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Near-field wavepackets and the far-field sound of a subsonic jet

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This paper details the analysis of the relationship between the near-field pressure fluctuations of an unforced, subsonic free jet ($0.4 \leq M \leq 0.6$) and its low-angle, far-field sound emissions. Azimuthal rings of six microphones recorded pressure fluctuations on a conical surface in the jet near field while an azimuthal ring of three microphones recorded fluctuations in the far field at $\theta = 20^\circ$ and $R/D = 47.1$. Recent measurements have shown close agreement between the velocity fluctuations up to the end of the potential core of the currently studied jet and predictions from the linear Parabolised Stability Equations (PSE), indicating the presence of linear wavepackets in the jet velocity field. Solutions of the Linearised Euler Equations (LEE) reported in the present paper also show good agreement with measurements, and provide a first step toward a time-domain description of the said wavepackets. Though the agreement for PSE in the velocity field breaks down downstream of the potential core, Proper Orthogonal Decomposition (POD) of the current results shows that the wavepackets do persist in this region and are clearly apparent in the near pressure field. Attention is then turned to establishing a relationship between these wavepackets and the radiated sound by comparing simultaneously-obtained measurements of the far-field pressure both directly to the near-field signature as well as to numerical predictions of the far-field emissions available from a recent technique using a tailored Green's function. The direct comparisons are made by correlations between the POD modes and the far-field sound. The first POD mode captures most of the flow energy for the frequency range studied, and the correlation between this mode and the far field is nearly identical to the correlation using the full near-field signal. Higher POD modes also show significant correlation to the far field with a different space-time structure than the first mode. The Green's function predictions are performed both statistically and in the time domain, and though they are shown to be valid for a near-field array with a long axial extent, the experimental limitation of a shorter array ($0.5 \leq x/D \leq 8.9$), which truncates the wavepacket source in the calculations, causes inaccurate predictions for the experimental data. This error is thought to be the result of a spurious source introduced by the truncation that interferes both constructively and destructively with the wavepacket source. A validation problem shows that this error would be smaller for a higher- M jet.

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I. Introduction

The analysis of noise generation from turbulence in a jet is a story of two different complexities. In the turbulent flow region, the fluctuations satisfy the full non-linear Navier–Stokes equations, numerical solutions of which are now possible in simple cases but not necessarily practical or informative for jet design. In the far pressure field, the pressure fluctuations follow simple linear acoustics equations, which can be easily solved for given boundary conditions (typically a pressure distribution on a surface surrounding the turbulent flow). This reduced complexity of the far field gives hope to researchers that the acoustically-important behaviour of jet turbulence can be also be captured with a reduced-order model of the jet dynamics.¹ This hope was tantalized by the discovery of coherent structures in turbulence in the early 1960s, a development from the early understanding of turbulence as a superposition of stochastic eddies. The role of coherent structures in turbulence and noise production has been a topic of intense research ever since. A candidate source structure that is consistent with many of the observed features of jet noise is an axially extended wavepacket.² Forerunners such as Mollo-Christensen,^{3,4} Crow & Champagne,⁵ and Crighton⁶ made early observations of these signatures and developed the mathematical basis for their sound production and prediction. In all likelihood, wavepackets will not tell the whole story of jet noise emissions, but they do appear to account for many of the salient features, especially at low emission angles. Jordan & Colonius¹ have presented a comprehensive review of the various works dealing with wavepackets and jet noise sources.

Crow⁷ (see Crighton⁶) was the first to show that the sound emissions from simple wavepacket sources could be calculated analytically, and more recent work has noted the effect on sound emissions of several wavepacket parameters including the spatial envelope,⁸ temporal growth and decay,⁹ and intermittency.¹⁰ Early experimental work in analysing wavepackets focused on forced jets because measurements could be phase-locked to an external trigger, but recent work has also found evidence of wavepackets in the velocity fields of natural jets¹¹ and provided a technique for educing a simplified structure from numerical simulation data.¹²

The physical basis for the appearance of wavepackets in a jet arises from a stability analysis of the shear layer,^{3,4} which can be made in a linear or non-linear framework. The resulting wavepackets—calculated on an azimuthal mode-by-mode, frequency-by-frequency basis—correspond to the unstable modes of the jet base flow. Good agreement between the stability predictions and the measured fluctuations verifies the presence of wavepackets in the jet, providing a basis on which to build wavepacket noise prediction schemes. The stability predictions in this work are calculated by the solution of the Parabolised Stability Equations (PSE)¹³ and the results are compared both with experimental measurements and with solutions of the Linearised Euler Equations (LEE) driven at the nozzle exit plane by eigenfunctions of a locally parallel stability calculation. The main motivation for the LEE approach is the need for time-domain reduced-order models.

Despite recent favourable agreement demonstrated between PSE predictions and both in-flow and near-field flow fluctuations,^{14,11} as well as wavepacket predictions and far-field pressure statistics,^{2,15} several questions remain unanswered about wavepackets and sound emissions in jets. For example, Cavalieri *et al.*¹¹ found that the agreement between stability predictions and experimental data deteriorated beyond the end of the potential core. Also, a clear relationship has not yet been determined between time-domain wavepacket fluctuations and far-field sound emissions. To this end, the objectives of this paper are to determine if wavepackets are still present beyond the potential core and to determine if experimental wavepacket signatures can be related to the observed far-field sound.

This paper reports the analysis of data obtained from a ring array of microphones in the jet-far field and by a conical array of microphones in the part of the near pressure field of the jet labelled the linear hydrodynamic region.¹⁶ In this region, the pressure fluctuations are dominated by waves with subsonic phase speed in the axial direction. These fluctuations are thus mostly of hydrodynamic nature, but they do not exhibit the rotational, turbulent nature of the fluctuations within the jet flow. The near-field array used in these experiments is similar to that used by Suzuki & Colonius¹⁶ and Gudmundsson & Colonius¹⁴ in their similar investigations. The unique aspect of this work is the simultaneous measurement of the far-field pressure, enabling analysis of the causal link between the near-field wavepackets and the emitted sound. The measured data are used to establish a relationship between the near-field fluctuations and the far-field pressure by means of correlation analysis and to characterise the wavepacket behaviour in the near field in terms of wavepacket parameters. A Green’s function tailored to the geometry of the conical microphone array is used to propagate the recorded near-field wavepacket signatures to the far field using both statistical and time-domain analysis. This method follows the approach of Reba *et al.*¹⁷ and has been implemented with

the code already developed and validated by Léon^{18,19} for far field sound estimates from PSE predictions.

