

17th AIAA/CEAS Aeroacoustic Conference and Exhibit, 6-8 June 2010, Portland, Oregon

Axisymmetric superdirectivity in subsonic jets

André V. G. Cavalieri*, Peter Jordan† and Yves Gervais‡

Institut Pprime, Poitiers, France

Tim Colonius§

California Institute of Technology, Pasadena, CA, USA

We present experimental results for the acoustic field of jets in the Mach number range $0.35 \leq M \leq 0.6$. Data acquired by means of an azimuthal ring of six microphones, whose polar angle, θ , was progressively varied, is decomposed into azimuthal Fourier modes. In agreement with past observations, the sound field for low polar angles (measured with respect to the jet axis) is found to be dominated by the axisymmetric mode, particularly at the peak Strouhal number. As θ is increased, modes 1 and 2 become increasingly important and dominate at angles greater than $\theta \approx 30^\circ$. A number of features of the axisymmetric mode of the acoustic field suggest that it can be associated with an axially *non-compact source*, in the form of a convected wave comprising amplification, saturation and decay, and whose axial extension is of the order of several jet diameters: (a) the sound pressure level for peak frequencies is shown to be *superdirective* for all Mach numbers considered, with exponential decay as a function of $(1 - M_c \cos \theta)^2$, in agreement with wave-packet models for an axially non-compact axisymmetric source; (b) while the mode $m = 1$ spectrum scales with Strouhal number, suggesting that its energy content is associated with turbulence scales, the axisymmetric mode scales with Helmholtz number—the ratio between source length scale and acoustic wavelength; (c) the axisymmetric radiation has a stronger velocity dependence than the higher order azimuthal modes, again in agreement with predictions of the said wave-packet models. We use such a wave-packet model to estimate that the axial extension of the source structure underpinning the axisymmetric component of the sound field is of the order of 6–8 jet diameters, and that the source comprises a convected wave with three spatial oscillations, weighted by a Gaussian envelope; such a source structure is in good agreement with past observations based on coherent structure eduction techniques. The present results show that the narrow-band spectrum of the axisymmetric mode contributes to the appearance of the characteristic jet-noise spectrum at low angles, an effect that becomes more marked as the Mach number is increased.

I. Introduction

The sound generation by subsonic turbulent jets is a problem characterised by the coupling between the turbulent motions of the jet and the less complex acoustic motions of the sound field. The said difference in complexity is not only apparent in the structure of the equations that model the two different kinds of motion, but also in their respective, experimentally-observed, kinematic structures.

If we consider, for instance, the azimuthal structure of the two, considerably fewer azimuthal Fourier modes are necessary for description of the sound field than for description of the turbulence;^{1,2} and many researchers have interpreted this low-order azimuthal structure of the sound field as evidence of a corresponding low-order *sound-producing* turbulence structure. One may indeed postulate that the coherent structures observed in jets (see for instance Crow & Champagne,³ Moore,⁴ Hussain & Zaman⁵ or Tinney & Jordan,⁶ among others) will produce such an azimuthally-coherent signature in the sound field. And it is this idea that

*PhD student, Institut Pprime, 43 rue de l'Aérodrome, 86036 Poitiers, France.

†Research Scientist, Institut Pprime, 43 rue de l'Aérodrome, 86036 Poitiers, France.

‡Professor, Institut Pprime, 40 avenue du Recteur Pineau, 86022 Poitiers, France.

§Professor, Department of Mechanical Engineering, California Institute of Technology

Copyright © 2011 by P. Jordan. Published by the American Institute of Aeronautics and Astronautics, Inc. with permission.

is frequently the motivation for decomposition of the sound field into azimuthal Fourier modes (Maestrello,⁷ Fuchs & Michel,² Juvé *et al.*,⁸ Brown & Bridges⁹ and Kopiev *et al.*¹⁰).

