Exotic Axion Cosmology in Theories with Phase Transitions below the QCD Scale

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We show that axion phenomenology may be significantly different than conventionally assumed in theories which exhibit late phase transitions (below the QCD scale). In such theories, one can find multiple pseudoscalars with axionlike couplings to matter, including a string scale axion, whose decay constant far exceeds the conventional cosmological bound. Such theories have several dark matter candidates.

Introduction.—The smallness of the experimentally determined upper bound on the strong *CP* violating parameter, $\bar{\theta} \lesssim 10^{-9}$, is an outstanding puzzle of the standard model. One can either assume *CP* to be an exact symmetry spontaneously broken in such a way as to ensure that $\bar{\theta}$ is naturally small, as in the Nelson-Barr mechanism [1,2], or one can introduce a $U(1)$ symmetry, known as the Peccei-Quinn (PQ) symmetry [3,4] to allow $\bar{\theta}$ to dynamically relax to zero. (Spontaneous breaking of parity has also been suggested [5,6].) An attractive feature of the PQ mechanism is that it divorces the strong *CP* problem from flavor physics—the masses and mixings of the quarks—whose origin remains a mystery. The PQ mechanism entails a global $U(1)$ symmetry which is exact up to a QCD (and possibly electromagnetic) anomaly. The symmetry breaks spontaneously at a scale f, giving rise to a pseudoscalar Goldstone boson, the axion, which couples to matter via the interaction $(a/f)G\tilde{G}$ [7–12]. Here a is the axion, f is its decay constant, $G_{\mu\nu}$ is the gluon field strength, and the ratio (a/f) should be thought of as an angle. This angle has a potential arising from instantons which causes (a/f) to select the vacuum $\bar{\theta} = 0$. The axion will, in general, have additional derivative couplings to matter, such as a model-dependent coupling to photons of the form $(a/f)F\tilde{F}$ [13,14]. The mass of the axion m_a satisfies $m_a \approx m_\pi f_\pi / f$, where $m_\pi \approx 140$ MeV and $f_\pi \approx 93$ MeV are the pion mass and decay constant, respectively.

The axion decay constant is bounded from below by collider experiments and astrophysical arguments. The latter are the more stringent: If f is too small, the coupling to ordinary matter is large enough to allow rapid axion production in red giants and supernovae, leading to an unacceptably large cooling rate. This yields the lower bound 10^9 GeV $\lesssim f$ [15]. Large f leads to copious production of cold, degenerate axions in the early Universe [16–18], so that $f \gtrsim 10^{12}$ GeV leads to unacceptably large $\Omega_{\rm dm}$, where $\Omega_{\rm dm}$ is the fraction of dark matter in the Universe today, measured to be 0.21 to within 4% [19]. Therefore, the axion decay constant is conventionally assumed to lie in the window 10^9 GeV $\lesssim f \lesssim 10^{12}$ GeV. At the upper end of this bound, axions are a viable dark matter candidate, and experimental attempts to detect their presence are in progress [20].

In this Letter, we will present an exotic cosmological scenario for axions, and so we first summarize the conventional picture. The PQ symmetry is assumed to break spontaneously at a temperature $T \sim f$ well above the QCD scale where the instanton induced axion potential turns on. Following this phase transition, (a/f) equals some random angle θ_i until $T \sim 1$ GeV, below which the axion potential develops rapidly. The axion field begins to oscillate at temperature $T_i \sim 1$ GeV when the axion mass comes within the horizon, $m_a(T_i) \approx H(T_i)$, where $H(T_i)$ is the Hubble parameter at temperature T_i, which is only weakly dependent on f. The coherent oscillation may be thought of as a gas of degenerate nonrelativistic axions with the number density of axions per comoving volume equal to $n_a = \theta_i^2 H(T_i) f^2$. This quantity remains constant, since annihilation rates are negligible. As a result, the subsequent axion energy density at temperature T is given by $\rho_a = m_a n_a (R(T_i)/R(T))^3$, where $R(T)$ is the Robertson-Walker scale factor at temperature T. The upper bound on f follows from observational limits on $\Omega_{\rm dm}$, since ρ_a varies almost linearly with f, assuming that $\theta_i = O(1)$.