II. Experimental setup

The experiments were carried out in an anechoic free jet facility with a cut-off frequency of 200 Hz at the *Centre d'Etudes Aérodynamiques et Thermiques* (CEAT), Institut Pprime, Poitiers, France. Two experimental campaigns obtained measurements in the pressure field generated by a jet with nozzle diameter $D = 50$ mm and Mach numbers in the range $0.4 \leq M \leq 0.6$. The corresponding Reynolds number range, $Re = \rho U D / \mu$, was 4.2×10^5 to 5.7×10^5 , where U is the velocity, ρ is the density, and μ is the air viscosity measured at the nozzle exit. A carborundum trip placed $2.7 D$ upstream of the nozzle lip ensured the boundary layer was turbulent at the jet exit. Both the acoustic far field² and the velocity field¹¹ of this jet have been previously examined, and more details are available in the cited papers. The potential core length of the jet ranged between $5 \leq x/D \leq 5.5$.

In both of the current campaigns, far-field pressure measurements were recorded by an azimuthal ring array of three 1/4" GRAS 40BP microphones at $\theta = 20^\circ$ and $R/D = 47.1$. The far-field array was limited to three microphones because the sound field of this jet at $\theta = 20^\circ$ has been shown to be dominated by fluctuations in the axisymmetric mode (azimuthal mode zero),² making resolution of higher azimuthal modes unnecessary for the Strouhal number range investigated here. The near-field array was made up of a number of azimuthal ring arrays. Azimuthal arrays were used to enable decomposition of the pressure signals into azimuthal Fourier modes, enabling mode-by-mode analysis consistent with the mode-by-mode calculation of the linear PSE solutions. An overview of the test setup is illustrated in figure 1 for one of the campaigns, and the experimental conditions for both campaigns are tabulated in table 1.

Figure 1: Overview of 4-ring setup

The focus of the first campaign was a simultaneous measurement of as much of the near field as possible to enable a time-domain comparison to the near-field fluctuations and the far-field sound. To that end the near-field array comprised seven azimuthal rings, each with six microphones distributed azimuthally. A schematic of the setup is given in figure 2. There were a total of 42 microphones in the near-field cone. Two types of microphones were used in the near-field array. The first three rings used 1/4" electret microphones, while the last four rings used 1/4" GRAS 40BP microphones. The frequency response of the electrets was found to closely match the GRAS microphones up to about 9 kHz ($St \approx 2.2$ for $M = 0.6$). The half-angle of the cone, $\alpha = 8^\circ$, matched the expansion of the jet so that each microphone measured pressure amplitudes of the same order of magnitude. The conical surface intersected the jet nozzle plane at radial distance labeled R_s , which was set between 0.6 and $0.8 D$ depending on the test configuration. The entire array was mounted on a traverse, which allowed the rings to span different portions of the jet near field ranging from X to $X_{\max}$. In this campaign, the radii of the rings were not adjusted when X was changed, meaning that each X corresponded to a different R_s as well as different axial coverage of the jet. Since the axial spacing was $0.75 D$, shifting X in multiples of this amount gave measurements at the same axial locations on different conical surfaces, allowing a partial evaluation of the radial decay of the near-field fluctuations. At each axial position, measurements were obtained in the velocity range $0.4 \leq M \leq 0.6$ in increments of 0.05 . An additional ring of 14 microphones was included close to the nozzle lip in order to investigate the relationship between nozzle fluctuations and downstream activity. These data have not been examined here and will be

considered in future work.

The second campaign, illustrated in figure 3, was designed to provide finer-resolution measurement of the statistics of the near-field fluctuations. This array consisted of four rings of six 1/4" GRAS 40BP microphones that could be moved independently in the axial direction. This setup does not allow the monitoring of the space-time structure of the near-field pressure, but it does allow the statistical shape and energy of the modes to be determined via two-point correlations, allowing the construction of the cross-spectral matrix describing the jet statistics. To ensure that all the measurements fell on the same conical surface ($R_s/D = 0.8$), the radial positions of the microphones were adjusted every time a ring was moved axially. The use of four rings—instead of the minimum two—simply reduced the number of acquisitions required to span the whole jet length. The final axial resolution of the measurements was $0.4 D$, and the measurements were made in the axial range $0.5 \leq x/D \leq 8.9$ for $M = 0.6$. Again, these parameters are summarised in table 1.

III. Parabolised Stability and Linearised Euler Equations

PSE and LEE are complementary approaches to explore and model wavepackets. PSE provides a means by which to demonstrate their existence, in a statistical sense, and may turn out to be a useful tool for low-cost prediction. LEE, on the other hand, serves as a basis for the development of time-domain, control-oriented, reduced-order models.

Herbert¹³ has given a full description of the development and application of PSE, and recent developments in the application of PSE to the near-field statistics of jet flows have been described by Gudmundsson & Colonius.¹⁴ In addition, the PSE computations used in this paper for comparison to the experimental results—which are limited to linear PSE—were already presented by Cavalieri *et al.*¹¹ This section therefore includes only the level of detail necessary to motivate the current work.

The stability computation begins with either a measured or calculated mean flow profile. For design purposes, this profile could even be calculated by a Reynolds-averaged Navier–Stokes simulation. Linear stability theory, which can only be applied to a parallel flow, takes this mean flow and gives solutions for the disturbances that correspond to a number of shape functions with corresponding growth rates and oscillation frequencies in each dimension. PSE extends this theory by allowing for a slowly spreading mean flow.¹⁴ For jet calculations, a PSE solution is obtained taking the linear stability solution at the nozzle exit as a boundary condition and then marching the solution downstream to solve for the shape functions throughout the whole jet flow. This is possible due to the parabolisation of the equations, with each step independent of the solution downstream. The outputs of the stability analysis are the amplitudes and relative phases of the flow variable fluctuations in the jet ($u', v', w', p', \rho', T'$) for a given frequency and azimuthal mode. In the linear framework, each mode amplitude includes a free constant that must be chosen after comparison with one of the flow variables. With the present jet, these constants have already been obtained by comparison with u' measured on the jet centreline.¹¹ PSE predicts fluctuations in the jet that are coherent over a large axial extent. In regions of the jet dominated by small-scale turbulence, the axially coherent fluctuations may be masked by these uncorrelated fluctuations. To compare PSE to the axially coherent fluctuations in the jet, Proper Orthogonal Decomposition (POD) is typically used to isolate the most energetic coherent fluctuations (the lowest POD modes).

