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Massive vector scattering in Lee-Wick gauge theory

Benjamín Grinstein,[1,*] Donal O'Connell,[2,3,†] and Mark B. Wise[4,‡]

[1]Department of Physics, University of California at San Diego, La Jolla, California 92093, USA
[2]Institute for Advanced Study, School of Natural Sciences, Einstein Drive, Princeton, New Jersey 08540, USA
[3]The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
[4]California Institute of Technology, Pasadena, California 91125, USA

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We demonstrate that amplitudes describing scattering of longitudinally polarized massive vector bosons present in non-Abelian Lee-Wick gauge theory do not grow with energy and, hence, satisfy the constraints imposed by perturbative unitarity. This result contrasts with the widely known violation of perturbative unitarity in the standard model with a very heavy Higgs boson. Our conclusions are valid to all orders of perturbation theory and depend on the existence of a formulation of the theory in which all operators are of dimension four or less. This can be thought of as a restriction on the kinds of higher dimension operator which can be included in the higher derivative formulation of the theory.

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I. INTRODUCTION

Non-Abelian Lee-Wick gauge theories were introduced in Ref. [1], building on pioneering work of Lee and Wick [2]. Lee and Wick introduced a finite theory of quantum electrodynamics, which includes extra degrees of freedom that cancel radiative divergences present in QED. These new degrees of freedom are associated with a nonpositive definite norm on the Hilbert space. Lee and Wick argued that the theory could nevertheless be unitary provided that the new Lee-Wick particles obtain decay widths. Another peculiar feature of the Lee-Wick theory is that it is classically unstable, so that a future boundary condition must be imposed to prevent exponential growth of some modes. This leads to causality violation in the theory [3]; however, this acausality is suppressed below the scales associated with the Lee-Wick particles.

There has been considerable discussion of this proposal in the literature, including debate about unitarity of the theory [4–7], possible applications to gravity [8], as well as nonperturbative studies of Lee-Wick theories [9,10]. In [1] a non-Abelian extension of the original proposal of Lee and Wick was described. Non-Abelian Lee-Wick gauge theories are not finite but the only divergences present are logarithmic. It was also shown in [1] how to build theories with Lee-Wick partners of chiral fermions. Consequently, it is possible to write down an extension of the standard model to include Lee-Wick fields. This Lee-Wick standard model has a stable Higgs mass and is consistent with current observations if the Lee-Wick particles present in the theory have masses of order the TeV scale. Recently, there has been further discussion of the Lee-Wick standard model, including aspects of the LHC (Large Hadron Collider) phenomenology of the model [11,12], automatic suppression of flavor changing neutral currents in the Lee-Wick standard model [13], gravitational Lee-Wick particles [14], and the possibility of coupling heavy physics to the model [15]. Similar higher derivative operators have recently been examined in [16].

In this work, we focus on an important conceptual issue in Lee-Wick gauge theory. These theories can be thought of as ordinary gauge theories with additional higher dimension operators present in the Lagrangian. A particular choice of higher dimension operator was made in [1] but this was not the most general choice. The higher dimension operator leads to the presence of interacting massive (Lee-Wick) vector bosons in the theory. It was shown in [1] that an equivalent form of the theory exists in which all operators are of dimension four or less. In this formulation, new fields are present which describe the Lee-Wick particles. One might think that scattering of massive Lee-Wick vector bosons will violate perturbative unitarity at some scale since their longitudinal polarizations grow with energy. We will demonstrate that this does not occur precisely for the very special choice of higher derivative operators that can be written in the form of a Lee-Wick gauge theory. The existence of the Lee-Wick form of the Lagrangian will be a crucial ingredient in our proof. Thus, a gauge theory with arbitrary higher dimension operators added is not unitary in perturbation theory. Only specific higher dimension operators are consistent with unitary scattering. These operators are such that a Lee-Wick form of the Lagrangian exists where only operators with dimension less than or equal to four are present.