There have been prior attempts to evade the cosmological bound, motivated in part by the fact that, in string theories, axions with $f = \sqrt{2}\alpha_U M_p \approx 10^{16}$ GeV, where α_U is the unified value of the fine structure constant, are generic. A trivial way to harmlessly incorporate an axion A with decay constant $F > 10^{12}$ GeV is to introduce a second pseudoscalar a so that the Lagrangian contains the term $(A/F + a/f)G\tilde{G}$, in which case the spectrum can consist of an innocuous massless boson and a conventional axion with decay constant f. Another resolution is to assume inflation and an ensemble of initial angles θ_i to choose from, invoking the anthropic principle to justify a value in our Universe of $\theta_i \approx 0$ [21]; recently, it was pointed out that such a scenario could be constrained by the Planck polarimetry experiment [22]. Alternatively, in Ref. [23] the axion is coupled to light quarks in such a way as to allow for exotic axion cosmology below the QCD scale.

In this Letter, we explore unconventional axion cosmology without invoking the anthropic principle and without convoluting the PQ mechanism with flavor physics; in common with Refs. [23,24], our scenario involves late evolution of the PQ symmetry breaking order parameter,

although it differs significantly in realization and phenomenology. The conventional cosmological bound on f results because the PQ symmetry spontaneously breaks at a temperature well before the QCD scale. However, if f only evolves to a large vacuum expectation value (vev) after the QCD time, such bounds may be evaded. This could be accomplished with a sufficiently flat PQ potential, so that the radial mode only evolves out to a large vev once its mass enters the horizon.

The fine-tuning associated with such a flat potential can be avoided only with supersymmetry (SUSY); but even then, SUSY breaking will, in general, generate a curvature for the PQ potential which forces PQ breaking at or above the weak scale, a transition which is too early to evade the cosmological constraints. This problem can be avoided with the introduction of a new sector which couples only weakly to the standard model via the conventional PQ sector. We show that such a coupling may be weak enough to shield the potential in the new sector from SUSY breaking effects, while still significantly affecting axion phenomenology. Our model evades the problems of Ref. [25], where it was shown that, in supersymmetric theories, cosmological overproduction of saxions is typically more of a problem than an excess of axions.

The spectrum of our model includes (i) an axion far lighter than conventionally allowed with no significant cosmological abundance; (ii) an additional pseudoscalar which is heavier than would be an axion with comparable decay constant; (iii) a light dilatonlike scalar particle. In the model presented here, the latter two are dark matter candidates. In the next sections, we describe a model which realizes this scenario. While not particularly compelling as a description of nature, the model has been constructed to illustrate how model-dependent axion cosmology and detection can be, within a framework that successfully addresses the strong CP problem.

A model.—Our starting point is to assume a viable supersymmetric theory which implements the conventional PQ mechanism. We assume that there exists a superfield ϕ_1 which carries PQ charge and couples to colored fermions in a real representation of the gauge group; at a temperature well above the QCD scale, this field acquires a vev $\langle \phi_1 \rangle = v_1/\sqrt{2}$, which lies within the conventional window 10^9 GeV $\lesssim v_1 \lesssim 10^{12}$ GeV. With this vev, the heavy colored fermions coupled to ϕ_1 develop a mass $M_Q = g v_1$. It is important for our modification of the theory that the saxion be light (to be specified below) so that it not communicate large SUSY breaking to a new sector we will be adding. A light saxion is expected in any theory of low energy SUSY breaking; it could also occur in gravity-mediated SUSY breaking models, so long as the PQ sector is sequestered from the SUSY breaking. An excess of saxion energy can be avoided in such models either by having relatively late inflation (below the PQ breaking scale but well above the QCD time) or by having the minimum of the saxion potential be at the same point as

preferred by finite temperature effects prior to an epoch of higher scale inflation.

To this theory, we now introduce the superpotential

$$\tilde{W} = \sqrt{2}\lambda A(h\phi_1\phi_2 - \phi_0^2) + \frac{1}{\sqrt{2}}\mu^2 B(2(\phi_0/v_0)^2 - 1),$$

(1)

where ϕ_2 carries opposite PQ charge from ϕ_1, while A, B, and ϕ_0 are PQ-invariant fields. The four parameters v_0^2, μ^2, h, and λ may all be taken to be real by redefinition of the phases of A, B, ϕ_0, and ϕ_2. The part of the scalar potential relevant to us is

$$\tilde{V}(\phi_1, \phi_2, \phi_0) = 2\lambda^2|h\phi_1\phi_2 - \phi_0^2|^2 + \frac{\mu^4}{2}|2(\phi_0/v_0)^2 - 1|^2.$$

(2)

The first term in $\tilde{V}$ exhibits a flat direction in ϕ_2 and ϕ_0, which is lifted slightly by the second term in $\tilde{V}$, with $\lambda \ll 1$ and $\mu \ll \Lambda_{\text{QCD}}$. The smallness of the couplings leaves $\sqrt{2}\langle\phi_1\rangle = v_1$ unaffected, and the minimum of the almost flat direction is at $\sqrt{2}\langle\phi_0\rangle = v_0 \gg v_1$, and $\sqrt{2}\langle\phi_2\rangle = v_2 = v_0^2/(hv_1) \gg v_0$.