A candidate source *Ansatz* where these coherent structures are concerned is the axially-extended wave packet. This simple model, which can be dynamically accounted for in the context of linear stability theory (based on a time-averaged mean flow), and physically justified by means of a scale-separation argument, comprises the acoustically-important features of axial amplification, saturation and downstream decay; the saturation is not necessarily associated with non-linearity, and can be accounted for in a linear framework by appealing to the slow spread of the mean flow. While Mollo-Christensen^{11,12} observed and discussed these features from the point of view of both hydrodynamic stability theory and aeroacoustics, Crow¹³ (see also Crighton¹⁴) was first to propose a model, within the framework of Lighthill's acoustic analogy.¹⁵ The radiation of such sources, for subsonic convection speeds, is highly directive and concentrated at low polar angles (measured with respect to the downstream jet axis).

Similar studies were undertaken by Crighton & Huerre,¹⁶ who evaluated the directivity pattern of different envelope functions for the convected wave, and by Sandham *et al.*,¹⁷ who showed that *temporal* modulation of such convected wave-packets can further enhance sound radiation. Other variants, proposed by Ffowcs Williams & Kempton¹⁸ and Cavalieri *et al.*,¹⁹ allow inclusion of the jitter that is observed in real turbulent jets: the intermittent appearance of trains of turbulent 'puffs', as observed by Crow & Champagne³ for instance.

All of the above models have in common their directivity: sound radiation is concentrated at low polar angles, polar decay being exponential. The term *superdirectivity* was coined by Crighton & Huerre¹⁶ to describe this characteristic of the sound field, and they showed that the requirement for such radiation is that the penetration zone of the source (i.e. the near pressure field) have an extent comparable to the acoustic wavelength; i.e. that acoustic non-compactness is the salient source feature.

Experimentally, there is not, for the moment, complete consensus regarding the relationship between the superdirectivity of wave-packet models and the sound field of subsonic jets; while the latter does present higher sound intensities at low polar angles, it does not have exponential decay as a function of θ , as predicted by the said models. Superdirectivity was detected in a forced jet by Laufer & Yen,²⁰ where forcing was effected at a Strouhal number, based on the momentum thickness, of $St_{\delta_2} = 0.017$. The excited jet comprised subharmonics of the forcing frequency, and the directivity of the subharmonic sound radiation was observed to decay exponentially with $(1 - M_c \cos\theta)^2$, in agreement with the directivity of the models of Crow¹³ and Ffowcs Williams & Kempton.¹⁸ However, the excitation frequency corresponds to a Strouhal number, based on the jet diameter, of $St = 5.8$, which is much higher than the spectral peak of the sound field radiated by free turbulent jets. It is therefore difficult to affirm that Laufer and Yen's experiment corresponds to practical unforced jet flows; the mechanism they studied may occur in low Mach number jets with laminar boundary layers at the nozzle exit, as discussed by Bridges & Hussain.²¹

On the other hand, Cavalieri *et al.*²² have shown, with numerical data from a LES of a Mach 0.9 jet, that a simplified wave-packet *Ansatz*, fitted with velocity data from the LES, can reproduce the radiated sound for the axisymmetric mode of the simulation to within 1.5dB at low polar angles. Furthermore, the axisymmetric mode was shown to be highly directive, dominating sound radiation at low polar angles, as found experimentally by Fuchs & Michel² and Juvé.⁸ These results suggest that the signature of a wave-packet source structure, such as postulated by the cited works,^{13,18,19} may be observable in high Reynolds number jets if the axisymmetric radiation is isolated from the other azimuthal modes present in the acoustic field.

The objective of the present work is to investigate, experimentally, if such wave-packet radiation and the associated superdirective sound field are present in the acoustic field of unforced subsonic jets. We decompose the acoustic field measured by a microphone ring array into azimuthal Fourier modes. We then examine the directivity and spectra of each azimuthal mode; polar spacings of $\Delta\theta = 5^\circ$ are used at low emission angles, in order to obtain good resolution of the directivity of the different azimuthal modes, and to detect the expected high variations of acoustic intensity. Finally, we focus our attention more particularly on the axisymmetric mode, in an effort to characterise its structure and ascertain if this is consistent with existing wave-packet models.