II. NON-ABELIAN LEE-WICK GAUGE THEORY

In this section we review the construction of non-Abelian Lee-Wick gauge theories. The Lagrangian is

$$\mathcal{L}_{\rm hd} = -\frac{1}{2}\,{\rm tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{M^2}\,{\rm tr}(\hat{D}^{\mu}\hat{F}_{\mu\nu})(\hat{D}^{\lambda}\hat{F}_{\lambda}{}^{\nu}), \quad (1)$$

*bgrinstein@ucsd.edu
†donal@ias.edu
‡wise@theory.caltech.edu

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ig[\hat{A}_\mu, \hat{A}_\nu]$, and $\hat{A}_\mu = \hat{A}^A_\mu T^A$ with T^A the generators of the gauge group G in the fundamental representation. We will refer to this as the higher derivative formulation of the theory. We can derive an equivalent formulation as follows. First, we introduce an auxiliary vector field $\tilde{A}_\mu$ so that we can write the Lagrangian of the theory as

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - M^2\,\mathrm{tr}\tilde{A}_\mu\tilde{A}^\mu + 2\,\mathrm{tr}\hat{F}_{\mu\nu}\hat{D}^\mu\tilde{A}^\nu, \quad (2)$$

where $\hat{D}_\mu \tilde{A}_\nu = \partial_\mu \tilde{A}_\nu - ig[\hat{A}_\mu, \tilde{A}_\nu]$. To diagonalize the kinetic terms, we introduce shifted fields defined by

$$\hat{A}_\mu = A_\mu + \tilde{A}_\mu. \quad (3)$$

The Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{LW}} = &-\tfrac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\mathrm{tr}(D_\mu\tilde{A}_\nu - D_\nu\tilde{A}_\mu)(D^\mu\tilde{A}^\nu - D^\nu\tilde{A}^\mu)\\ &- ig\,\mathrm{tr}([\tilde{A}_\mu, \tilde{A}_\nu]F^{\mu\nu}) - \tfrac{3}{2}g^2\,\mathrm{tr}([\tilde{A}_\mu, \tilde{A}_\nu][\tilde{A}^\mu, \tilde{A}^\nu])\\ &- 4ig\,\mathrm{tr}([\tilde{A}_\mu, \tilde{A}_\nu]D^\mu\tilde{A}^\nu) - M^2\,\mathrm{tr}(\tilde{A}_\mu\tilde{A}^\mu). \quad (4)\end{aligned}$$

Note that in this (Lee-Wick) formulation only dimension four operators appear in the Lagrangian. It is also evident in this form that the theory is unstable because of the wrong sign kinetic terms for the field $\tilde{A}_\mu$. We impose a future boundary condition that there is no exponential growth of any mode to deal with this instability.

The Lagrangian given in Eq. (4) contains an interacting massive Lee-Wick vector boson $\tilde{A}_\mu$. These massive vectors obtain widths since they can decay to ordinary gauge bosons. This width is necessary to remove unphysical cuts in Feynman diagrams associated with single Lee-Wick particles which would otherwise violate unitarity.[1] In this paper, we focus on constraints unitarity places on the growth of amplitudes with energy and so we neglect the widths of Lee-Wick vectors.

Consider the scattering of four Lee-Wick gauge bosons. The scattering amplitude $\mathcal{M}$ is of the form

$$\mathcal{M} = \epsilon(p_1)^\mu \epsilon(p_2)^\nu \epsilon(q_1)^\rho \epsilon(q_2)^\sigma \mathcal{M}_{\mu\nu\rho\sigma}, \quad (5)$$

where $\epsilon(p)$ is a polarization vector associated with momentum p and $\mathcal{M}_{\mu\nu\rho\sigma}$ is a dimensionless quantity built from the Feynman rules of the theory. Since the Lagrangian in Lee-Wick form contains only dimension four operators, $\mathcal{M}_{\mu\nu\rho\sigma}$ does not grow at high energies with fixed scattering angle. Longitudinal polarization vectors, on the other hand, do grow at high energy: the longitudinal polarization vector associated with a particle of four-momentum $(E, p, 0, 0)$ is given by

$$\epsilon_L = (p, E, 0, 0)/M. \quad (6)$$

When the growth of longitudinal polarization vectors is taken into account, we see that the amplitude $\mathcal{M}$ could grow as quickly as E^4 at large energy. This kind of growth would be a disaster for the theory. In a theory of a massive vector boson with Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + M^2\,\mathrm{tr}A_\mu A^\mu, \quad (7)$$

it is known that the amplitude describing four longitudinal vector boson scattering grows as E^2. The possible E^4 growth is removed because to a first approximation one can ignore the mass M occurring in propagators at high energies; then the Ward identity of gauge-invariant theories acts to remove the largest growth. Nevertheless, the amplitudes still grow too quickly with energy to be consistent with unitarity. Thus, the theory must either become strongly coupled so that perturbative computations are misleading, or new degrees of freedom must appear around the scale M to restore unitarity. In non-Abelian Lee-Wick gauge theories, the Ward identity is exact despite the presence of massive vector bosons. We will show that the exact Ward identity prevents the amplitudes growing on account of large polarization vectors.