Cavalieri *et al.*¹¹ analysed the velocity field of the currently studied jet for the presence of wavepackets using the PSE approach and found strong evidence that up to the end of the potential core for $St \gtrsim 0.3$ the velocity fluctuations are dominated by wavepacket activity. Some of these results are reproduced in figures 4 & 5, where figure 4 also includes the current LEE result. The close agreement up until $x/D \approx 5$ indicates the effectiveness of PSE for predicting flow fluctuations in this region, but the deterioration in agreement further downstream remained unexplained. Cavalieri *et al.*¹¹ proposed three possible hypotheses to explain this:

- H1. Downstream of the end of the potential core nonlinear effects become significant in the development of wavepackets, invalidating any linear instability-wave model;
- H2. Linear instability waves persist for high x , but account only for a small part of the azimuthally-coherent overall energy;
- H3. The instability-wave *Ansatz* no longer applies to describe velocity fluctuations in this region: downstream of the potential core the wavepackets degenerate into turbulence.

Table 1: Experiment conditions for both experimental campaigns

Campaign	X/D	$X_{\max}/D$	R_s/D	$\Delta X/D$	M
7 Ring	1.25	5.75	0.8		
	2	6.5	0.7	0.75	0.4, 0.45, \dots, 0.6
	2.75	7.25	0.6		
4 Ring	0.5	8.9	0.8	0.4	0.6

Figure 2: 7-ring near-field azimuthal array setup (7pt correlation)

Figure 3: 4-ring near-field azimuthal array setup (2pt correlation)

The data set available at the time did not allow the identification of the responsible effect. This work aims to further this investigation by examining the pressure field to clarify the fate of wavepackets downstream and by providing an opportunity to test these hypotheses.

The Linearised Euler Equations computation was performed again using the experimentally measured mean flow as a base flow, and prescribing the eigenfunction from the linearised stability analysis as a periodic disturbance at the nozzle exit plane. Since only the axisymmetric mode is considered in the present case, the code is written in 2D for simplicity. The simulation employed a sponge zone and a characteristic boundary condition in the far field with a pole condition to treat singularities. The code is based on that of Freund²⁰ and further details of the numerics can be found in that paper. As can be seen in figure 4, the preliminary LEE results agree well with those of the PSE.

Figure 4: Comparison between linear PSE (—) and LEE (---) for the axisymmetric mode and u' (□) on the jet centreline for $M = 0.4$ and $0.4 \leq St \leq 0.6$. Reproduced from Cavalieri *et al.*¹¹

Figure 5: Comparison between linear PSE and u' for $M = 0.4$ at axial stations $3.5 \leq x/D \leq 8$. — PSE, ---- Experimental u' , Axisymmetric mode of u' , -.-.- First POD mode of axisymmetric mode of u' . Reproduced from Cavalieri *et al.*¹¹

IV. Analysis and results

IV.A. POD mode analysis

First, the pressure signals are decomposed by Fourier series into azimuthal modes and frequency components as

$$\tilde{p}_m(x, t) = \tilde{p}(x, m, t) = \int p'(x, \phi, t) e^{im\phi} d\phi \quad (1)$$

$$P_{m,\omega}(x) = P(x, m, \omega) = \int \tilde{p}_m(x, t) e^{-i\omega t} dt. \quad (2)$$

The 2-point cross-spectral matrix (CSM), $\mathbf{R}_{m,\omega}(x, x')$, is then formed as

$$\mathbf{R}_{m,\omega}(x, x') = \langle P_{m,\omega}(x) P_{m,\omega}^*(x') \rangle, \quad (3)$$

where $*$ indicates complex conjugation, so that the POD problem on the cone surface is represented by the Fredholm integral equation

$$\frac{2\pi}{\cos \alpha} \int \mathbf{R}_{m,\omega}(x, x', r, r') \xi_{m,\omega}(x', r') r' dx' = \lambda_{m,\omega} \xi_{m,\omega}(x, r), \quad (4)$$

with $r = (x - X_0) \tan \alpha$, where X_0 is the virtual origin of the antenna array and α is the cone half-angle. $\lambda_{m,\omega}^{(i)}$ and $\xi_{m,\omega}^{(i)}(x)$ are the eigenvalues and eigenvectors of the matrix $\mathbf{R}_{m,\omega}(x, x')$, respectively. The kernel $r' \mathbf{R}_{m,\omega}(x, x', r, r')$ is not Hermitian, but following Jung *et al.*²¹ and Cavalieri *et al.*,¹¹ it can be made Hermitian by multiplying equation (4) by $\sqrt{r}$ and considering the kernel $\sqrt{r' r} \mathbf{R}_{m,\omega}(x, x', r, r')$ with the eigenvector $\sqrt{r} \xi_{m,\omega}^{(i)}(x)$. The constant in front of the integral can also be absorbed into the eigenvalues. Now that the kernel is Hermitian and positive semi-definite, the eigenvalues are real and greater than or equal to zero, while the eigenvectors are complex-valued. In practise, small errors in the CSM can result in some negative eigenvalues, which are considered to be zero. In the discrete case, the matrix has dimensions $I \times I$, where I is the number of axial observation locations. The calculation is performed separately for each combination of frequency and azimuthal mode. An individual POD mode is obtained by

$$POD_{m,\omega}^{(i)}(x) = \sqrt{\lambda_{m,\omega}^{(i)}} \xi_{m,\omega}^{(i)}, \quad i = 1, \dots, I. \quad (5)$$

By construction, each set $\{\xi_{m,\omega}^{(i)}(x), i = 1, \dots, I\}$ forms a complete basis for $\tilde{p}_m(x, t)$, but including only those modes for which $\lambda_{m,\omega}^{(i)} > 0$ has the added advantage of reducing the span of the basis to the span of the realizations that were used to create the eigenvectors,²² forcing all relevant boundary and physical constraints to be met automatically in any reconstruction. Figure 6 shows the energy distribution of the various POD modes at selected frequencies obtained with the high-resolution 4-ring array measurements. Most of the energy of the axisymmetric mode (45-65%) is captured in its first POD mode, and this proportion is lowest for higher St. This first POD mode is the focus of the comparison to PSE, but higher POD modes are also considered in the correlation results of §§ IV.B & IV.C.

