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Passive Force Closure and its Computation in Compliant-Rigid Grasps

Amir Shapiro, Dept. of ME, Technion
Elon Rimon, Dept. of ME, Technion
Joel W. Burdick, Dept. of ME, Caltech

Abstract *The classical notion of* force closure *is formulated for multi-fingered hands, where the fingers* actively *apply any desired force consistent with friction constraints at the contacts. This paper considers a simpler notion of* passive force closure, *where each finger obeys some force-displacement law that depends on the finger's joint parameters. The fingers apply initial preload grasping forces, and the grasped object is stabilized against external disturbances by the automatic response of the grasping fingers. After motivating the usefulness of passive force closure, we characterize the conditions for its existence. Then we introduce the* passive stability set, *defined as the collection of external wrenches that can be passively resisted by a given grasp. We introduce a class of grasp arrangements where the grasping mechanism is compliant while the grasped object is rigid. Such compliant-rigid systems are common, and for these systems the passive closure set can be computed in closed form. Simulation results demonstrate the computation of the passive closure set for two and three-finger planar grasps.*

1 Introduction

The notion of *force closure* was originally formulated for multi-fingered robot hands [7, 14]. This notion should be called *active* force closure, since it requires that the fingers be able to actively balance any disturbing wrench (i.e. force and torque) acting on the grasped object. Active force closure requires sophisticated contact-force sensors and agile contact-force controllers whose action must be precisely coordinated. However, in applications such as fixturing the grasping elements are simple devices that are preloaded against an object with initial grasping forces [11]. Physical processes at the contacts, such as friction and compliance, provide passive stabilization of the object against external disturbances. Another important application concerns multi-fingered mechanisms that establish an initial grasp of an object. Using decoupled position-based controllers for the individual fingers, the effective compliance of the grasping mechanism together with friction at the contacts provide passive stabilization of the grasped object (Figure 1(a)). A related application is a multi-limbed robot bracing against a tunnel-like environment in

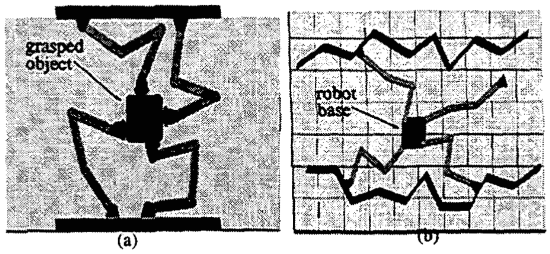

Figure 1: (a) A multi-fingered hand grasping an object. (b) A multi-limbed robot bracing against tunnel walls.

static equilibrium (Figure 1(b)). Here the tunnel walls play the role of the grasped object, and the robot passively stabilizes itself by pushing against the walls using decentralized position-based controllers. In all of these examples stabilization is achieved by passive means, without active control or coordination of the contact forces.

Our notion of passive force closure refines a passivity notion introduced by Yoshikawa [18]. Consider a grasp arrangement where each finger or contacting body obeys its own force-displacement law. In particular, some fingers may apply a fixed force on the object. The grasp is *passive force closure* if for suitably selected initial grasping forces, the fingers or bodies contacting the object passively balance any external wrench in a neighborhood about the origin. The literature on active force closure is only partially relevant for studying passive force closure[1]. Examples of works on friction-based active force closure are [10, 13, 17], and examples of works that additionally consider the structure of the grasping mechanism are [2, 4, 5, 9, 12]. The notion of passive force closure considered here should not be confused with the notion of passive internal forces discussed in whole-arm manipulation [1].

This paper makes three contributions. First, it provides necessary and sufficient conditions for passive force closure. In particular, *the geometrical condition for active force closure is necessary but not sufficient for passive force closure.* Second, the paper characterizes the set of external wrenches that can be passively resisted by a given grasp. This set,

[1] *Form closure* can be interpreted as passive force closure specialized to frictionless contacts [17].

called the ***passive closure set***, depends on the grasp geometry, the amount of friction at the contacts, the kinematics and dynamics of the grasping mechanism, as well as the preload forces. Third, the paper describes how to explicitly compute the passive closure for grasp arrangements where a compliant mechanism holds a rigid object. Such *compliant-rigid systems* arise in multi-fingered hands and multi-limbed robots that interact with rigid objects using simple position-based controllers [3]. Finally, we demonstrate the computation of the passive closure set for 2-finger and 3-finger linearly compliant planar grasps.