We assume that following inflation and reheating the Universe sits away from the minimum of the potential with $\sqrt{2}\langle\phi_1\rangle = v_1$, and $\langle A \rangle = \langle B \rangle = \langle\phi_0\rangle = \langle\phi_2\rangle = 0$, the latter being determined by high temperature effects due to interactions with unspecified heavy fields prior to inflation. This field configuration persists down to a temperature $T_0 < \Lambda_{\text{QCD}}$, satisfying $\mu^2/v_0 \approx H(T_0)$, when the curvature lifting the flat direction is sufficiently strong to overcome the Hubble friction. Then ϕ_0 will roll out to its minimum at $\langle\phi_0\rangle = v_0 \gg v_1$, causing ϕ_2 to follow its flat direction out to $v_2 = v_0^2/(hv_1) \gg v_0$. With $\mu \ll \Lambda_{\text{QCD}}$, this phase transition will occur at $T_0 < \Lambda_{\text{QCD}}$, and, as we discuss further below, the dark matter produced in this transition can be made acceptable.

To understand how the pseudoscalars behave in this model, we expand around the minimum of the potential, writing $\phi_i = \frac{1}{\sqrt{2}}(v_i + \sigma_i)e^{i\pi_i/v_1}$, with $i = 0, 1, 2$. The fields σ_i and π_i are the scalar and pseudoscalar excitations, respectively. After adding to $\tilde{V}$ the QCD contribution to the π_1 potential, the complete pseudoscalar potential is

$$V_\pi = -m_\pi^2 f_\pi^2 \cos\frac{\pi_1}{v_1} - \lambda^2 v_0^4 \cos\left(\frac{\pi_1}{v_1} + \frac{\pi_2}{v_2} - \frac{2\pi_0}{v_0}\right)$$
$$- \mu^4 \cos\left(\frac{2\pi_0}{v_0}\right)$$

(3)

[up to an overall $O(1)$ factor [13] in front of the first term which does not concern us here]. Since π_1 is the only field that couples to ordinary matter, to understand phenomenology we must decompose π_1 into mass eigenstates $a_{1,2,3}$. We diagonalize the mass matrix to leading nonzero order in $\delta = v_1/v_0$ and $\epsilon = \mu^4/(\lambda^2 v_0^4)$ and to all orders in $x = \lambda^2 v_0^4/(m_\pi^2 f_\pi^2)$, finding the masses and decay constants

$$m_{a_1}^2 = \frac{\lambda^2 v_0^4}{v_1^2}\left(\frac{x}{1+x}\right), \qquad f_1 = v_1,$$

$$m_{a_2}^2 = \frac{m_\pi^2 f_\pi^2}{v_0^2}\left(\frac{x}{1+x}\right), \qquad f_2 = v_0\left(\frac{1+x}{2x}\right), \quad (4)$$

$$m_{a_3}^2 = \frac{\mu^4}{v_0^2},$$

where the decay constants are defined by

$$\mathcal{L}_{\text{QCD}} = \frac{\pi_1}{v_1}G\tilde{G} = \left(\frac{a_1}{f_1} + \frac{a_2}{f_2}\right)G\tilde{G}. \quad (5)$$

In these formulas, we have neglected terms of order v_0/v_2; the a_3 pseudoscalar decouples from the standard model to the order we work. Note that x may be $\gg 1$.

This model evades the conventional constraints on the axion decay constant as the axion a_2 has a decay constant far in excess of 10^{12} GeV—it is potentially an axion with string scale PQ constant $f_2 \approx 10^{16}$ GeV—yet is cosmologically unpopulated. The pseudoscalar a_1 also couples to $G\tilde{G}$, but, unlike an axion, its mass does not vanish in the limit that its coupling to $G\tilde{G}$ vanishes; its mass may, in fact, be far in excess of that of a conventional axion. The energy originally in π_1 is primarily transferred into the a_1 field, making it a good dark matter candidate; the light a_2 axion receives only a small fraction of that energy, suppressed by $(v_1/v_0)^2 \ll 1$. The axionlike pseudoscalar a_1 may be detectable by experiments searching for cosmologically abundant axions; however, its mass may lie outside the window in which these experiments are currently searching.