Figure 6: Energy distribution (bars) and cumulative sum (lines) for the first 10 POD modes for $m = 0$ from the 4-ring array measurements. St = 0.2 black & —, St = 0.5 gray & — —, and St = 0.8 white & · · ·.

Figures 7 & 8 compare the PSE predictions of the azimuthal mode with the measured amplitude and phases of both the original signal and the first POD mode for different frequencies. The PSE predictions are taken from Cavalieri *et al.*¹¹ The data from the 7-ring array are also included to show that the same amplitudes and mode shapes are obtained for the coarser-resolution array. The reported amplitudes are scaled estimates of $\langle P_{m,\omega}(x) P_{m,\omega}(x) \rangle$, and the phases are obtained using

$$\text{Phase angle} = \arg(P_{m,\omega}(x)) - \arg(P_{m,\omega}(x_{\phi\text{ref}})), \quad (6)$$

where the phase reference location is $x_{\phi\text{ref}}/D = 2$. The scale factors applied to the PSE amplitudes to match the measurements were obtained already from the hot-wire measurements of Cavalieri *et al.*;¹¹ however, it was necessary to apply an additional factor of 10 to the PSE estimates to produce the current plots. The reason for this discrepancy is uncertain, but it likely results from an inconsistency in scaling between the calculation of the hot-wire results and the current results.

In figure 7 for all cases, the PSE predictions beyond the end of the potential core underestimate the fluctuation intensity, but for $St \geq 0.3$ the first POD mode matches the PSE prediction nearly exactly. For $St \geq 0.7$, this agreement breaks down at a distance well beyond the potential core length ($x/D \approx 5$). The coarser 7-ring array also captures this behaviour. For the phase estimates in figure 8, the agreement is also striking for $St > 0.1$. Again, for $St \geq 0.7$ there is a breakdown well beyond the potential core, but the agreement is otherwise excellent. The good agreement between the 7-ring and 4-ring measurements in figure 8 also shows that the phase response of the electret microphones used in the first three near-field azimuthal rings is sufficient to provide accurate measurements. The implication of this downstream

Figure 7: Comparison of PSE estimates and auto-spectra magnitudes for $m = 0$ and $M = 0.6$. — PSE, $\square$ 4-ring full signal, $\circ$ 4-ring first POD mode, $*$ 7-ring full signal, $+$ 7-ring first POD mode

Figure 8: Comparison of PSE estimates and auto-spectra phases for $m = 0$ and $M = 0.6$. Reference is $x/D = 2$. See figure 7 for legend.

agreement is that linear wavepackets do persist in this region. Since POD is efficient at filtering spatially incoherent fluctuations, the good collapse downstream with just the first POD mode shows that incoherent fluctuations are responsible for the high downstream magnitudes observed before POD is applied. Thus the PSE predictions accurately estimated the coherent part of the fluctuations verifying the presence of linear wavepackets downstream. This is in contrast to the results of Cavalieri *et al.*¹¹ that are reproduced in figure 5, particularly in subplot (c), in which the POD did not collapse well to the PSE estimate at $x/D = 8$. A possible explanation for this is that the incoherent in-flow velocity fluctuations are much greater than the coherent wavepacket-related fluctuations for POD to efficiently isolate them. This explanation corresponds to Cavalieri *et al.*'s¹¹ hypothesis H2 reproduced in § III. The current results make it difficult to accept hypothesis H3, because the wavepackets—though not apparent in the velocity field—are still observable in the pressure field measurements.

IV.B. Projection of time signals onto POD modes

Since the POD modes form a complex-valued orthonormal basis for the measured pressure fluctuations,²² complex-valued coefficients of each POD mode, $b_{m,\omega}^{(i)}$, can be determined by projecting the Fourier transformed time signals of the azimuthal coefficients, $P_{m,\omega}(x)$, onto the basis:

$$b_{m,\omega}^{(i)} = \int P_{m,\omega}(x) \xi_{m,\omega}^{(i)*}(x) dx. \quad (7)$$

A low-order estimate of the time-domain signal for an azimuthal mode m can then be formed by

$$\hat{p}_m(x, t) = \frac{1}{2\pi} \int \left[\sum_{i=1}^J b_{m,\omega}^{(i)} \xi_{m,\omega}^{(i)}(x) \right] e^{i\omega t} d\omega, \quad (8)$$

where the approximation becomes an equality when $J = I$ or at least the number of $\lambda_{m,\omega}^{(i)} > 0$. For discrete computations, equations (7) and (8) can be applied sequentially to blocks of data where the number of points in each block should be the same as in equation (2). For consistency, when calculating ensemble averages with the reconstructed data, the blocks should be the same as in the reconstruction.

In principle, for a full time-domain reconstruction using a particular POD basis, it is necessary to have simultaneous measurements at all the original ring locations used for obtaining that basis. This means that a time-domain decomposition of all 22 POD modes available (per azimuthal mode) using the 4-ring array is not possible, and only statistical information is available. However, since the 7-ring and 4-ring setups measure the same jet, the POD modes are the same and the 7-ring data can be projected onto the higher resolution basis. To determine $b_{m,\omega}^{(i)}$, the 4-ring basis must be re-sampled at the same locations as the 7-ring data. Both the re-sampled basis and the original basis should be scaled by the same factor such that $|\xi_{m,\omega}^{(i)}(x)| = 1$ for the re-sampled basis. This rescaling ensures that fluctuation energy is conserved—barring small errors—in the projection. In practise, since the 7-ring data have fewer degrees of freedom and modes higher than 7 will be aliased into the lower modes causing discrepancies in the modes obtained by 4-ring and 7-ring arrays, this technique is only valid to the extent that the obtained POD modes have the same shape. Figure 9 shows that this is valid over the frequency range of interest for the first POD mode and that the validity is reasonable but deteriorating for the next two POD modes. These modes account for nearly all of the flow energy as indicated by figure 6. Figure 10 shows an example of the reconstruction for $m = 0$ using just the first POD mode in comparison to the full 7-ring signal in retarded time coordinates to align with the far-field pressure. For POD reconstruction in figure 10, the block size for the Fourier transform was $N = 2440$ points ($\Delta St = 0.01$). The results in figure 10 highlight the wavepacket structure of the near pressure field, which is not only present statistically (as previously shown by Suzuki & Colonius¹⁶ and Tinney & Jordan²³), but also in the time domain.