2 Definition of Passive Force Closure

In this section we introduce terminology for frictional grasps and review the notion of active force closure. Then we define passive force closure and describe necessary and sufficient conditions for its existence.

2.1 Frictional Grasps Terminology

We study 2D or 3D grasps, where a rigid object $\mathcal{B}$ is held in frictional point contact by k rigid bodies $\mathcal{A}_1,...,\mathcal{A}_k$. The bodies $\mathcal{A}_1,...,\mathcal{A}_k$ represent fixturing elements or the fingertips of a multi-fingered hand. Although we use the language of grasping, these bodies can also represent the footpads of a multi-limbed robot. The contact point between $\mathcal{A}_i$ and $\mathcal{B}$ is denoted r_i when expressed in $\mathcal{B}$'s body frame, and x_i when expressed in a fixed world frame (Figure 2). The two representations of the i^{th} contact point are related by the rigid-body transformation: $x_i = Rr_i + d$, where d and R are the position and orientation of $\mathcal{B}$ with respect to a fixed world frame. The orientation matrix R is parametrized by the exponential map, $R(\theta) = \exp(\theta)$, where $\theta \in \mathbb{R}$ in 2D and $\theta \in \mathbb{R}^3$ in 3D. The object configuration is parametrized by $q = (d, \theta) \in \mathbb{R}^m$, where $m = 3$ in 2D and $m = 6$ in 3D. The wrench generated by a force F_i acting on $\mathcal{B}$ at x_i is given by the familiar formula:

$$w_i = \begin{pmatrix} F_i \\ \rho_i \times F_i \end{pmatrix} \quad \text{where } \rho_i = R(\theta)r_i.$$

The collection of wrenches that act on $\mathcal{B}$ at a particular configuration q is called the *wrench space* at q. This space can be identified with $\mathbb{R}^m$.

We assume the standard Coulomb friction model: $|F_i^t| \le \mu|F_i^n|$, where F_i^t and F_i^n are the tangent and inward normal components of F_i, and μ is the coefficient of Coulomb friction[2]. The force F_i can only push on the object, and this constraint is described by the inequality $F_i^n \ge 0$. The friction cone at the i^{th} contact, denoted FC_i, is the collection of all frictional forces that can be applied to $\mathcal{B}$ at x_i, and it is given by

$$FC_i = \{F_i \colon\ F_i^n \ge 0 \text{ and } -\mu F_i^n \le F_i^t \le \mu F_i^n\}.$$

The set of wrenches generated by all forces in FC_i forms a cone of feasible wrenches. The i^{th} feasible wrench cone, denoted $\mathcal{W}_i$, is given by

$$\mathcal{W}_i = \{w_i \colon\ w_i = \begin{pmatrix} F_i \\ \rho_i \times F_i \end{pmatrix},\ \forall F_i \in FC_i\}.$$

When $\mathcal{B}$ is held by k fingers, we say that $\mathcal{B}$ is in ***equilibrium*** if in the absence of any external wrench there exist feasible wrenches $w_i \in \mathcal{W}_i$ for $i = 1, \ldots, k$ such that $\sum_{i=1}^{k} w_i = \vec{0}$.

[2] In 3D, under a soft-contact model there is also frictional torque about the contact normal.

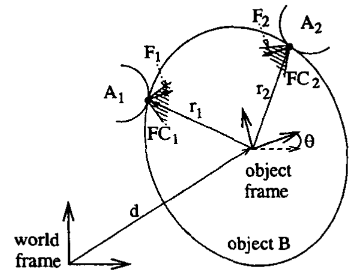

Figure 2: Basic notation for frictional grasps.

2.2 Review of Active Force Closure

Active force closure is the standard notion of force closure [13, 17]. The collection of wrenches that can be generated by k frictional contacts is given by the set sum: $\mathcal{W}_1 + \cdots + \mathcal{W}_k = \{w_1 + \cdots + w_k : w_i \in \mathcal{W}_i \text{ for } i = 1, ..., k\}$. This notation is used in the following standard definition.