In our evaluation of the experimental data we consider a model problem wherein the free-space wave equation is driven by a simplified line source; the form of the source is consistent with Lighthill's acoustic analogy.¹⁵ The model problem considered is not, of course, intended to correspond to the real flow, or to contain all of the physics of jet noise production; its purpose is to allow us to test hypotheses. On one

hand we want to check for consistency between experimentally-observed features of the sound field and certain, hypothesised, acoustically-important, features of the flow; and, on the other, we wish to rule out source features that are not consistent with the sound field: for example, a superdirective sound field at low Mach number cannot be produced by an acoustically compact source; an extended axial source region, with significant interference effects, is required to generate such an acoustic field,¹⁶ and one of the conclusions of the analysis is that the sound radiated to low polar angles is dominated by such as source.

The paper is organised as follows. In section II we describe the experimental setup. There is then a brief recapitulation, in section III, of some pertinent results where wave-packet sound radiation is concerned. This is followed by a presentation and general discussion of the experimental results in section IV, while in section IV.A we focus on the results for the Mach 0.6 jet. It is here that the axisymmetric mode is shown to dominate sound radiation at low angles, in particular for the peak frequencies, and, furthermore, to comprise a marked directivity: a 7.8dB decrease in OASPL from $\theta = 20^\circ$ to $\theta = 45^\circ$ and a 15.4dB decrease in SPL for $St=0.2$ over the same polar range. The SPL for $St=0.2$ is shown to be in agreement with the superdirectivity predicted by non-compact wave-packet models. The same characteristics are then shown, in section IV.B, to be endemic to lower Mach number jets also ($M = 0.4$, $M = 0.5$), and the axial extension of the source is here estimated using Crow’s wave-packet model.¹³ In section IV.C we show that while the spectra for azimuthal mode 1 scale with the Strouhal number, the axisymmetric mode spectra scale with the Helmholtz number, again suggesting that the non-compactness of the source is a salient feature where sound generation is concerned. We then further explore the spectrum in section IV.D, showing that the LSS similarity spectra²³ is a result of the dominance, and narrow-band character, of the axisymmetric mode. Finally, in section IV.E we present some results regarding the velocity dependence of the different azimuthal modes; the axisymmetric radiation is found to have a stronger velocity dependence than the higher order modes at low polar angles, especially for the peak frequency. Extrapolation of the present results to higher subsonic Mach numbers suggests that the dominance of the axisymmetric mode will be further enhanced as the Mach number is increased.

II. Experimental description

The experiments were performed in the ‘Bruit et Vent’ anechoic facility at the Centre d’Etudes Aérodynamiques et Thermiques (CEAT), Institut Pprime, Poitiers, France. A photo of the experimental setup is shown in figure 2(a). Acoustic measurements were made for unheated jets, with acoustic Mach numbers ($M = U/c$, where U is the jet exit velocity and c the ambient sound speed) in the range $0.35 \leq M \leq 0.6$; the Mach number increment was 0.05. The nozzle diameter, D , is 0.05m. With these conditions, the Reynolds number, $\rho U D / \mu$, varies from 3.7×10^5 to 5.7×10^5 , where ρ and μ are, respectively, the density and the viscosity at the nozzle exit.

A convergent section was located upstream of the jet exit, with area contraction of 31. This was followed by a straight circular section of length 150mm; inside this section, a boundary layer trip was used to force transition 135mm upstream ($2.7D$) of the nozzle exit. Velocity measurements were performed, with a hot wire, in order to characterise both the global structure of the flow, and the local, exit-plane conditions. Where the latter is concerned, high-density radial profiles were measured both upstream and downstream of the exit plane. The resulting profiles, representative of the nozzle boundary layer, are shown in figure 1. The results are typical of turbulent boundary layers.²¹ The calculated values for the boundary layer thickness, δ , and for the momentum thickness, δ_2 , at the nozzle exit for the $M = 0.4$, 0.5 and 0.6 jets are shown in table 1. We have used the Crocco-Busemann relation for unitary Prandtl number to determine the density across the boundary layer; however, as the calculated density changes are small for the present range of Mach numbers, the compressibility plays a minor role in the determination of the momentum thickness.