Constraints.—At temperatures below the QCD scale but above the secondary transition, the Universe has a background density of cold π_1 pseudoscalars which behave like a conventional axion with decay constant v_1. Subsequently, the ϕ_0 field rolls out to its minimum $\langle\phi_0\rangle = v_0$ at the temperature T_0 where $H(T_0) \sim \mu^2/v_0$. At this point (i) the energy in π_1 pseudoscalars is redistributed among the a_1 and a_2 mass eigenstates following Eq. (4), and (ii) an energy density μ^4 is released and is mostly transferred into σ_0 (radial) oscillations of the ϕ_0 field. There are two cosmological constraints on the late transition. The first is that the combined energy density in the exotic a_1 pseudoscalar and σ_0 scalar not exceed the observed dark matter density today. A second constraint is that the phase transition occur after the QCD time but before today (or well before matter-radiation equality if either a_1 or σ_0 contribute an appreciable fraction of the dark matter). In addition, there is the astrophysical constraint that, to prevent copious production of any of the light pseudoscalars in supernovae, we require $f_1, f_2 \gtrsim 10^9$ GeV. We now discuss these constraints in detail and map out the corresponding parameter space.

In order to ensure no excess of dark matter, we must limit the energy in π_1 by requiring $f_1 \lesssim 10^{12}$ GeV. We also require that the energy ρ_0 in the form of σ_0 oscillations

produced at the secondary transition not dominate the Universe at the epoch of matter-radiation equality, $T_{\text{eq}} \approx 1$ eV:

$$\rho_0(T_{\text{eq}}) \approx \mu^4\left(\frac{T_{\text{eq}}}{T_0}\right)^3 \lesssim T_{\text{eq}}^4, \qquad H(T_0) \approx \frac{\mu^2}{v_0}. \quad (6)$$

A complication arises from the fact that energy in the early π_1 oscillations is transferred primarily into a_1 oscillations, while a_1 becomes heavier than π_1 by a factor of $\sqrt{1+x}$ as ϕ_0 rolls from $\phi_0 = 0$ to $\phi_0 = v_0$. This increase of energy must come from the energy released during the secondary transition; it can be represented by a contribution to the potential for ϕ_0 due to the ϕ_0-dependent energy density of the a_1 pseudoscalar condensate, of the form

$$\rho_1(\phi_0, T) = n_{\pi_1}(T)m_{\pi_1}\sqrt{1 + x(\phi_0)}, \quad (7)$$

where $T_i \sim 1$ GeV is the temperature when π_1 begins to oscillate, $n_{\pi_1}(T) = (\theta_i^2 H_i f_1^2)(T/T_i)^3$ is the number density of π_1 bosons in the temperature range $\Lambda_{\text{QCD}} \gtrsim T \gtrsim T_0$, $m_{\pi_1} = (m_\pi f_\pi/f_1)$ is the π_1 mass before ϕ_0 rolls out, and $x(\phi_0) = (\phi_0/v_0)^4 x$ controls how the π_1 mass becomes the a_1 mass as ϕ_0 increases. If $\rho_1(v_0, T_0) > \mu^4$, this potential delays the secondary transition, so that ϕ_0 only gains its vev at some lower temperature $T = T_0'$ satisfying $\rho_1(v_0, T_0') \approx \rho_0(T_0') \approx \mu^4$. After this delayed transition, the energy density in both ϕ_0 and a_1 oscillations remains comparable, diluting $\propto T^{-3}$. After a little algebra, one finds that Eq. (6) still holds in this case but is augmented by the constraint that there not be too much energy in the cosmological a_1 abundance,

$$T_{\text{eq}}^4 \gtrsim \left(\frac{T_{\text{eq}}}{T_0}\right)^3 n_{\pi_1}(T_0)m_{a_1}, \quad (8)$$

with m_{a_1} defined in Eq. (4) and $T_0 \approx \sqrt{M_p\mu^2/v_0}$ from Eq. (6). If we now assume $\rho_1 + \rho_0 = \rho_{\text{DM}}$, that is, that a_1 and σ_0 together compose the dark matter, we can use Eq. (8) to compute allowed masses and couplings, f_1, m_{a_1}, for the observable dark matter candidate a_1. The result is shown in Fig. 1; while the a_1 pseudoscalar couples to ordinary matter as a conventional axion would, its mass generally exceeds that of an axion, a result of its coupling with the sector that generates the exotic axion a_2.