Definition 1. *Let an object $\mathcal{B}$ be held in equilibrium grasp by k frictional point contacts. Let $\mathcal{W}_i$ be the feasible wrench cone of the i^{th} contact. Then the grasp is* **active force closure** *if the sum of the wrench cones $\mathcal{W}_1 + \cdots + \mathcal{W}_k$ spans the entire wrench space $\mathbb{R}^m$, where $m=3$ in 2D and $m=6$ in 3D.*

The active aspect of the grasp lies in the assumption that the grasping bodies can generate any contact force within the respective friction cones. The following theorem gives a simple rule for determining active force closure [13, 18]. By definition, a grasp is *non-marginal* when the contact forces are non-zero and lie in the interior of their respective friction cones.

Theorem 1 (Active force closure). *Let a 2D or 3D object $\mathcal{B}$ be grasped by k frictional contacts, such*

that the contacts do not lie along the same spatial line when the grasp is 3D. Then the grasp is* active force closure *iff it is possible to establish a non-marginal equilibrium grasp of $\mathcal{B}$.

2.3 Passive Force Closure

Active force closure is based on the assumption that the contact forces can be freely modified within the respective friction cones. Passive force closure is based on the assumption that each contact force obeys some force-displacement relationship subject to friction constraints at the contacts. To formalize this notion, we define three types of contacts that encapsulate three common types of force-displacement laws and other modeling idealizations.

Definition 2. *• A* **rigid-body contact** *is a stationary rigid-body that passively interacts with* $\mathcal{B}$ *through a frictional contact. • A* **fixed-force contact** *is a frictional point contact that applies a specific force at the contact point. • A* **compliant contact** *is a frictional contact that applies force according to a force-displacement relationship of the contact point.*

Let us give examples of these types of contacts. Passive rigid-body contacts are commonly used in fixturing applications to restrict the motions of a workpiece. Fixed-force contacts are generated by mechanisms such as pressure-controlled hydraulic fixels and force-controlled robot grippers. Compliant contacts are generated by finger and limb mechanisms whose joints are controlled by position-servoed controllers [3, 14]. A more complex type of contact occurs when several contacts are coupled together by the grasping mechanism. Such coupled contacts often occur in power or enveloping grasps [18]. In order to avoid such coupled contacts, we assume that *each contact is generated by its own independent mechanism.* We are now ready to define passive force closure.

Definition 3. *Let an object* $\mathcal{B}$ *be held in equilibrium grasp by k independent frictional contacts of the types defined above. Then the grasp is* **passive force closure** *if any external wrench in a neighborhood about the origin is passively balanced by the contacts.*

Passive grasps can be implemented with controllers that simply maintain fixed joint torques or fixed joint positions, while the balancing of external wrenches is performed automatically by the contacts. Let us discuss several properties of passive force closure grasps. First, the condition for *active force closure is necessary for the existence of passive force closure,* since a neighborhood about the origin in wrench-space can be expanded to the entire space by actively increasing the contact-force magnitudes. However, *active force closure does not automatically imply passive force closure.* Figure 3(a) shows a 2-fingered frictional grasp of a rectangular object. If the two contacts are fully active the grasp is active force closure. But when the two contacts apply fixed forces the grasp is not passive force closure, since the contacts cannot generate any net horizontal force on $\mathcal{B}$. (The mechanisms generating the fixed forces must move in response to a horizontal force acting on $\mathcal{B}$.) On the other hand, if one contact is a passive rigid body while the other applies a fixed force, the grasp becomes passive force closure. Another 2-fingered frictional grasp is depicted in Figure 3(b). If the two contacts are fully active the grasp is active force closure. But when the contacts apply compliant forces along the horizontal direction and zero forces along the vertical direction, the grasp is not passive force closure.

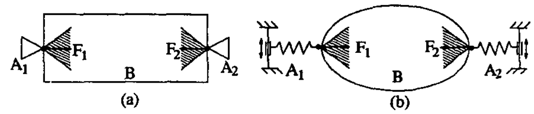

Figure 3: (a) A grasp which is not passive force closure when $\mathcal{A}_1$ and $\mathcal{A}_2$ apply fixed forces. (b) A grasp which is not passive force closure when $\mathcal{A}_1$ and $\mathcal{A}_2$ apply horizontally compliant forces.