Table 1. Boundary layer thickness δ and momentum thickness δ_2 at the nozzle exit

M	δ (mm)	δ/D	δ_2 (mm)	δ_2/D
0.4	4.5	$9.0 \cdot 10^{-2}$	0.477	$9.5 \cdot 10^{-3}$
0.5	4.25	$8.5 \cdot 10^{-2}$	0.401	$8.0 \cdot 10^{-3}$
0.6	4.25	$8.5 \cdot 10^{-2}$	0.396	$7.9 \cdot 10^{-3}$

Six microphones were deployed on an azimuthal ring in the acoustic field at constant angle θ to the downstream jet axis. The setup is shown in figure 2(a). The ring has a fixed diameter of $35D$. The ring

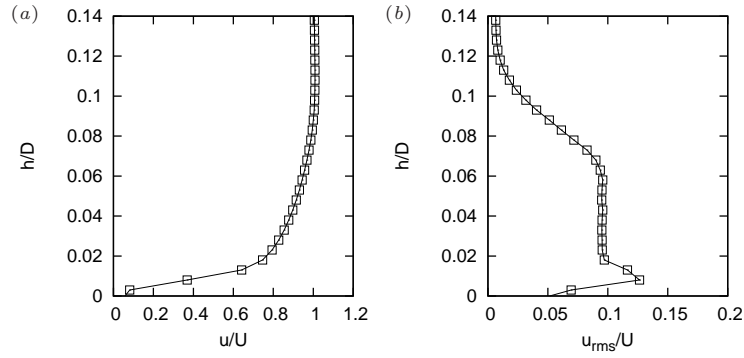


Figure 1. Boundary layer profiles at the nozzle exit for the Mach 0.6 jet: (a) mean velocity and (b) rms value

array was displaced incrementally along the jet axis in order to characterise the sound field as a function of polar angle, θ . On account of the resultant differences in the distance, r , between the nozzle exit and the microphones, a $1/r$ scaling is applied for the acoustic pressure so as to correct the measurements to a fixed distance of $r = 35D$.

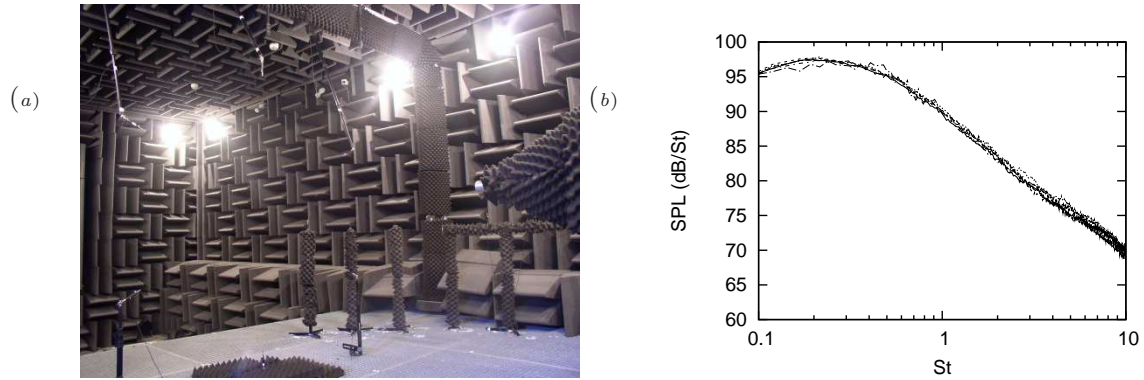


Figure 2. (a) Experimental setup; (b) spectra of the six microphones at $\theta = 30^\circ$ and $M = 0.6$

It is assumed that the jets comprise circumferential homogeneity.¹ A verification of this hypothesis in the acoustic field was performed by comparing spectra of the individual microphones, shown in figure 2(b). The close agreement indicates that there is no preferred direction for sound radiation.

