Compressive Direct Measurement of the Transverse Photonic Wavefunction

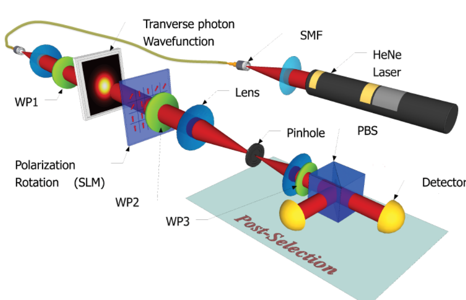
Mohammad Mirhosseini[1,*], **Omar S. Magaña-Loaiza**[1], **S. M. Hashemi Rafsanjani**[2], **and Robert W. Boyd**[1,3]

[1] *The Institute of Optics, University of Rochester, Rochester, New York 14627, USA*
[2] *Center for Coherence and Quantum Optics and the Department of Physics & Astronomy, University of Rochester, Rochester, New York 14627, USA*
[3] *Department of Physics, University of Ottawa, Ottawa ON K1N 6N5, Canada*
[*] *mirhosse@optics.rochester.edu*

Abstract: We generalize the method of direct measurement and combine it with compressive sensing. Using our method, we measure a 19200-dimensional state using only 20% of the total required measurements.

OCIS codes: 270.0270, 270.5585, 100.5070.

Determining an unknown wavefunction is of fundamental importance in quantum mechanics. Despite many seminal contributions, this task remains challenging for high-dimensional states. The direct measurement (DM) approach, introduced by Lundeen *et. al*, has provided a ground for meeting the high-dimensionality challenge [1]. Contrary to state tomography, this methods does not require a time-consuming post-processing. Nevertheless, the number of measurements required by the direct measurement protocol grows linearly with the dimensionality of the measured state. Here we combine a novel computational method known as compressive sensing with the direct measurement technique. Utilizing our approach, the wavefunction of a high-dimensional state can be estimated with a high fidelity using much fewer number of measurements compared to the standard direct measurement.



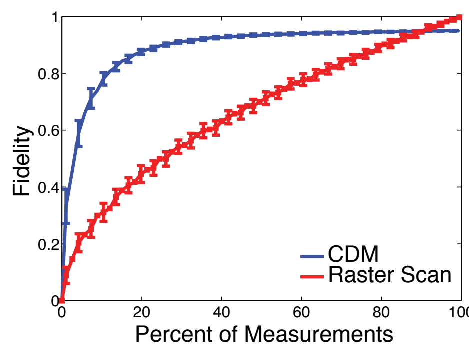


Fig. 1. A schematic illustration of the experimental setup (left). The fidelity of a reconstructed Gaussian state with the target wavefunction, shown in blue, as a function of the percentage of the total measurements (right). The fidelity of the state reconstructed from a partial pixel-by-pixel scan with the same number of measurements is shown in red for comparison.

A weak value is the expectation value of a weak measurement that is followed by a post-selection [2]. Consider a weak measurement of the position projector $\hat{\pi}_j = |x_j\rangle\langle x_j|$ at point x_j followed by a post-selection on the zeroth component of the Fourier transform of the spatial wavefunction, which we denote by $|o\rangle$. The complex wavefunction of a photon can be calculated at each point by measuring the real and imaginary part of the weak value as

$$\pi_w = \frac{\langle o|x_j\rangle\langle x_j|\psi\rangle}{\langle o|\psi\rangle} = \frac{\psi(x_j)}{\phi_0\sqrt{N}}. \tag{1}$$

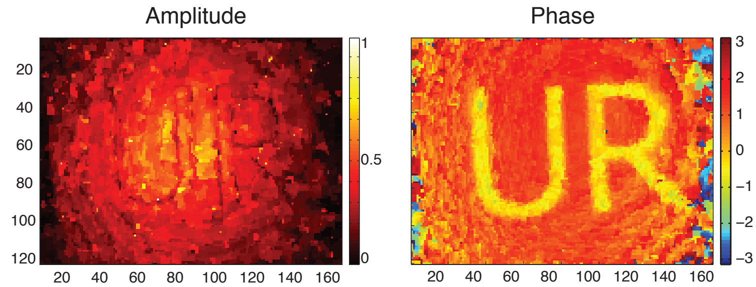


Fig. 2. The amplitude and phase of a Gaussian mode illuminating a custom phase mask (the initials of the University of Rochester). The data is reconstructed by the CDM method with N=19200, and $M/N = 20\%$ of total measurements.