We now give necessary and sufficient conditions for passive force closure of grasps having frictional compliant or fixed-force contacts, as well as frictionless rigid-body contacts. The conditions are based on the following notion of potential energy function. The wrench generated by a compliant contact can be written as $w_i = -\nabla U_i(q)$, where $U_i(q)$ is the elastic potential energy function induced on $\mathcal{B}$ by the i^{th} compliant contact[3]. Similarly, the wrench generated by a fixed-force contact is induced by a potential function which is linear in x_i, where $x_i = R(\theta)r_i + d$. The wrench generated by a frictionless passive rigid-body contact also has the form $w_i = -\nabla U_i(q)$, where the elastic energy function is given by the Hertz formula from elasticity theory [6]. (This theory treats the contacting bodies as quasi-rigid.) The total potential energy of $\mathcal{B}$ is the sum $U(q) = \sum_{i=1}^{k} U_i(q)$.

Proposition 2.1. *Let a 2D or 3D object* $\mathcal{B}$ *be held in equilibrium grasp by k independent compliant frictional contacts, fixed-force frictional contacts, or frictionless rigid-body contacts. Let* q_0 *be* $\mathcal{B}$*'s equilibrium configuration, and let* $U(q)$ *be the potential energy induced on* $\mathcal{B}$ *by the contacts. Then the following two conditions are sufficient for* **passive force closure***:*
1. The initial equilibrium grasp is non-marginal. (In

[3] $U_i(q)$ is identically zero when the i^{th} contact is broken.

3

3D the contacts must not lie along a common line.)

2. *The equilibrium q_0 is a non-degenerate local minimum of the potential energy function $U(q)$.*

Moreover, in all generic grasps conditions (1) and (2) are also necessary for passive force closure.

A proof of the proposition is sketched in the appendix. The first condition of the proposition states that the grasp must satisfy the condition for active force closure. I.e., the grasp must be active force closure if the contacts are made fully active. The second condition is the standard stability condition for compliant grasps [5, 9]. This condition is a key to understanding the difference between active and passive closure. *The stability condition ensures that when an external wrench acts on $\mathcal{B}$, the object would automatically settle at a new equilibrium in the vicinity of q_0 where the contact forces balance the external wrench.* Note that two issues play a role in this convergence. First, the equilibrium induced by the external wrench must be locally stable. Second, the original unperturbed equilibrium must lie in the basin of attraction of the new equilibrium. Finally, the proposition generalizes to any type of contact whose dynamics varies smoothly with the external wrench acting on $\mathcal{B}$.

3 The Passive Closure Set of Compliant-Rigid Grasps

Given a passive force closure grasp, the *passive closure set* is the collection of external wrenches which are automatically balanced by the contacts. In this section we characterize the passive closure set of compliant-rigid grasps. Before describing this class of grasps, let us depict a fundamental difficulty in computing the passive closure set. The Coulomb friction model allows generation of tangential forces at the contacts up to a limit determined by μ times the normal component of the contact forces. In passive grasps the normal component of the contact forces is determined by the initial preload of the grasp, and can change only in response to an external wrench $\boldsymbol{w}_{ext}$ acting on $\mathcal{B}$. In other words, the normal loadings at the contacts cannot "spontaneously" change as they do in fully active contacts. Thus we write the normal loading at the i^{th} contact as $F_i^n(\boldsymbol{w}_{ext})$. The friction cone at the i^{th} contact is determined by the inequality $|F_i^t| \leq \mu F_i^n(\boldsymbol{w}_{ext})$, and this friction cone determines a $\boldsymbol{w}_{ext}$-dependant feasible wrench cone denoted $\mathcal{W}_i(\boldsymbol{w}_{ext})$. An external wrench can be possibly balanced by the contacts only when the recursive relation $\boldsymbol{w}_{ext} \in \mathcal{W}_1(\boldsymbol{w}_{ext}) + \cdots + \mathcal{W}_k(\boldsymbol{w}_{ext})$ holds true. The solution of this recursive relation is a key step in computing the closure stability set.

