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Reconstruction of large-scale structures and acoustic radiation from a turbulent M=0.9 jet using the proper orthogonal decomposition

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The Proper Orthogonal Decomposition (POD) uses data to generate an optimal set of basis functions that represent the "energy" of the data, defined by a user-selected norm. This basis is optimal in the sense that a finite number of these modes represent more of the energy than any other set of orthogonal modes. The POD can be seen, on one hand, as a way to *define energetic structures in a flow*, essentially a generalization of the traditional Fourier-based spectrum to treat data that is inhomogeneous in one or more coordinate directions. But the POD modes are perhaps more useful in *quantitatively modeling the dynamics of the flow*, via Galerkin projection of the governing equations onto a relatively small number of modes in order to generate a reduced-order model.

Here we document the three-dimensional POD modes, based on a varitety of norms, of a round, subsonic, turbulent jet (with Mach number 0.9 and Reynolds number 3600) that was recently computed with well-validated DNS[1]. Of special interest is the three-dimensional structure of the POD modes that involve all near field quantities (velocities and thermodynamic variables), which has not been possible to measure directly. Instead, experiments have concentrated either on slices of the jet (normal to the flow direction) in which the streamwise velocity alone is used to compute the POD modes (e.g. [2]), or pressure measurements along the streamwise direction at a position just outside the jet (e.g. [3]). We investigate the connection between energetic structures (defined with POD) in the near field and the far-field radiated sound. An intriguing question is whether an appropriate norm can be defined that would efficiently represent the sound producing dynamics of the flow, and a long term goal of the present work is to

address this issue. In other words, we seek to determine what is a good norm to define such that relatively few modes contribute to the generation of radiated sound. This, we hope, will in the long run lead to phenomenological models for sound radiation by large scale structures in turbulent jets and other flows.

Computational considerations

POD provides a set of scalar or vector basis functions that optimally represent, with a given number of modes, a user-defined norm of a set of data. For incompressible three-dimensional datasets, this norm is typically taken to be the fluctuation kinetic energy, integrated over the entire flow volume, and therefore the vector basis function represents the three components of velocity (in an appropriately defined coordinate system). In general, however, we can quite arbitrarily specify which vector of flow quantities (including scalars), the region over which the norm is defined, and how the individual components of a vector of quantities are weighted in the norm. For compressible flows, the choices of variables, norms, and weightings are not as obvious, especially when the governing equations are to be projected on to the modes. Some generalizations of the POD to this case were considered by Rowley[4].

In the present work, we have sidestepped this issue, and simply defined a general vector of variables that include 3 components of velocity, sound speed, and pressure, from which all other flow quantities may be computed. We define an arbitrary scaling factor as part of the norm, so that individual components of vector are weighted differently in the integration, and so that a consistent non-dimensionalization of different flow variables is implied. Then, for example, if we weight the sound speed and pressure with zero, we obtain the standard kinetic energy norm used in incomrpessible flow. We have used that norm, along with norms based on enthalpy (pressure weighted to zero, sound speed weighted by a factor involving the ratio of specific heats), the streamwise velocity along, and the pressure alone. The integrations are defined over (i) "freespace", which is truncated with little error at the edges of the open computational domain, (ii) slices normal to the jet axis, and (iii) a portion of a large sphere 60 jet radii in radius in the acoustic field. We note that the acoustic field (all flow variables) at that location was determined by extrapolation of the wave field from the smaller computational domain using a method similar to the so-called Kirchhoff surface.

For homogeneous (periodic) coordinate directions, Fourier modes are identical to POD modes. We therefore use an azimuthal Fourier transform of the flow data and compute the POD modes of each Fourier mode (as functions of x, the streamwise coordinate and r, the radial coordinate) independently. The POD modes are computed using the method of snapshots[5] from 2333 snapshots of the DNS data saved every 5 time steps.