III. WARD IDENTITIES IN THE HIGHER DERIVATIVE FORMULATION

We now turn to the proof that scattering amplitudes in Lee-Wick gauge theory do not increase too quickly with energy. The key to the proof is the following observation. For a massive vector boson with very large momentum p with respect to some reference frame, the associated longitudinal polarization vector is proportional to the momentum plus a residual vector δ which does not grow with energy. For example, if the momentum is $(E, p, 0, 0)$ then

$$\begin{aligned}\delta^\mu &= (p - E, E - p, 0, 0)/M,\\ \lim_{p\to\infty}&(p - E, E - p, 0, 0) = 0.\end{aligned} \quad (8)$$

The Ward identities (WI) provide us with nonperturbative information on the structure of amplitudes with external momenta contracted into a leg. We study the Ward identities in higher derivative gauge theories, and show that they force amplitudes to vanish if an external momentum is contracted in. This happens for ordinary gauge bosons as in normal gauge theories, and also for the new Lee-Wick poles. Thus, because of gauge invariance, we may always replace external longitudinal polarizations with the residual vector δ^μ:

$$\epsilon_L(p) \sim \delta(p) = \epsilon_L - p/M. \quad (9)$$

Since δ does not grow with energy, the high energy behavior of the amplitudes is given by dimensional analysis of the vertices in the Lee-Wick form of the theory.

Let us now begin our study of the Ward identities. We will work in background field gauge, so we derive the identities for one particle irreducible (1PI) functions from

[1]Further adjustments to the theory are necessary to ensure that the only cuts present in loop graphs are associated with physical states; for a pedagogical discussion of this topic, see [3].

the effective action.[2] Recall that this quantity is related to the 1PI functions by

$$\Gamma(A) = \sum_n \frac{1}{n!} \int \left(\prod_{i=1}^{n} d^4x_i\right) A^{a_1}_{\mu_1}(x_1)\cdots A^{a_n}_{\mu_n}(x_n) \times \Gamma^{(n)a_1\cdots a_n}{}_{\mu_1\cdots\mu_n}(x_1,\ldots,x_n). \quad (10)$$

Now, this is a gauge-invariant quantity, so that

$$\Gamma\left(A_\mu + \frac{1}{ig} D_\mu \omega\right) = \Gamma(A_\mu) \quad (11)$$

for infinitesimal $\omega^a(x)$. Consequently, the 1PI functions obey

$$\sum_n \frac{1}{(n-1)!} \int \left(\prod_{i=1}^{n} d^4x_i\right) (D_{\mu_1}\omega)^{a_1}(x_1) A^{a_2}_{\mu_2}(x_2)\cdots A^{a_n}_{\mu_n}(x_n) \times \Gamma^{(n)a_1\cdots a_n}{}_{\mu_1\cdots\mu_n}(x_1,\ldots,x_n) = 0. \quad (12)$$

This implies the WIs, which are obtained by taking one functional derivative with respect to ω, $(n-1)$ functional derivatives with respect to A and setting ω and A to zero. Carrying out the algebra we obtain

$$\begin{aligned} &\partial^{\mu_1}\Gamma^{(n)a_1\cdots a_n}{}_{\mu_1\cdots\mu_n}(x_1,\ldots,x_n) \\ &= g\sum_{i=2}^{n} \delta^{(4)}(x_1 - x_i) f^{a_1 a_i b} \\ &\quad \times \Gamma^{(n-1)a_1\widehat{a_2\cdots a_n}}{}_{\mu_1\widehat{\mu_2\cdots\mu_n}}(x_1, x_2, \widehat{\ldots}, x_n), \end{aligned} \quad (13)$$