IV.C. Near field–Far field correlation

Figure 11 shows the normalised cross-correlation, $C(\tau)$, between the axisymmetric modes for the near-field and far-field signals ($\tilde{p}_0$). All the correlation plots shown here are plotted with respect to the retarded time, $\tau_r = \tau - R/c_0$, accounting for an estimate of the sound propagation time to the observation location. The extended nature of the peak signature indicates that the flow phenomena related to sound production are

Figure 9: $m = 0$ POD modes for 4-ring (—) and 7-ring ($\times$ and $\circ$) setups. Arbitrary scaling between St .

correlated over a large spatial extent.²⁴ If such a phenomenon also has high energy compared to other flow fluctuations, it should be detectable in the POD modes. As proposed in § IV.B, the correlation between the reconstructed POD modes and the far field can indicate if POD efficiently collapses the sound-producing phenomena. If the correlation for any particular mode is significantly higher than the baseline correlations in figure 11, then that mode is a good candidate for a sound generation mechanism and a possible target for control schemes. If the correlation magnitudes are similar across all POD modes, then POD—while being optimal in terms of energy in the near field—is inefficient for isolating sound-producing fluctuations. Since the first POD mode was shown in § IV.A to be nearly equivalent to the PSE prediction, the correlation between the first POD mode and the far field also constitutes a correlation between a time-domain projection of the near-field array onto the wavepacket model and the far-field pressure. If this correlation is significantly higher than the full signal correlation, then this projection is a suitable denoising method to remove fluctuations uncorrelated to the far field and it provides strong evidence that wavepackets are the main source of sound at $\theta = 20^\circ$.

With the current data, the correlation can be made directly with both the 4-ring and 7-ring data for the full signal for each $\tilde{p}_m(x, t)$, but only the axisymmetric mode for the 4-ring data is shown here. For the 7-ring data, the contributions of the POD modes to the nearfield-farfield correlation are presented in figures 12 & 13 for the individual modes and the cumulative modes, respectively. Subplot 13(g) corresponds to the full correlation with $\tilde{p}_0(x, t)$, which is consistent with figure 11. For the 4-ring data, this POD mode-by-POD mode correlation for all POD modes is impossible because not all axial locations were recorded simultaneously. However, using the reconstruction method of § IV.B, the 7-ring data was projected to the 4-ring POD modes, allowing the higher-resolution, longer axial extent correlations to be estimated. Figures 14 & 15 show these POD mode correlations both individually and cumulatively for the first three POD modes using this projection.

Figure 14(a) is nearly identical to the full correlation result plotted in figure 11, which shows that the first POD mode captures the salient correlation features. Interestingly, the correlations for the other modes show different space-time structures than for the first mode. However, this should not be considered evidence of independent sound generation mechanisms because the POD reconstruction includes a wide range of frequencies, over which it is unlikely that such concerted fluctuations occur in both time and space. Rather, the various correlation features of the higher modes may be better described as coherent deviations from the underlying coherent wavepacket structure. It is unclear why these deviations are as well-correlated to the far-field sound as the POD mode, so this result is rather inconclusive. Either the wavepacket model fails to denoise the non-sound-related fluctuations, or the components of the signal that do not fit well with the wavepacket model ($St < 0.3$ and possibly $St > 0.9$) affect the correlation result here. Another possible

Figure 10: Comparison between near-field and far-field $m = 0$ pressure mode signals for both full signal and POD reconstruction. $2\sigma_p$ (—, where σ_p is the standard deviation of $\tilde{p}_{0,FF}$) is also plotted to show peaks of the far-field signal.

Figure 11: Normalised retarded correlation between near-field $\tilde{p}_0$ and far-field $\tilde{p}_0$.

Figure 12: Individual $m = 0$ POD mode contributions to near field–far field correlation ($C_{\tilde{p}_{NF}\tilde{p}_{FF}}$) for 7-ring data. The absolute maximum for each subplot is indicated in brackets.

explanation is that the higher POD modes are capturing jitter about the first POD mode because jitter has been shown to have an impact on the efficiency of wavepacket sources.¹⁰ Regardless of the specific explanation, it would seem that a single POD mode is insufficient to extract all the acoustically important fluctuations in the flow.

IV.D. Far-field prediction from near-field wavepackets

Reba *et al.*^{25,17} showed, by changing coordinates to a spherical system $(\mathcal{R}, \Theta, \varphi)$ centred at the virtual origin of the conical array, that it is possible to write the far acoustic field due to a pressure distribution on the near-field conical surface as

$$P_{m,\omega}(\mathcal{R}', \Theta') = 2\pi \sin \alpha \int_0^\infty P_{m,\omega}(\mathcal{R}, \alpha) \frac{\partial g_{m,\omega}}{\partial \Theta}(\mathcal{R}, \alpha; \mathcal{R}', \Theta') d\mathcal{R}, \quad (9)$$

where $g_{m,\omega}$ is a tailored Green's function for the reduced wave equation and the $'$ indicates the observer coordinate location. Equation (9) can be formulated for either statistical or time-domain predictions, both of which are performed below. A brief validation of these prediction techniques is presented in appendix A.

Figure 13: Cumulative $m = 0$ POD mode near field-far field correlations ($C_{\tilde{p}_{\text{NF}}\tilde{p}_{\text{FF}}}$) for 7-ring data. The absolute maximum for each subplot is indicated in brackets.

Figure 14: Individual $m = 0$ POD mode contributions to near field-far field correlation ($C_{\tilde{p}_{\text{NF}}\tilde{p}_{\text{FF}}}$) for projection using equation (8). The absolute maximum for each subplot is indicated in brackets.

Figure 15: Cumulative $m = 0$ POD mode near field-far field correlations ($C_{\tilde{p}_{\text{NF}}\tilde{p}_{\text{FF}}}$) for projection using equation (8). The absolute maximum for each subplot is indicated in brackets.