Results

For most norms we consider, we find a significant amount of energy in relatively few POD modes. The norms based on surface-integrals converge more rapidly, in general, requiring 0.6% of all the modes to capture 85% of the energy, for the case of streamwise velocity at a position near the end of the potential core. By contrast, 3% of all the three-dimensional modes are required to capture the volume integral of pressure or kinetic energy to around 80%.

For the pressure norm, we find in general that the most energetic POD modes have a wave packet structure, and that pairs of such modes represent advection of the structures at nearly constant phase speed. The most energetic modes are dominated by the axisymmetric and first helical modes and bear a strong resemblance to the growth and decay of instability waves. The firts and second most energetic of these appears to coincide with the formation of vortices (both axisymmetric and helical). The third and forth most energetic modes (both axisymmetric and helical) shown a streamwise period doubling consistent with a vortex-pairing phenomena near the end of the potential core.

The most energetic mode based on kinetic energy, by contrast, appears as a long, slowly rotating structure with azimuthal wavenumber two, and it is most intense past the end of the potential core. There is little discernable "propagation" of the kinetic energy modes.

We have examined the extent to which a portion of the POD modes are able to "reconstruct" various statistical quantities in the flow, such as (local) turbulent kinetic energy, Reynolds stresses, and pressure fluctuations both near and far (acoustic) field. For the pointwise near field quantities, the results are for the most part consistent with the rate at which the eigenvalues converge to the total (integrated) energy. One surprising result is that the pressure modes seem to have better overall pointwise convergence in the near field even for quantities that do not explicitly appear in the norm, such as turbulent kinetic energy.

In particular, we have found that all reconstructions based on norms defined over the near field give very poor convergence for the far acoustic field. While the basic directivity of the acoustic field is well represented by about 0.3% of the modes, it requires virtually all the modes to determine the sound pressure level to within one dB. These results are shown in Figure 1 (note the logarithmic scale). For reference, we also show reconstruction of the far field pressure based on pressure integrated on the surface of a large sphere in the far field. Only a few modes in that case capture the full acoustic field, and this indicates (not surprisingly) that the structure of the acoustic field is not particularly complex, only its relation with the near field dynamics.

We have also examined the structure of POD modes (in the near-field) when the norm is based solely on the acoustic field (i.e. the large surface integral described above). The most energetic modes from this norm continue to display a strong wave packet structure in the near field. The wave packets seem to have an abrupt change in structure near the close of the potential core that may give rise to the acoustic radiation.
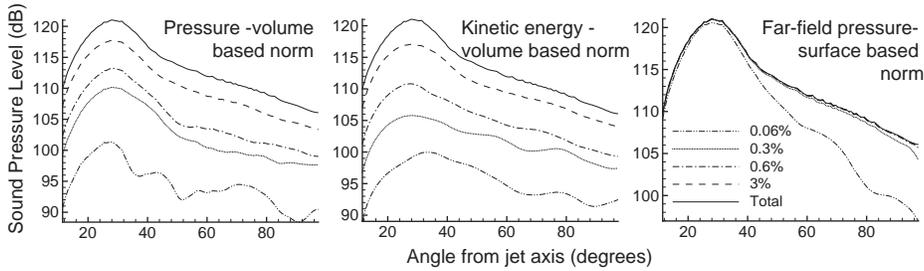
Figure 1: Reconstruction of the far field sound pressure level from POD modes defined with various norms.

Summary

We have computed POD modes for the turbulent jet using a variety of norms that highlight different aspects of the structure of the flow, and differ in the extent to which near and far field statistics can be reconstructed with limited numbers of modes. While reconstruction of the far-field sound pressure level based on near field POD modes converges disappointingly slowly (and vice versa) our preliminary results show some interesting connections between the structure of the modes and the radiated acoustic field. Work is needed to further elucidate the connections between the POD modes, in various norms, and the sources of sound. Ultimately, our goal is to use that connection to develop reduced-order models of sound generation for the purpose of noise control. JBF gratefully acknowledges the support of NASA.

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