where the hat over the last $(n-2)$ entries of the lists indicates that the ith entry should be removed. Fourier transforming to momentum space, we find

$$\begin{aligned} &p_1^{\mu_1}\Gamma^{(n)a_1\cdots a_n}{}_{\mu_1\cdots\mu_n}(p_1,\ldots,p_n) \\ &= -ig\sum_{i=2}^{n} f^{a_1 a_i b}\Gamma^{(n-1)a_1\widehat{a_2\cdots a_n}}{}_{\mu_1\widehat{\mu_2\cdots\mu_n}}(p_1 + p_i, p_2, \widehat{\ldots}, p_n), \end{aligned} \quad (14)$$

where the momenta satisfy the condition $\sum p_i = 0$. We now move on to examine Ward identities explicitly in several cases.

A. Uses of WIs I: The two point function

First, we apply Eq. (14) to the case $n = 2$. This is simple since there is no $n = 1$ 1PI function:

$$p^\mu \Gamma^{(2)ab}{}_{\mu\nu}(p) = 0, \quad (15)$$

which implies

$$\Gamma^{(2)ab}{}_{\mu\nu}(p) = -i\delta^{ab}(p^2 g_{\mu\nu} - p_\mu p_\nu)\Pi(p^2). \quad (16)$$

[2] We assume that the procedure used to define amplitudes that are unitary and contain no cuts or poles from the Lee-Wick vector bosons does not violate gauge invariance.

More precisely, the WI does not determine the dependence on color indices, but one can easily derive that separately. Note that in the higher derivative theory, any zero of $\Pi(p^2)$ corresponds to an on-shell degree of freedom, unlike in ordinary gauge theories for which $p^2 = 0$ is the on-shell condition.

B. Uses of WIs II: The three point function

The case $n = 3$ is, explicitly,

$$\begin{aligned} k^\mu \Gamma^{(3)abc}{}_{\mu\nu\lambda}(k,p,q) &= -igf^{abe}\Gamma^{(2)ec}{}_{\nu\lambda}(k+p) \\ &\quad - igf^{ace}\Gamma^{(2)eb}{}_{\lambda\nu}(k+q). \end{aligned} \quad (17)$$

Now we can use the results in Eqs. (15) and (16) on the right-hand side of Eq. (17). It follows that if we put the momenta p and q on shell and contract with either polarization vectors or momenta this vanishes. Let us see how this works in some detail.

First, consider the case $q^2 = p^2 = 0$. Then the right-hand side of Eq. (17) is

$$-gf^{abc}(p_\lambda p_\nu - q_\lambda q_\nu)\Pi(0). \quad (18)$$

If we now contract this with two polarization vectors, $\epsilon^\nu(p)\epsilon^\lambda(q)$, or with the two remaining external momenta $p^\nu q^\lambda$, the result is obviously zero. The same result holds if we contract with one polarization and one momentum, and use the condition $q^2 = p^2 = 0$.

The case with $p^2 = M^2$ and $q^2 = 0$ has the right-hand side

$$\begin{aligned} &gf^{abc}[(M^2 g_{\lambda\nu} - p_\lambda p_\nu)\Pi(M^2) + q_\lambda q_\nu \Pi(0)] \\ &\quad = gf^{abc} q_\lambda q_\nu \Pi(0) \end{aligned} \quad (19)$$

and this obviously vanishes if one contracts with $\epsilon^\lambda(q)$ or q^λ. Finally, the case with $p^2 = q^2 = M^2$ is trivial because $\Pi(M^2) = 0$.

C. Uses of WIs III: The four point function and gauge invariance of the *S*-matrix

In this case we find an identity which has a less obvious interpretation:

$$\begin{aligned} &k^\mu \Gamma^{(4)abcd}{}_{\mu\nu\lambda\sigma}(k,p,q,r) \\ &= -ig[f^{abe}\Gamma^{(3)ecd}{}_{\nu\lambda\sigma}(k+p,q,r) \\ &\quad + f^{ace}\Gamma^{(3)bed}{}_{\nu\lambda\sigma}(p,k+q,r) \\ &\quad + f^{ade}\Gamma^{(3)bce}{}_{\nu\lambda\sigma}(p,q,k+r)]. \end{aligned} \quad (20)$$

The key to understanding the use of this identity is to write the scattering amplitude, which is the sum of $\Gamma^{(4)}$ plus three more terms corresponding to the s-, t-, and u-channel exchanges of a gauge boson between $\Gamma^{(3)}$ vertices. What we will show is that when we contract those with k^μ, put the other particles on shell, and also contract with external polarizations or momenta, then the sum precisely cancels the three terms of $k^\mu \Gamma^{(4)}_{\mu\nu\lambda\sigma}$ above.