The *compliant-rigid grasps* are defined as grasps where a rigid object $\mathcal{B}$ is held by compliant finger mechanisms. This class of grasps also includes multi-limbed robots bracing against a rigid environment. The rigidity of $\mathcal{B}$ is an excellent approximation—although all objects exhibit some natural compliance at the contacts, this compliance is negligible relative to the compliance induced by the joints of the grasping mechanism. For example, consider our experimental multi-limbed robot depicted in Figure 1(b) [16]. Each limb of this robot has four joints actuated by Maxon motors that generate a stiffness of 2 N/mm at the footpads. In contrast, the stiffness of objects made of Aluminum is $4.5 \cdot 10^3$ N/mm.

We make the following simplifying assumptions. First, each finger mechanism is assumed to interact with $\mathcal{B}$ through a pointed finger-tip. This assumption implies that when a finger-tip locally rolls on the surface of $\mathcal{B}$, the location of the contact point remains fixed in $\mathcal{B}$'s body frame. Second, we assume that each finger mechanism is fully actuated, so that it can generate any force in $\mathbb{R}^n$, where $n=2$ in 2D and $n=3$ in 3D. Our third assumption is that each finger generates a force-displacement law of the form:

$$F_i = F_i^0 + \boldsymbol{f}_i(x_i), \tag{1}$$

where F_i^0 and x_i^0 are the contact forces and contact points at the initial equilibrium grasp, and $\boldsymbol{f}_i$ is a smooth function such that $\boldsymbol{f}_i(x_i) = 0$ when $x_i = x_i^0$.

Our first step in the characterization of the passive closure set is to express the contact forces as a function of the object configuration $q = (d, \theta)$. The i^{th} contact point is given by $x_i = R(\theta)r_i + d$, where r_i is the description of x_i in $\mathcal{B}$'s body coordinates. Let r_i^0 denote the coordinates of r_i at the initial grasp. Let $\mathcal{FQ}$ denote the collection of $\mathcal{B}$'s configurations where the contact forces lie in their respective friction cones. (The set $\mathcal{FQ}$ is considered below.) Then the pointed-finger assumption together with the rigidity of $\mathcal{B}$ guarantee that *the points r_i remain fixed in $\mathcal{B}$'s body frame,* for all configurations $q \in \mathcal{FQ}$. Thus we may write $x_i = R(\theta)r_i^0 + d$ for $i = 1, ..., k$. Substituting for the x_i's in (1) gives the desired expression for the contact forces:

$$F_i(d,\theta) = F_i^0 + \boldsymbol{f}_i(x_i(d,\theta)) \quad i = 1, ..., k. \tag{2}$$

The approach presented here for computing the contact forces was originally proposed by Bicchi [1]. However, Bicchi assumes only a small change of Δq in the object's configuration, with a linear force-displacement law. Our formulation generalizes Bicchi's approach to any object configuration and any force-displacement law.

Next we write an expression for the set of feasible configurations $\mathcal{FQ}$. This set is given by the intersection $\mathcal{FQ} = \cap_{i=1}^k \mathcal{FQ}_i$, where $\mathcal{FQ}_i$ denotes the collection of $\mathcal{B}$'s configurations where the i^{th} contact force

$F_i(q)$ lies in the friction cone FC_i. Let n_i denote the inward normal to the boundary of $\mathcal{B}$ at r_i, written in $\mathcal{B}$'s body coordinates. And let N_i be the inward unit normal to the boundary of $\mathcal{B}$ at x_i, expressed in world coordinates. Then $N_i = R(\theta)n_i$, and the normal component of the i^{th} contact force is: $F_i^n = F_i \cdot N_i = F_i \cdot (R(\theta)n_i)$. The tangential component of F_i is: $F_i^t = \|[I - N_i N_i^T]F_i\| = \|[I - n_i n_i^T]R(\theta)^T F_i\|$. Substituting for F_i^n and F_i^t in the inequalities that define FC_i gives:

$$\begin{aligned}\mathcal{F}Q_i = \ & \{q = (d,\theta):\ F_i \cdot (R(\theta)n_i) \geq 0 \ \text{ and} \\ & \|[I - n_i n_i^T]R(\theta)^T F_i\| \leq \mu F_i \cdot (R(\theta)n_i)\},\end{aligned}$$

where μ is the coefficient of friction. Substituting for the forces F_i according to (2) gives:

$$\begin{aligned}\mathcal{F}Q_i = \{q = (d,\theta):\ & F_i(d,\theta) \cdot (R(\theta)n_i) \geq 0 \ \text{ and} \\ \|[I - n_i n_i^T]R(\theta)^T F_i(d,\theta)\| & \leq \mu F_i(d,\theta) \cdot (R(\theta)n_i)\}.\end{aligned}$$