Here we have used the Fourier transform property $\langle o|x_j\rangle = 1/\sqrt{N}$ where N is the dimension of the Hilbert space and $\phi_0 = \langle o|\psi\rangle$.

We generalize the DM to a form suitable for compressive sensing. Let the initial system-pointer state be $|\Omega\rangle = |\psi\rangle \otimes |V\rangle = \sum_{i=1}^{N} \psi_i |x_i\rangle \otimes |V\rangle$, where we have assumed to have a discrete Hilbert space for the spatial degree of freedom $|\psi\rangle$ and a two-level system such as the polarization of a single photon for the pointer state $|V\rangle$. We consider a situation where instead of a measuring a projector $\hat{\pi}_j$ we perform a weak measurement of the operator $\hat{Q}_m = \sum_j Q_{m,j} \hat{\pi}_j$ where the coefficients $Q_{m,j} \in \mathbb{R}$. In this situation the imaginary and the real part of ψ_j, $\Im[\psi_j]$ and $\Re[\psi_j]$, can be related to the expectation values of the polarization of the post-selected state $\bar{\sigma}_{x,m}$ and $\bar{\sigma}_{y,m}$ via a linear set of equations $\phi = \mathbf{Q}\ \psi$.

Here, $\phi_m = \frac{1}{\kappa}[\bar{\sigma}_{x,m} + i\bar{\sigma}_{y,m}]$ and $\kappa = \frac{2\alpha}{\phi_0\sqrt{N}}$. The numbers $m \in \{1:M\}$ and $n \in \{1:N\}$, where M is the total number of sensing operators and N is the dimension of the Hilbert state of the unknown wavefunction. To find the wavefunction ψ we need to (approximately) solve this linear system of equations in the case where $M \ll N$. A nonlinear strategy can be used to recover ψ with a high quality using the idea of compressive sensing (CS). If the wavefunction under the experiment ψ is known to have very few non-zero coefficients under a linear transformation $\mathbf{T}$, it can be reconstructed with a high probability by solving the convex optimization problem [3]

$$\min_{\psi'} ||\mathbf{T}\psi'||_{\ell_1} \text{ , subject to } \mathbf{Q}\psi' = \phi. \tag{2}$$

Fig. 1 shows the schematics of the experiment. A vertically polarized Gaussian mode is prepared by spatially filtering a He-Ne laser beam with a single mode fiber and passing it through a polarizer. A random polarization rotation at each point is performed using a spatial light modulator (SLM) in combination with two quarter wave plates (QWP) [4]. To provide a quantitive comparison of the two methods we calculate the fidelity between a retrieved Gaussian state $|\psi'\rangle$ and the state $|\psi\rangle$ from a full pixel-by-pixel scan (See Fig. 1). We prepare a custom target state by illuminating phase mask depicting letters U and R with a phase jump of $\pi/2$ with a Gaussian beam. Figure 2 shows the amplitude and the phase of the reconstructed state with $M/N = 20\%$ of the total measurements. Notice that while the amplitude is relatively uniform, the phase shows the letters U and R with a remarkable accuracy.

To conclude, we have demonstrated high fidelity reconstruction of spatial states using the compressive direct measurement (CDM) method. This technique can be used for measurement of high-dimensional quantum states as well as classical applications such as wavefront sensing.

References

1. J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart, and C. Bamber, "Direct measurement of the quantum wavefunction." Nature **474**, 188–191 (2011).
2. Y. Aharonov, D. Albert, and L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," Physical Review Letters **60**, 1351–1354 (1988).
3. J. Romberg, "Imaging via Compressive Sampling," IEEE Signal Processing Magazine **25**, 14–20 (2008).
4. M. Malik, M. Mirhosseini, M. P. J. Lavery, J. Leach, M. J. Padgett, and R. W. Boyd, "Direct measurement of a 27-dimensional orbital-angular-momentum state vector," Nature Communications **5**, 3115 (2014).