Finally, we obtain a constraint from requiring ϕ_0 to roll after the QCD phase transition, but somewhat before matter-radiation equilibrium if either a_1 or σ_0 are to be the dark matter:

$$T_{\text{eq}}\left(\frac{v_0}{M_{pl}}\right)^{1/2} \lesssim \mu \lesssim T_{\text{QCD}}\left(\frac{v_0}{M_{pl}}\right)^{1/2}. \quad (9)$$

If the origin of dark matter lies elsewhere, then T_{eq} in the above equation is replaced by today's temperature.

The model as it stands possesses an exact discrete symmetry $\phi_0 \to -\phi_0$ which is spontaneously broken and leads to domain walls. This can be avoided by breaking the symmetry explicitly, either with a small linear term in the low energy superpotential Eq. (1) or in the high energy
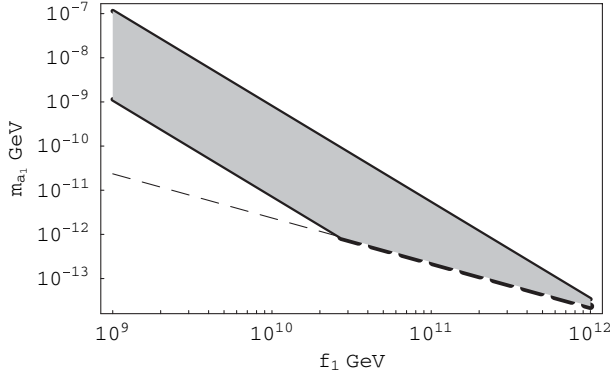
FIG. 1. The mass of the exotic pseudoscalar a_1 vs its decay constant f_1. Two solid parallel lines give $m_{a_1}(f_1)$, assuming a_1 comprises half the dark matter for initial misalignment angle $\theta_i = 0.1$ (upper line) and $\theta_i = 1$ (lower line). The dashed line is the pseudoscalar mass in standard PQ models, with the heavy dashed line giving the region where conventional axions could be the dark matter.

TABLE I. Two sets of parameters (I, II) allowed by the constraints Eqs. (6) and (8)–(10). Set I gives parameters for a prompt transition at temperature T_0; set II gives parameters for a delayed transition at T_0'. All parameters are in GeV except the pseudoscalar masses, which are in eV.

	m_{a_1}	f_1	m_{a_2}	f_2	x	μ	$T_0(T_0')$
I	10^{-2}	10^{10}	10^{-5}	10^{15}	10	10^{-3}	10^{-2}
II	1	10^{10}	10^{-9}	10^{16}	10^3	10^{-6}	10^{-6}

interactions such that $\langle \phi_0 \rangle \neq 0$ (but $\lesssim \nu_1$) after postinflationary reheating.

Aside from cosmological constraints, there is also a naturalness constraint. Having a flat potential is critical for the late phase transition, so we require that SUSY breaking terms induced by the interactions between ϕ_1, ϕ_2, and ϕ_0 do not lift the flat direction too much. The most important term is the soft mass generated for ϕ_2 from the SUSY breaking ϕ_1 mass, for which we require

$$\Delta m_2^2 f_2^2 \approx \frac{m_{a_1}^2 m_{\phi_1}^2}{16\pi^2} \lesssim \mu^4. \qquad (10)$$

This imposes a significant new constraint on μ. In addition, we are assuming this sector is sequestered from gravity-mediated SUSY breaking [26].

The combination of constraints Eqs. (6) and (8)–(10) yields a parameter space too large to explore in detail here. We give here instead two representative sets of values in Table I satisfying the constraints.

Phenomenology.—Conventional axion models have fairly circumscribed phenomenology; if one assumes that the dark matter consists of a conventional axion, then the mass and coupling of the axion are related in a direct way, and both lie in a fairly model independent range about $m_a \sim 10^{-5}$ eV, $f_a \sim 10^{12}$ GeV determined by the initial axion misalignment θ_i. If the axion is not the dark matter, its mass can be heavier.

In contrast, we have shown how these simple relations can be greatly modified in a theory with a late phase transition below the QCD scale, as shown in Table I. Dark matter in this theory can be comprised of roughly equal parts of remnant pseudoscalar and scalar particles.

It will be interesting to investigate the consequences of such late phase transitions for structure formation.

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