The desired set $\mathcal{F}Q$ is the intersection of the sets $\mathcal{F}Q_i$ for $i = 1, ..., k$. Our third step is to identify the configurations that guarantee stable convergence of $\mathcal{B}$ to the equilibrium induced by an external wrench. This condition is captured by the requirement that the second-derivative matrix of the grasp potential energy function, $D^2U(q)$, be positive definite. The set of configurations that satisfy this stability condition, denoted $\mathcal{P}$, is given by

$$\mathcal{P} = \{q = (d,\theta):\ \lambda_{min}(D^2U(q)) > 0\}, \qquad (3)$$

where λ_{min} denotes the minimal eigenvalue of a matrix. Condition (3) guarantees local stability of the equilibrium induced by w_{ext} at q. However, it does not guarantee that $\mathcal{B}$'s original equilibrium at q_0 lies in the basin of attraction of the new equilibrium at q. The condition for global convergence from q_0 to q is currently under investigation [15].

Finally, the net wrench generated on $\mathcal{B}$ by the contact forces is $w = \sum_{i=1}^{k}(F_i, \rho_i \times F_i)$. Since F_i and ρ_i are functions of q, w can be interpreted as a mapping from configuration space to wrench space. The passive closure set, denoted $\mathcal{W}_{passive}$, is the image in wrench-space of the configurations q in $\mathcal{F}Q \cap \mathcal{P}$ under the mapping $w(q)$:

$$\mathcal{W}_{passive} = \left\{ w = \sum_{i=1}^{k} \begin{pmatrix} F_i(q) \\ \rho_i(q) \times F_i(q) \end{pmatrix} :\ q \in \mathcal{F}Q \cap \mathcal{P} \right\}.$$

Any wrench w_{ext} in $\mathcal{W}_{passive}$ would be automatically balanced by the contacts of the grasp.

4 Computation of the Passive Closure Set

In this section we compute the passive closure set of compliant-rigid grasps whose contacts specifically obey a *linear* compliance law of the form:

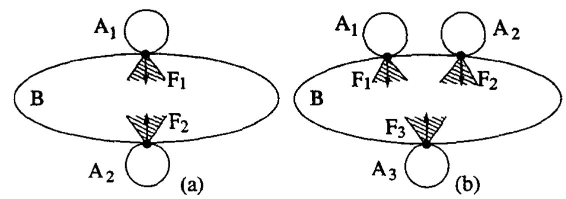

Figure 4: Passive force closure grasps of an elliptical object using (a) two and (b) three fingers.

$$F_i = F_i^0 - K_i(x_i - x_i^0), \qquad (4)$$

where F_i^0 and x_i^0 are the contact forces and contact points at the initial equilibrium grasp, and K_i is an $n \times n$ positive semi-definite matrix ($n = 2$ in 2D and $n = 3$ in 3D). First we substitute the linear law (4) into (2):

$$F_i(d,\theta) = F_i^0 - K_i\big((R(\theta)r_i^0 + d) - x_i^0\big) \quad i = 1, ..., k.$$

Next we substitute for the contact forces $F_i(d,\theta)$ in the inequalities that define the sets $\mathcal{F}Q_i$. This substitution yields a closed-form expression for the set of feasible configurations, $\mathcal{F}Q = \cap_{i=1}^{k}\mathcal{F}Q_i$. Finally, the inequality that defines the locally stable configurations requires a formula for $D^2U(q)$. This formula is provided in the following lemma. Given a vector $u \in \mathbb{R}^3$, $[u\times]$ denotes the 3×3 skew-symmetric matrix satisfying $[u\times]v = u \times v$ for all $v \in \mathbb{R}^3$.