IV.D.1. Formulation for statistical predictions

Reba *et al.*^{25,17} continued with equation (9) to write the auto-spectral density of the far-field pressure as

$$P_{m,\omega}(\mathcal{R}', \Theta') = 4\pi^2 \sin^2 \alpha \iint_0^\infty \mathbf{R}_{m,\omega}(\mathcal{R}_1, \mathcal{R}_2) \frac{\partial g_{m,\omega}}{\partial \Theta}(\mathcal{R}_1, \alpha; \mathcal{R}', \Theta') \frac{\partial g_{m,\omega}^*}{\partial \Theta}(\mathcal{R}_2, \alpha; \mathcal{R}', \Theta') d\mathcal{R}_1 d\mathcal{R}_2, \quad (10)$$

which gives the far-field Sound Pressure Level (SPL). Their predictions fit well for high-speed ($M \approx 1.6$) jets up to about $\theta < 45^\circ$ at $St = 0.25$, but the results were significantly worse for slower jets $M \approx 0.9$ and higher frequency ($St = 0.40$).

IV.D.2. Formulation for time-domain predictions

Though the statistical prediction technique has been used before, a prediction of the time-domain far-field pressure has not been attempted by any investigators. Taking the inverse Fourier transform of equation (9) gives

$$\tilde{p}_m(\mathcal{R}', \Theta', t) = \int P_{m,\omega}(\mathcal{R}', \Theta') e^{i\omega t} d\omega, \quad (11)$$

which is the time-resolved far-field pressure mode. In conjunction with the POD mode projection of § IV.B, this technique can give time-domain far-field predictions from near-field microphone measurements. To implement this with discrete data, equation (9) must first be solved for every frequency for the number of Fast Fourier Transform (FFT) points N used to calculate $P_{m,\omega}(\mathcal{R}', \Theta')$, which allows the inverse FFT (IFFT) to be applied. In practice for each far-field observation location, the signal at each time-step is dependent on a number of time steps in the near field. Based on causality, since the fluid everywhere outside of the array cone is quiescent, the contribution from each near-field location takes $\tau = |R' - R|/c_0$ to arrive, making each far-field point dependent on a duration of data equal to $\tau_{\max} - \tau_{\min}$, where $\tau_{\min}$ and $\tau_{\max}$ are the minimum and maximum propagation times, respectively, from locations with significant contribution to the far field. So in practice the number of valid time series points is limited by the physical layout of the array and far-field observer because the discontinuity at the edge of the FFT window causes an error in the far-field prediction. Since the propagation distances are finite (see figure 16) and the sound speed is constant in the ambient fluid, the number of affected points is limited, and the number of valid points can be estimated by

$$N_{\text{Valid}} = N - (\tau_{\max} - \tau_{\min})f_s, \quad (12)$$

When calculating a long time series, the series can be broken into blocks, and overlapping the blocks by at least N_{Valid} points overcomes the issue of corrupted points.

Figure 16: Geometric considerations for time-domain predictions with a finite window Fourier transform.

IV.D.3. Evaluation of experimental limitations

Since the experimental near-field array extent is limited to $0.5 \leq x/D \leq 8.9$, the predictions with the real data have an additional source of error. To estimate the effect of this truncation, a validation problem, described in appendix A was attempted where only those near-field locations falling in $0.5 \leq x/D \leq x_{\max}$ were used in the calculation. Figure 17 shows that the choice of $x_{\max}$ has a major effect on the predicted directivity by making calculations for $x_{\max}/D = 8.9, 20$, and 30 . For reference, the result for $X_0/D \leq x/D \leq 70$ is also plotted. A direct interpretation of this result would imply that for accurate predictions at least a $20D$ array is necessary, which would be surprising because the decay of fluctuation energy beyond the potential core is rapid, and the fluctuations well beyond the end of the potential core are not expected to contribute greatly to the far-field sound. However, a more likely explanation is that the sudden truncation of the data constitutes a spurious, highly localised source at the array cut-off that is perfectly correlated with the wavepacket source. The interaction between this source and the wavepacket would result in both constructive and destructive interference yielding both underprediction and overprediction of the emissions at different angles, a result consistent with figure 17. The strongest evidence for this is in figure 17(a), where for high θ , both the results for $X_0/D \leq x/D \leq x_{\max}/D$ and $x_{\max}/D \leq x/D \leq 70$ overpredict the directivity by ≈ 20 dB, while both together give the correct result. This is only possible if the two solutions are highly correlated and interfere destructively. Further evidence for this explanation is provided by comparing the prediction at $\theta = 20^\circ$ in figure 17(a) for the $X_0/D \leq x/D \leq x_{\max}/D$ array, which is off by ≈ 6 dB, to the time-domain prediction in figure 18(a), where although the amplitude is off, most of the phase information is still correctly predicted in time domain; that is the peaks and troughs of the signal are still roughly aligned. This raises the question of how to quantitatively evaluate how ‘similar’ two time-domain signals are, and whether a definition of ‘similar’ can be developed that best aids the evaluation of noise source modelling schemes. A first step in this direction has been presented by Breakey & Meskell.²⁶ A better understanding of this effect—and possible alleviations of it—will be available from viewing time-resolved spatial plots of the sound predictions near the array edges, a result available from the current time-domain prediction technique. These plots will be considered in future work.

The other experimental factors tested were the axial spacing of the microphone rings, $\Delta x/D$ (0.4 in the experiments); the first microphone ring location, $X_{\min}/D$ (0.5 in the experiments); and the initial radius of the conical array, R_s/D (0.8 in the experiments). For values close the experimental parameters, none had a significant effect on the results. In fact, for all $\Delta x/D \lesssim 1.2$ and $St < 0.7$, the results were practically identical, meaning that a sparser microphone experimental array would be practical. Beyond this limit, the directivity was erroneously predicted, particularly for higher St and higher θ .

Figure 17: Effect of experimental near-field array truncation on the statistical far-field prediction. — Lighthill solution and Green’s function solutions with $\circ$ $X_0/D \leq x/D \leq 70$, $\bullet$ $X_0/D \leq x/D \leq x_{\max}/D$, and $\square$ $x_{\max}/D \leq x/D \leq 70$ array extents.