III. Sound radiation by a wave-packet

In this section we recall the results of Crow¹³ (see also Crighton¹⁴) for a simple wave-packet source. The results of this model are used for analysis of the experimental results presented in the following sections.

The model is based on Lighthill's analogy, the free-space wave-equation being driven by a simplified line source, constituted of the T_{11} term alone (i.e. an axial distribution of axially-aligned, longitudinal quadrupoles), and comprising of a convected wave of frequency ω and wavenumber k , modulated by a gaussian with characteristic length L :

$$T_{11}(\mathbf{y}, \tau) = 2\rho_0 U \tilde{u} \frac{\pi D^2}{4} \delta(y_2) \delta(y_3) e^{i(\omega\tau - ky_1)} e^{-\frac{y_2^2}{L^2}} \quad (1)$$

where ρ_0 is the density of the undisturbed fluid and \tilde{u} is the streamwise velocity fluctuation amplitude.

Evaluation of the far field pressure leads to

$$p(\mathbf{x}, t) = -\frac{\rho_0 U \tilde{u} M_c^2 (kD)^2 L \sqrt{\pi} \cos^2 \theta}{8|\mathbf{x}|} e^{-\frac{L^2 k^2 (1 - M_c \cos \theta)^2}{4}} e^{i\omega(t - \frac{|\mathbf{x}|}{c})}, \quad (2)$$

where M_c is the Mach number based on the phase velocity U_c of the convected wave.

The models of Ffowcs Williams & Kempton¹⁸ and Cavalieri *et al.*,¹⁹ which include jitter in this source shape, also present the same exponential function $\exp(-L^2 k^2 (1 - M_c \cos \theta)^2 / 4)$ for the pressure. This exponential polar variation is referred to as *superdirectivity*.¹⁶

We note that superdirectivity is present if the characteristic length, L , of the gaussian is large compared to the convected wavelength, λ_c , for $k = 2\pi/\lambda_c$ and thus $kL = 2\pi L/\lambda_c$. The superdirectivity can thus be seen thus to be a result of axial interference in an axially-extended source comprising more than one spatial oscillation. The interference between regions of positive and negative source strength results in the sound field being beamed towards low angles, an almost complete cut-off occurring at high polar angles. This is illustrated in figure 3, where source shapes and corresponding directivities are plotted for different values of kL , considering $M_c = 0.36$. For the compact limit, $kL \rightarrow 0$, the directivity of the source is given by $\cos^4 \theta$ for the acoustic intensity. For small values of the characteristic length, L , the dependence of the directivity on L is weak. However, as the axial interference becomes significant, the directivity changes considerably, becoming increasingly concentrated at low axial angles, as can be seen in figure 3(b) for $kL = 6$. For this source extent, as shown in figure 3(a), there is interference between three neighbouring wavefronts in the source, leading to the observed superdirectivity.