Lemma 4.1 ([13, 15]). *Let a rigid object $\mathcal{B}$ be grasped by k compliant contacts each satisfying the linear compliance law (4). Then the formula for $D^2U(q)$ in the 3D case is:*

$$D^2U(q) = \sum_{i=1}^{k} \begin{bmatrix} K_i & K_i[\rho_i\times] \\ [\rho_i\times]^T K_i & [\rho_i\times]^T K_i [\rho_i\times] + ([F_i\times][\rho_i\times])_s \end{bmatrix}$$

where for a given matrix A, $A_s = \frac{1}{2}(A + A^T)$. The formula for $D^2U(q)$ in the 2D case is:

$$D^2U(q) = \sum_{i=1}^{k} \begin{bmatrix} K_i & -K_i J\rho_i \\ -(J\rho_i)^T K_i & (J\rho_i)^T K_i J\rho_i + F_i \cdot \rho_i \end{bmatrix},$$

where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

We now depict the passive closure set of the two passive force closure grasps shown in Figure 4. The object $\mathcal{B}$ is an ellipse with minor and major axes of length two and four length-units. The contacts are frictional with a coefficient of friction $\mu = 0.3$. In both grasps the fingers apply linear-compliance forces, with a stiffness matrix $K_i = I$ for $i = 1, ..., k$. That is, each finger applies a uniform one-unit force per one-unit of deflection of the respective contact point. In the 2-finger grasp the magnitudes of the

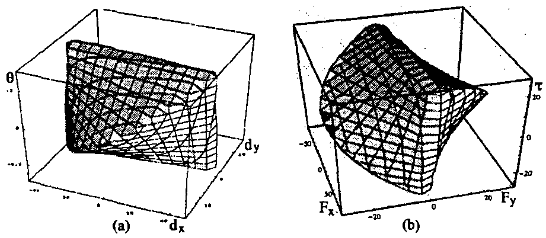

Figure 5: The feasible configurations set and the passive stability set of the 2-finger grasp.

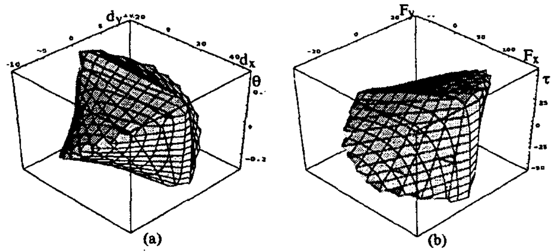

Figure 6: The feasible configurations set and the passive stability set of the 3-finger grasp.

initial forces are set to $\|F_1^0\| = \|F_2^0\| = 50$ force units. Figure 5(a) shows the collection of feasible configurations $\mathcal{FQ} \cap \mathcal{P}$. The coordinates in this figure are (d_x, d_y, θ), where (d_x, d_y) are in length-units and θ in radians. Note that the d_y-coordinate of $\mathcal{FQ} \cap \mathcal{P}$ varies in the interval $[-10, 10]$, while the d_x-coordinate of this set varies in the interval $[-20, 20]$, where both intervals are measured in length-units. This difference in the range of the d_x and d_y coordinates can be explained by the fact the deflection of the ellipse along the y-axis generates pure tangential forces which are bounded by μ times the normal forces generated by deflection of the ellipse along the x-axis. The passive closure set $\mathcal{W}_{passive}$ of the 2-finger grasp is depicted in Figure 5(b). The coordinates in this figure are (F_x, F_y, τ). Note that the asymmetry of $\mathcal{FQ} \cap \mathcal{P}$ now appears as an asymmetry of $\mathcal{W}_{passive}$ along the F_x and F_y axes. Finally, the magnitudes of the initial forces in the 3-finger grasp are set to $\|F_1^0\| = \|F_2^0\| = 25$ and $\|F_3^0\| = 50$ force units. Figure 6(a) depicts the set of feasible configurations $\mathcal{FQ} \cap \mathcal{P}$. Figure 6(b) shows the set $\mathcal{W}_{passive}$ for the 3-finger grasp.