For example, the s-channel exchange amplitude is

$$-\Gamma^{(3)abe}{}_{\mu\nu\rho}(k, p, -k-p) G^{(2)eh}{}_{\rho\eta}(k+p) \times \Gamma^{(3)cdh}{}_{\lambda\sigma\eta}(q, r, k+p), \tag{21}$$

where we have used the same combination of momenta and indices as in the four point functions above. Also we have introduced the full propagator $G^{(2)}$. This, of course only makes sense if it is gauge fixed but the WIs of the previous section show that the gauge fixing term gives no contribution once the external legs are put on shell and contracted with polarizations or momenta. Now, contract this with k^μ and use

$$k^\mu \Gamma^{(3)abe}{}_{\mu\nu\rho}(k, p, -k-p) = ig[f^{hba}\Gamma^{(2)he}{}_{\nu\rho}(k+p) + f^{hea}\Gamma^{(2)hb}{}_{\rho\nu}(-p)]. \tag{22}$$

We can ignore the second term, since $\Gamma^{(2)hb}{}_{\rho\nu}(-p)\epsilon^\nu(p) = 0$ and $\Gamma^{(2)hb}{}_{\rho\nu}(-p)p^\nu = 0$. The first term remains. Let us contract it with the propagator:

$$igf^{hba}\Gamma^{(2)he}{}_{\nu\rho}(k+p)G^{(2)el}{}_{\rho\eta}(k+p) = igf^{lba}\left(g_{\nu\eta} - \frac{(k+p)_\nu(k+p)_\eta}{(k+p)^2}\right). \tag{23}$$

We have used the fact that $\Gamma^{(2)}$ is the inverse of $G^{(2)}$, but only on the space projected out by the former. Hence the s-channel exchange contracted with k^μ gives

$$-igf^{hba}\left(g_{\nu\eta} - \frac{(k+p)_\nu(k+p)_\eta}{(k+p)^2}\right)\Gamma^{(3)cdh}{}_{\lambda\sigma\eta}(q, r, k+p) = -igf^{hba}\Gamma^{(3)cdh}{}_{\lambda\sigma\nu}(q, r, k+p) \tag{24}$$

up to terms that vanish when external legs go on shell and are contracted with polarization vectors or momenta. This term cancels the first term in (20). The second and third terms in (20) are canceled by the t- and u-channel exchanges. Thus, we see that the Ward identity forces four particle scattering amplitudes to vanish for on-shell external particles when one or more external momenta are contracted into the legs. This removes the growth of scattering amplitudes associated with large polarization vectors.

IV. SUCCESSES AND FAILURES OF DIMENSIONAL ANALYSIS

We have now seen that the growth of longitudinal polarization vectors in the higher derivative theory is not important in scattering amplitudes. However, the amplitudes could still grow if the uncontracted amplitude $\mathcal{M}_{\mu\nu\rho\sigma}$ grows. In fact, in the higher derivative theory of Eq. (1) one might expect this amplitude to grow. The reason is that the four particle scattering amplitude must be dimensionless. However, there are vertices in this theory which are proportional to $1/M^2$. Thus, by dimensional analysis (DA) the rest of the interaction must have mass dimension 2, so that terms like E^2 are allowed. These terms would eventually lead to violation of perturbative unitarity.

However, the Lee-Wick description of the theory shown in Eq. (4) is equivalent to the higher derivative formulation. In the Lee-Wick description, only operators of dimension four are present. Thus, the four vector boson vertex, for example, is dimensionless. Now DA indicates that at high energies, $E \gg M$, the rest of the interaction consists of dimensionless ratios formed from the momenta in the problem, so that the uncontracted amplitude does not grow at high energies with fixed nonzero scattering angle. Of course, the on-shell scattering amplitudes in the higher derivative and Lee-Wick forms of the theory are the same. Since we can compute the scattering amplitude appropriate for the higher derivative theory from the Lee-Wick form, we conclude that the full amplitude does not grow with energy. Now when we contract in the polarization vectors, we see that their growth is also unimportant. Putting it all together, we find that the on-shell scattering amplitudes cannot grow at high energies.