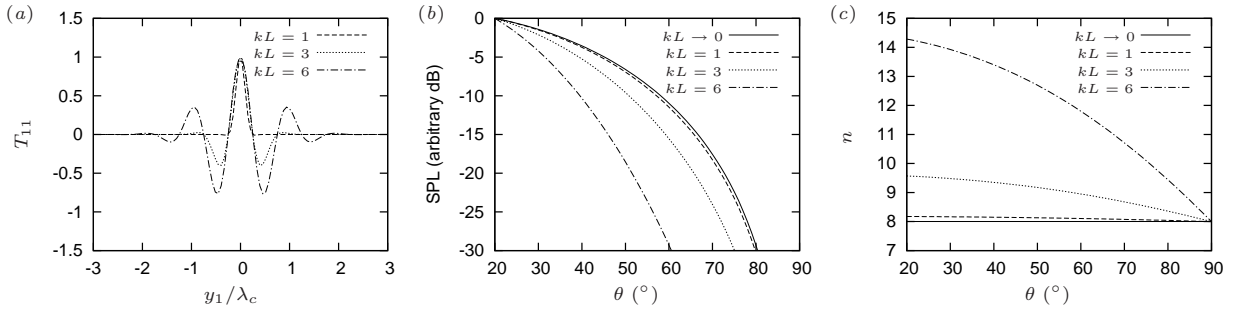


Figure 3. (a) Wave-packet shapes, (b) corresponding directivities for $M_c = 0.36$ with values at $\theta = 20^\circ$ were fixed at 0dB, and (c) velocity exponents, taken with a derivative around $M_c = 0.3$.

A further effect of non-compactness of the source is noticeable in the velocity dependence of sound radiation. A compact quadrupole-like source will lead to a U^8 velocity dependence. But as non-compact effects become significant, the velocity dependence changes, and may even be other than a power law; indeed, the expression in eq. (2) is not a power law in the velocity. In the compact limit it reduces to a U^8 velocity dependence, as expected. Figure 3(c) shows the velocity exponent, n , of the acoustic intensity for $M = 0.5$, evaluated using eq. (2). For this calculation we assume constant Strouhal number and source extent, L/D . We note that a compact wave-packet, such as the case for $kL = 1$, has a velocity exponent close to 8 for all angles; increases in L lead to higher velocity exponents, especially for lower axial angles.

IV. Experimental results and analysis

IV.A. Mach 0.6 jet

Figure 4 shows the directivity of the Mach 0.6 jet for the measured angles, as well as the contributions of the different azimuthal modes.

We note that the axisymmetric mode presents a marked directivity towards the low axial angles. Indeed, there is a 7.8 dB increase in the sound intensity between 45° and 20° . The other azimuthal modes increase more gradually over $45^\circ \leq \theta \leq 90^\circ$, with a slope close that of mode 0 in the same angular sector. For lower angles, modes 1 and 2 decay. Similar directivities for the azimuthal modes 0, 1 and 2 have been observed in a large eddy simulation of a Mach 0.9 jet.²²

Spectra for angles 20° , 30° and 40° are shown in figure 5. The increase of mode 0 is mostly concentrated in the lower frequencies. For Strouhal numbers greater than 1, there is still a dominance in the total spectra of modes 1 and 2.

To evaluate the directivity of the spectral peak, the SPL for $St=0.2$ is shown in figure 6. We see that for this frequency there is an even higher directivity of mode 0, with an increase of 15.4 dB between 45° and 20° , i.e. a factor of 34 in the acoustic intensity.

As presented in §III, models representing the wave-packet form of axisymmetric coherent structures in jets predict an exponential change of sound intensity with $(1 - M_c \cos \theta)^2$. Figure 6(b) presents the SPL at

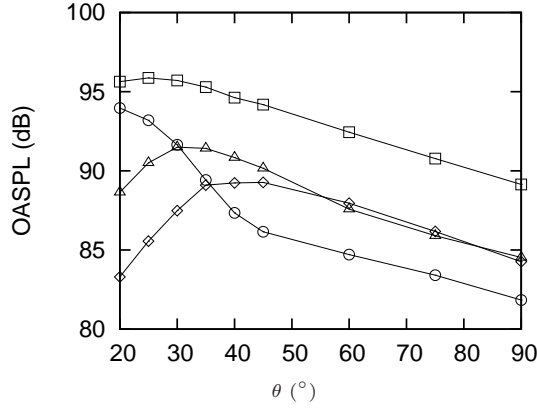


Figure 4. Directivity for $M = 0.6$: squares, total; circles, mode 0; triangles, mode 1; and diamonds, mode 2.