5 Conclusion

In active force closure the fingers resist external wrenches by actively applying the required forces at the contacts. Active grasping requires sophisticated contact-force sensors and contact-force controllers whose action must be precisely coordinated. In passive force closure each contact satisfies some fixed force-displacement law. The contacts apply preload grasping forces, and the balancing of external wrenches is performed automatically by the contacts. Passive grasping can be implemented with controllers that simply maintain fixed joint torques or fixed joint positions, without any coordination of the individual contacts.

We formally defined ***passive force closure*** and provided necessary and sufficient conditions for generic passive force closure grasps. In particular, the geometrical condition for active force closure is necessary but not sufficient for passive force closure. To guarantee passive force closure, the grasped object must automatically converge to a nearby equilibrium where the contact forces balance the external wrench. Next we characterized the ***passive closure set*** of compliant-rigid grasps. In these grasps a rigid object $\mathcal{B}$ is held by compliant grasping mechanisms. We derived analytic expressions for the passive closure set of such grasps. Finally, we used linear force-displacement laws to depict the passive closure set of 2-finger and 3-finger planar grasps.

Future extensions of this work will include the following topics. First, we have stated only a local stability criterion for the passive closure set. A global stability criterion ensuring that the original unperturbed equilibrium would converge to the new equilibrium induced by the external wrench must be added. A second topic is how to characterize the passive closure set for fingers having any shape at the contacts. Last, we wish to obtain conditions for passive form closure of grasps having ***frictional*** passive rigid-body contacts. This type of contact requires a consideration of complex phenomena such as microslip and hysteresis.

A Conditions for Passive Force Closure

Proof sketch of Proposition 2.1: Let $\mathcal{N}$ be a small neighborhood of configurations about q_0. As $\mathcal{B}$'s configuration varies in $\mathcal{N}$, the contact forces vary in a neighborhood about the contact forces of the initial grasp. Since the initial grasp is non-marginal, by a continuity argument all contact forces generated by varying $\mathcal{B}$'s configuration in $\mathcal{N}$ still lie in their respective friction cones. (This statement holds true even when the location of some contact points changes due to local rolling of $\mathcal{B}$.)

Next we establish that any external wrench in a neighborhood about the origin can be balanced by feasible contact forces. When $\mathcal{B}$ is at a configuration $q \in \mathcal{N}$, the net wrench generated by the contacts is given by the negated gradient $-\nabla U(q)$. Consider now the gradient $\nabla U(q)$ as a mapping from configuration space to wrench space. By assumption $\nabla U(q_0) = 0$. According to the Inverse Function Theorem, ∇U maps an open neighborhood about q_0 to an open neighborhood about the zero wrench if the derivative of ∇U at q_0, $D^2U(q_0)$, is non-singular. Since q_0 is a non-degenerate local minimum of U, $D^2U(q_0)$ is non-singular as required.

Finally we establish that $\mathcal{B}$ would automatically settle at a configuration where the contact forces balance the external wrench acting on $\mathcal{B}$. Let w_{ext} denote a fixed external wrench acting on $\mathcal{B}$. The dynamics of $\mathcal{B}$ is governed by the equation: $M(q)\ddot{q} + B(q,\dot{q}) = -\nabla U(q) + w_{ext}$. (The contacts also apply damping forces which we ignore for simplicity.) The external influences on $\mathcal{B}$ can be written as the negated gradient of a composite potential function: $\Phi(q) = U(q) - w_{ext} \cdot q$. A general result concerning the dynamics of mechanical systems states that the flow of a damped mechanical system governed by a potential function Φ is attracted to the local minima of Φ [8]. We have already shown that for every w_{ext} in a neighborhood about the origin there exists a configuration q_1 such that $\nabla\Phi(q_1) = 0$. The equilibrium point q_1 is a stable attractor if it is a local minimum of Φ i.e., if $D^2\Phi(q_1) > 0$. But $D^2\Phi(q) = D^2U(q)$, and the entries of $D^2U(q)$ vary continuously with q. Since the eigenvalues of a matrix vary continuously with its entries, $D^2U(q)$ remains positive definite in a neighborhood of q_0. By shrinking $\mathcal{N}$ if necessary, we conclude that q_1 is a local minimum of Φ, and $\mathcal{B}$ would automatically settle at q_1. □

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