distance a of the center is therefore:

$$\dot{M} \approx a \int A(E) \mathcal{M}_{\text{chaos}}(E) dE \quad (11)$$

where $\mathcal{M}_{\text{chaos}}(E)dE$ is the mass on chaotic orbits in the energy range E to $E + dE$. In a triaxial nucleus with density $\rho \sim r^{-2}$ and central mass M_{bh} , the numerical integrations reveal:

$$A(E) \approx 5 \frac{\sigma^5}{G^2 M_{\text{bh}}^2} e^{-(E - \Phi(r_{\text{bh}}))/\sigma^2}, \quad r \gtrsim r_{\text{bh}} \equiv \frac{GM_{\text{bh}}}{2\sigma^2}. \quad (12)$$

The feeding rate due to stars with energies $E > \Phi(r_{\text{bh}})$ is then

$$\dot{M} \approx \mathcal{F}_{\text{chaos}} \frac{\sigma^3}{G} \frac{a}{r_{\text{bh}}} \approx 10^3 M_{\odot} \text{ yr}^{-1} \frac{\mathcal{F}_{\text{chaos}}}{0.5} \left(\frac{\sigma}{200 \text{ km s}^{-1}} \right)^3 \frac{a}{r_{\text{bh}}} \quad (13)$$

with $\mathcal{F}_{\text{chaos}}$ the fraction of stars on chaotic orbits. Even a small $\mathcal{F}_{\text{chaos}}$ implies a substantial rate of supply to a MBHB when it first forms, with $a \sim a_{\text{hard}} \sim (\mu/2M_{\text{bh}})r_{\text{bh}}$. Such high feeding rates would imply substantial changes in MBHB separations even if triaxial distortions to the potential were transient, due for instance to mergers or accretion events.

N-BODY SIMULATIONS

Several attempts have been made to model the formation and the long-term evolution of MBHBs using large-scale N -body simulations [37, 33, 10, 38]. Because of the discreteness effects associated with approximating a galaxy with $N \gtrsim 10^9$ stars by a model consisting of only, at best, $N \lesssim 10^6$ particles, the applicability of direct N -body simulations appears to be limited to the early stage of the MBHB evolution. The rapidity with which a galactic merger proceeds guarantees that the discreteness effects do not affect the state of the galaxy just after the merger is complete. That point marks the transition to a much more gradual decay in the MBH separation when the effective relaxation time in the simulation can easily become shorter than the decay time.

Simulations fail to reproduce the long-term evolution correctly because in the simulations the loss cone is almost completely full, while in real galaxies it is largely empty. In the general case there exists a critical stellar orbital energy E_{crit} separating the full region $E < E_{\text{crit}}$ (large radii) from the empty region $E > E_{\text{crit}}$ (small radii). Assuming a density profile $\rho \sim r^{-2}$ and a potential of the form $2\sigma^2 \ln(r/r_0)$ such that $r_0 = 10^3 GM_{\text{bh}}/2\sigma^2$ we find [15]:

$$E_{\text{crit}} \approx 2\sigma^2 \ln \left(\frac{7.5 \times 10^4 GM_{\text{bh}}/8\sigma^2}{\mathcal{N}_{\text{bh}} a} \right). \quad (14)$$

where $\mathcal{N}_{\text{bh}} = M_{\text{bh}}/m_*$ is the number of stars that make the mass of the black hole. The transition from a full to an empty loss cone happens when E_{crit} becomes smaller than a few $\times 2\sigma^2$, implying that $\mathcal{N}_{\text{bh}} \sim 10^4 - 10^5$. Since a typical MBH contains 0.1% of its host galaxy's mass [39], and thus $N \sim 10^3 \mathcal{N}_{\text{bh}}$, an N -body simulation would have to contain $10^{4-5} \times 10^3 = 10^{7-8}$ bodies to reproduce the correct, diffusive behavior of a real galaxy. This requirement is a severe one for direct-summation N -body codes, which rarely exceed particle numbers of $\sim 10^6$ even on parallel hardware [32]. One route might be to couple the special purpose GRAPE hardware¹, which is limited to $N \lesssim 10^6$, to algorithms that can handle large particle numbers by swapping with a fast front end.

OUTSTANDING PROBLEMS

We conclude by mentioning two outstanding problems related to the dynamics of MBHB binaries. Although the gravitational back-reaction tends to circularize binary black holes,

¹ <http://grape.astron.s.u-tokyo.ac.jp/grape/>

some residual eccentricity can remain, especially if stellar dynamical processes prior to the emission of gravitational waves act to amplify the eccentricity to large values. Residual eccentricities result in the excitation of higher harmonics in the signal [40], thereby complicating the detection of a gravitational wave event severely. In spite of the importance of eccentricity for detection, an accurate and general evaluation of the MBHB eccentricity evolution remains a challenge.

Similarly, large-mass-ratio black hole binaries deserve extra attention. Although the best understood MBHBs are those of consisting of nearly equal-mass black holes, it is probable that most coalescences involve black holes of very unequal mass. For example, intermediate-mass black holes (IMBHs) with masses of $10^{2-4}M_{\odot}$ may be able to form in young star clusters—such as the Arches and the Quintuplet clusters in the Milky Way bulge [41, 42]—via the segregation of massive stars to the cluster center [43], followed by the runaway growth in stellar mergers and the collapse of the agglomerate star into an IMBH [44, 45]. An IMBH makes its way to the galaxy center and forms a binary with the nuclear MBH [46]. Therefore, large-mass-ratio MBHBs are expected to exist even in galaxies that had not experienced recent mergers. The orbital evolution of these systems remains another challenge to dynamical exploration.

CONCLUSIONS

We have focused on stellar dynamical mechanisms for extracting energy from a MBHB, but other schemes are possible and even likely. We note a close parallel between the “final parsec problem” and the problem of quasar fueling: in both cases, a quantity of mass of order $\sim 10^8 M_{\odot}$ must be supplied to the inner parsec in a time much shorter than the age of the universe. Nature clearly accomplishes this in the case of the quasars, probably through gas flows driven by torques from stellar bars [47]. The same inflow of gas could contribute to the decay of a MBHB, by leading to renewed star formation or rapid accretion of gas [48]. Similarly, the presence of a third black hole in a galactic nucleus could accelerate the decay by increasing the MBHB’s eccentricity through the Kozai mechanism [49], or by extracting the binary’s energy via the triple black hole slingshot interaction [50].

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