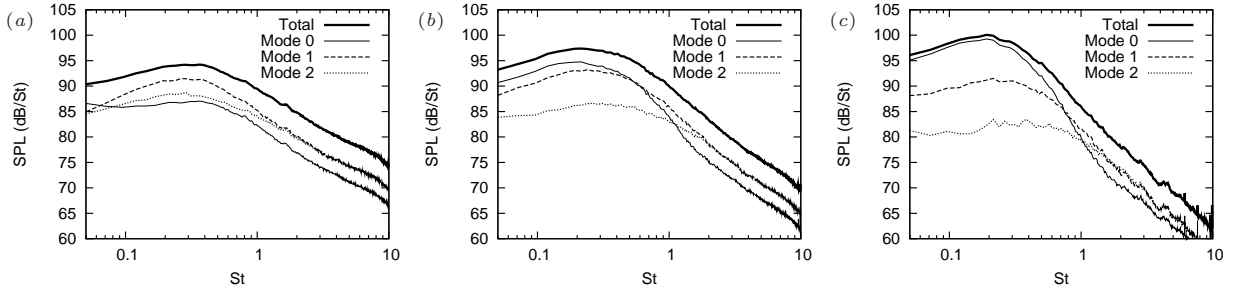


Figure 5. Spectra of individual modes for (a) $\theta = 40^\circ$, (b) $\theta = 30^\circ$ and (c) $\theta = 20^\circ$

$St=0.2$ as a function of this parameter, considering M_c to be equal to $0.6M$. The constant slope in the sector $20^\circ \leq \theta \leq 45^\circ$ indicates that there is indeed an exponential decay, in agreement with the superdirectivity of the cited models. Furthermore, since these models are based on a line source distribution, the radiated sound field is axisymmetric. The comparison with the experimental mode 0 is thus justified.

The superdirectivity observed for the axisymmetric mode is present for a frequency range around the peak; this can be seen in figure 7. We note that for $0.1 \leq St \leq 0.3$ the directivity changes very little, and in figure 7(b) the linear fit made for $St = 0.2$ closely matches the directivity for $St = 0.1$ and $St = 0.3$. The exponential decay is thus observed for a frequency range around the peak. For higher frequencies, we note that at low angles the SPL values are lower than the peak, but as the angle is increased the SPL seems to join, with a similar slope, the exponential decay observed for the peak frequency. As the frequency is increased this decay is progressively less significant: whereas a decay of 15.4dB between 20° and 45° is observed for $St = 0.2$, for $St = 0.4$ we have a decay of 10.7dB, and for $St = 0.6$ we have 7.7dB (now between 25° and 45° , for the maximum level is obtained for $\theta = 25^\circ$).

IV.B. Lower Mach numbers

The trends observed in the $M = 0.6$ jet were also found for the lower Mach number flows. Figures 8 and 9 show spectra for $M = 0.4$ and $M = 0.5$, respectively. The results are remarkably similar to the $M = 0.6$ jet. However, we note that as the Mach number is reduced, the dominance of the axisymmetric mode at, say, $\theta = 20^\circ$ or $\theta = 30^\circ$ is slightly lower. This effect in the OASPL is shown in figure 10, and for the SPL at $St=0.2$ in figure 11. For convenience, we replot, in both figures, the results for the $M = 0.6$ jet.

For $M = 0.4$ and $M = 0.5$, the SPL at $St=0.2$ is shown as a function of $(1 - M_c \cos \theta)^2$ in figure 12, where the results for $M = 0.6$ are again repeated for convenience. We note once more the same trends for the three jets, with an exponential decay of the acoustic intensity as a function of $(1 - M_c \cos \theta)^2$, indicating again the superdirectivity of the axisymmetric mode.