The Lagrangian given in Eq. (1) is not the most general Lagrangian including dimension six operators. One could also add a term

$$\Delta\mathcal{L} = \frac{i\gamma g}{M^2}\,\mathrm{tr}(\hat{F}_{\lambda\mu}[\hat{F}^{\lambda\nu}, \hat{F}^\mu{}_\nu]). \tag{25}$$

It is still possible to construct a Lee-Wick Lagrangian to describe the theory including this operator in the same way as described in Sec. II. However, the resulting Lagrangian contains dimension six operators and so we expect the amplitudes to grow with energy. We have confirmed that this is the case for the operator in $\Delta\mathcal{L}$ by explicit calculation. Thus, internal consistency of Lee-Wick gauge theories requires the special choice of higher dimension operator shown in Eq. (1).

V. EXPLICIT CALCULATIONS OF SCATTERING AMPLITUDES

We have demonstrated that the amplitudes for vector boson scattering in Lee-Wick theory do not grow with energy. Since our argument for acceptable high energy behavior uses a combination of the two formulations of the theory it is worth presenting the results of some explicit calculations that support our conclusions. For definiteness we consider in this section the scattering $\tilde{W}^1(p_1)\tilde{W}^2(p_2) \to \tilde{W}^2(q_1)\tilde{W}^1(q_2)$ of $SU(2)$ LW-gauge bosons. Here the superscripts denote the adjoint gauge indices associated with the particles being scattered. We work in the center of mass frame and take the incoming particles to have energy E and let θ be the angle between the momenta $\mathbf{p}_1$ and $\mathbf{q}_1$.

For the case where all the LW bosons are longitudinally polarized, we find that the leading behavior of the scattering amplitude at high energy E and fixed scattering angle θ is

$$\mathcal{M}(LL \to LL) \simeq g^2 \frac{1 + \cos\theta}{(1 - \cos\theta)^2}\left(\frac{M}{E}\right)^2((19 - 9\cos\theta) \times \cos\theta + 14), \tag{26}$$

where g is the $SU(2)$ gauge coupling, and the symbol $\simeq$ indicates that we have shown only the leading behavior in an expansion in powers of M/E. We have performed the tree-level calculation of this amplitude both in the higher derivative formulation of the theory and the formulation with LW fields. In the LW-field formulation the particular values of the coefficients of the operators in Eq. (4) encode the fact that the massive LW-vector bosons are associated with poles in a gauge field and are crucial for obtaining a scattering amplitude that does not grow with energy. If the Lagrange density,

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{tr}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + M^2\,\mathrm{tr}\tilde{A}_\mu\tilde{A}^\mu, \tag{27}$$

where $\tilde{F}_{\mu\nu} = \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu - ig[\tilde{A}_\mu, \tilde{A}_\nu]$, was used to calculate this scattering amplitude it would grow proportional to E^2 resulting at high energies in a theory that is either strongly coupled or violates unitarity.

Next we consider the case where the final LW-gauge boson with four-momentum q_2 is transversely polarized in the plane of the scattering. In that case the high energy behavior of the scattering amplitude is

$$\mathcal{M}(LL \to TL) \simeq g^2\cos\theta\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}}\frac{M}{E}. \tag{28}$$

Not all the amplitudes fall with increasing energy. For example, the scattering of two longitudinal LW-vector bosons into two that are transversely polarized in the plane of scattering does not grow at high energies,

$$\mathcal{M}(LL \to TT) \simeq g^2(1 + \cos\theta). \tag{29}$$

VI. CONCLUDING REMARKS

Non-Abelian Lee-Wick gauge theories contain massive vector bosons whose scattering does not violate perturbative unitarity, unlike ordinary gauge theories with mass terms. There still remains the question of whether Lee-Wick theories are Lorentz invariant and unitary to all orders of perturbation theory, and of whether this can be done while preserving gauge invariance. So far, only specific examples have been studied [4,8,10]. It would be very interesting to answer this question.

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