IV.D.4. Application to experimental data

The directivity results for the 4-ring data are compared to the data measured previously for this jet by Cavalieri *et al.*¹¹ in figure 19. Clearly, the Green’s function predicted directivity is not accurate for any of the CSM representations. Across all frequencies the full data prediction is closest in terms of magnitude,

Figure 18: Effect of experimental near-field array truncation on the time-domain far-field prediction at $\theta = 20^\circ$. — Lighthill solution and Green's function solutions with $\bullet X_0/D \leq x/D \leq x_{\max}/D$, and $\square x_{\max}/D \leq x/D \leq 70$ array extents. Every fifth Green's function data point is shown and the data between the lines denoted by $\tau_{\min}$ and $\tau_{\max}$ are known to be invalid based on equation (12).

but it still does not fit the experimental directivity. Using just the PSE or first POD mode shape reduces the prediction by as much as 20 dB over all angles, which is surprising. This effect could be attributed to either the large difference in fluctuation magnitude beyond the potential core between the full data and the POD and PSE shapes (see figure 7) or the effect of a non-unit coherence off the main diagonal of the CSM for the full data. In fact non-unit coherence is one key difference between the experimental data and the validation problem. Though figures 7 & 8 indicate there is very little difference in the CSM between the first POD mode and PSE, there is still a significant deviation in the directivity in figure 19(b), indicating the prediction is sensitive to small deviations in the CSM.

As discussed in § IV.D.3, the effect of the axial truncation of the near-field array is expected to contribute to this error. To estimate this effect, the PSE CSM was also truncated to $0.5 \leq x/D \leq 8.9$, and the result is also presented in figure 19. The truncation does cause a significant deviation in the predicted directivity shape, though it is not great enough to account for the large deviation between the predictions and the experimental results. Reba *et al.*²⁵ overcame the truncation limitation by modelling the axial decay of the CSM beyond the experimental array. This was also attempted with these data, but even after testing several models for the wavepacket decay, no significant improvement to the predictions was achieved. This is despite the fact that Reba *et al.*'s²⁵ predictions were accurate at least for low θ and low St . The difference here seems to be the much lower M in this case. Reba *et al.*'s²⁵ best agreement was for $M = 1.6$, and at $M = 0.9$, the errors were much more significant. In the context of the described spurious source explanation of the truncation effect (§ IV.D.3), the effect of the spurious source at the array edge is less noticeable because, for a higher- M jet, the upstream source is more efficient due to the fact that more of the fluctuations reside in the supersonic portion of the wavenumber–frequency spectrum.¹ A fuller discussion of the CSM modelling process, and the effect of the chosen model as well as M and R_s on the predictions is presented in appendix B, the results of which also indicate that for higher- M jets, accurate predictions are available even without treating the problem of array truncation at $x_{\max}/D = 8.9$.

Figure 19: Comparison of Green's function predicted directivities from the current 4-ring measurements. $\square$ Full prediction, $\circ$ POD mode 1 prediction, $--$ PSE prediction, $*$ truncated PSE prediction ($0.5 \leq x/D \leq 8.9$), $—$ Cavalieri *et al.*'s¹¹ experimental data.

V. Conclusions

In recent years, wavepacket models for jets have shown considerable promise for the prediction of acoustically-important flow fluctuations, particularly for fluctuations that are azimuthally and axially coherent, which have been associated with low-angle sound emissions. Recent analysis of the far-field sound emissions and the in-flow velocity field of a subsonic jet by Cavalieri *et al.*¹¹ found close agreement between wavepacket predictions based on PSE and experimental fluctuation measurements on the jet centreline up until the end of the potential core, indicating the presence of wavepackets at least up to this point. From their measurements, the mechanism responsible for this breakdown was unclear and three potential hypotheses were proposed, one of which was that the wavepackets do continue beyond the potential core, but are dominated by incoherent fluctuations. The current investigation analysed the near-field pressure signatures of the same jet to test this hypothesis and to form a link between the near-field pressure fluctuations and the far-field sound emissions.

The results indicate that linear wavepackets persist in the jet beyond the end of the potential core. The presence of these wavepacket fluctuations is masked by incoherent fluctuations in this region, but wavepackets can be recovered by axial POD filtering. A time-domain reconstruction of the near-field fluctuations for each POD mode has been performed, allowing the near-field fluctuations to be projected onto an empirically-obtained wavepacket (POD mode). This made it possible to consider the correlation between the POD modes and the far-field sound. The correlation indicated that the first POD mode captures the salient features of the space-time correlation to the far-field sound, but the higher POD modes also showed significant correlations to the far-field, indicating that a single POD mode is insufficient to extract all the acoustically important flow fluctuations.

The statistical far-field Green's function prediction of the near-field data from a conical microphone array was shown to be valid—a result already known from the work of Reba *et al.*²⁵—but the application to the experimental data gave poor directivity predictions. The Green's function prediction method was also extended to make time-domain predictions of the far-field sound, a quantity that could be compared directly with the current experimental dataset. A comparison of the both the directivity and time-domain predictions for a validation problem, in which it was possible to consider arrays matching the experimental extent as well as those with much longer extents, gave clear evidence that the poor predictions result from the finite extent of the near-field array. This truncation creates a spurious source at the array edge that is perfectly correlated with the wavepacket source, resulting in constructive and destructive interference and leading to poor predictions. The effect of this spurious source was also shown to be less important for higher- M jets, which is consistent with Reba *et al.*'s²⁵ successful predictions at high M .

The analysis toolbox presented in the current paper contains many of the tools necessary to make statistical and time-domain predictions of far-field sound emissions from real jets based on a wavepacket understanding of the dominant sound source. These tools also make it possible to address the problem of the finite extent of the near-field array, which will be pursued in future work. Wavepackets remain a promising candidate for a sound source structure in jets. Their existence within the jet near-field is apparent from quantitative comparisons, and the current results indicate they indeed persist beyond the potential core. In order to finish the jigsaw puzzle that links wavepacket sources to far-field jet noise emissions, it will be necessary to address the outstanding questions of how to address the experimental limitation of a finite microphone array and what effect non-unit coherence of the CSM has on jet noise emissions.

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A. Far-field Prediction Validation

The code developed for the calculation of the far-field pressure has been extensively validated by Léon¹⁸ and Léon & Brazier,¹⁹ and a modified validation suitable for time-domain signals is presented here. The validation problem is generated for the azimuthal pressure mode by prescribing a simple line source for the Lighthill stress tensor given by^a

$$T_{11}(x, t) = A(x)\tilde{p}_{\text{exp}}(t - x/U_c),$$

where $A(x)$ represents an arbitrary spatial envelope, $U_c = 0.6U$ is an estimated wavepacket convection velocity, and $\tilde{p}_{\text{exp}}(t)$ is an experimental pressure signal taken from the azimuthal mode of the microphone ring at $x/D = 4.1$. This was chosen so that the frequency content of the validation problem would be similar to the real case. $A(x)$ was taken to correspond roughly to the amplitude envelope measured on the microphone array using the model function

$$A(x) = a \exp \left[-\frac{(x-b)^2}{(c(x-b)+d)^2} \right], \quad (\text{A.1})$$

^a $T_{11}(x, t) = \delta(y)\delta(z)T_{11}(x, y, z, t)$ where $\delta(\cdot)$ is the Dirac delta function.

where the coefficients a , b , c , and d were determined from a least-squares fit of the experimental data.

The Lighthill integral for this line source is solved using a high spatio-temporal resolution numerical calculation in the time domain by

$$\tilde{p}_m(x', y', t) = \frac{1}{4\pi} \int_0^\infty \frac{1}{\sqrt{(x' - x)^2 - y'^2}} \frac{\partial^2 T_{11} \left(x, t - \sqrt{(x' - x)^2 - y'^2} \right)}{\partial x \partial x} dx, \quad (\text{A.2})$$

where ' again indicates the observer location. The integral is solved for observers at positions along the near-field array from the array origin (X_0) to $x/D = 70$ with a spacing of $0.4D$ as well as at the position of each far-field microphone ($R/D = 47.1$ and $15^\circ \leq \theta \leq 80^\circ$). To avoid edge effects from the finite time window, the Lighthill solution is calculated for a larger time window and only the central points are used in the predictions. The Green's function prediction for the far field is made by propagating the near-field Lighthill solution from equation (A.2) using the Green's function technique. This prediction is then compared directly to far-field Lighthill solution calculated directly from equation (A.2). Figure 20 shows the results for both the statistical prediction and the time-domain prediction. Both sets of results used blocks of $N = 2440$ points corresponding to $\Delta St = 0.01$, where the time-domain result was calculated from a single block, and the statistical results were calculated by a series of blocks with 50% overlap. Since the statistical results compare only single frequencies, a Hann window was used in the calculation of $\mathbf{R}_{m,\omega}$ to avoid spectral leakage. No windowing was used for the time-domain result. The Green's function predictions faithfully reproduce the Lighthill solution in both cases, verifying the propagation technique. In the time-domain case, the result shown is one block of $N = 2440$ points, and the points between $\tau_{\min}$ and $\tau_{\max}$ are corrupted as predicted.

Figure 20: Validation results for the far-field Green's function predictions from the near-field array with $X_0 \leq x/D \leq 70$. — Lighthill solution, $\circ$ Green's function solution.

B. Modelling the near-field Cross-Spectral Matrix (CSM)

In Reba *et al.*'s²⁵ work, the authors used a curve fit with a model function to extend the CSM beyond the length of the experimental array. Their resulting far-field predictions were accurate for low θ . The same

technique was also attempted here with both the validation problem and the experimental data, but without such success. This is thought to be because of the effect a the spurious source created by the finite axial extent of the near-field microphone array, an effect that is less noticeable for higher M . These comparisons are made with the data generated by the validation problem presented in appendix A since M cannot be varied for the experimental data. Several different model functions were tested to extend the CSM. These are tabulated in table 2. For the results presented here, only the generic surface phase model was used.

Amplitude Models	$ \mathbf{R}_{m,\omega}(x, x') $	Definitions
Coherent Analytical	$M_1(x)M_1(x')$	$M_1(x) = a \exp[-(x - b)^2/(cx + L)^2]$
Incoherent Analytical	$M_2(x, x')$	$M_2(x, x') = a \exp \left[-\frac{(x - b)^2}{(cx + L)^2} - \frac{(x' - b)^2}{(dx' + L')^2} \right]$
Generic Surface	$\mathcal{M}_3(x, x')$	$\mathcal{M}_3(x, x')$ is a generic non-linear surface fit
Reba ²⁵	$\sqrt{M_4(x)M_4(x')} \exp \left[-\frac{s}{L_{\text{corr}}(\bar{x})} \right]^2$	$M_4(x) = A(x/L_s)^{\gamma-1} e^{-\gamma x/L_s}$, $L_{\text{corr}}(\bar{x}) = B\bar{x} + C$, $\bar{x} = (x + x')/2$, $s = (x - x')/2$
Phase Models	$\arg(\mathbf{R}_{m,\omega}(x, x'))$	Definitions
Generic surface	$\text{unwrap}[\phi(x, x')] = \mathcal{P}_1(x, x')$	$\mathcal{P}_1(x, x')$ is a generic non-linear surface fit with linear extrapolation
Reba ²⁵	$\phi(\bar{x}) = 2s\kappa(\bar{x})$	$s = (x - x')/2$ and $\bar{x} = (x + x')/2$ with $\kappa(\bar{x})$ solved from a linear least-squares problem.

Table 2: Tested models for the CSM beyond the experimental array.

As the principal difference between the current dataset and Reba *et al.*'s,²⁵ M was a natural parameter to investigate in the validation problem. R_s (the radius of the conical array at the jet exit) was also pertinent because it affects the rate of decay with increasing x of the pressure fluctuations on the near-field array, thereby changing the shape of the curve that needs to be modelled. Figure 21 shows the effect of both M and R_s on the accuracy of the directivity predictions. For the experimental conditions (figure 21(a)), none of the models is able to accurately reproduce the directivity, though near $\theta = 40^\circ$, all the predictions are accurate. While R_s has a noticeable effect for some models for low M , the most significant thing to note is that for higher M all the models collapse to the right directivity. This is despite the fact that the truncated, un-modelled estimate shows only moderate improvement at higher M , implying that the chosen model does have an effect on the spurious source. In fact, for $M = 1.2$ (figure 21(c)), the coherent model would be sufficient to accurately predict the directivity. This helps explain part of the discrepancy between Reba *et al.*'s²⁵ application of this technique and the current results.

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Figure 21: Effect of M and R_s on the far-field statistical predictions with different CSM models. — Lighthill solution, $\circ$ $0 \leq x/D \leq 70$, $--$ $0.5 \leq x/D \leq 8.9$, $+$ Coherent analytical, $\diamond$ Incoherent analytical, $\square$ Generic surface, $\triangleleft$ Reba²⁵ model.

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