Since the directivity for $St=0.2$ is exponential between $\theta = 20^\circ$ and $\theta = 45^\circ$ for the three jet Mach numbers considered, we can estimate the wave-packet envelope size, and thus the number of oscillations that

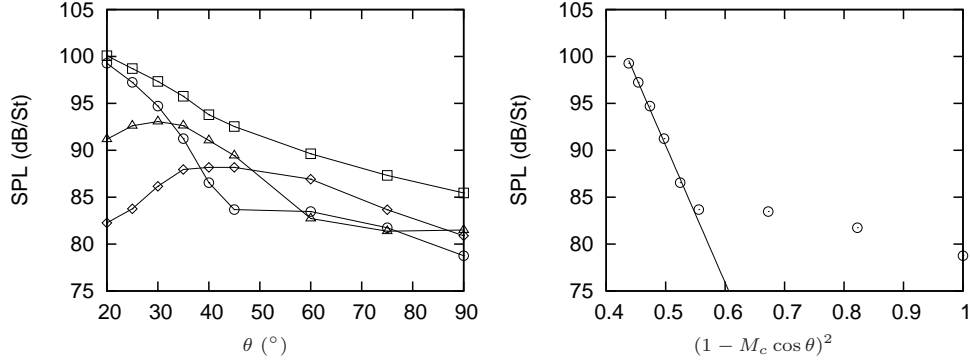


Figure 6. SPL for $St=0.2$ for the Mach 0.6 jet as a function of (a) θ and (b) $(1 - M_c \cos \theta)^2$. Same conventions of figure 4.

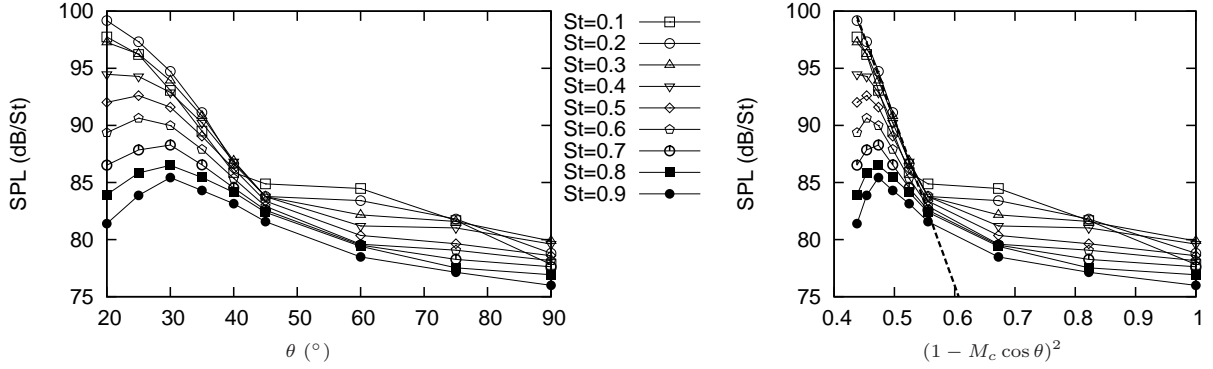


Figure 7. Directivity for the axisymmetric mode as a function of Strouhal number and of (a) θ and (b) $(1 - M_c \cos \theta)^2$.

participate in the source interference, by evaluating the ratio L/D using the wave-packet *Ansatz* described in section III. Since

$$Lk = \frac{2\pi St}{U_c/U} \frac{L}{D}, \quad (3)$$

if the directivity in pressure is described, as in eq. (2), by $\cos^2 \theta \exp(-L^2 k^2 (1 - M_c \cos \theta)^2 / 4)$, with $U_c = 0.6U$ we obtain the results shown in table 2.

Table 2. Estimation of source extent using the axisymmetric mode at $St = 0.2$

M	$SPL(\theta = 20^\circ) - SPL(\theta = 45^\circ)$ (dB)	kL	L/D
0.4	13.2	6.50	3.10
0.5	14.1	6.34	3.03
0.6	15.4	6.40	3.06

The use of Crow's wave-packet model with the three Mach numbers results in a consistent estimation of L/D for all cases, with values of 3–3.1. The values of L/D are related to the gaussian envelope in eq. (1), and indicate that this wave-packet *Ansatz* extends over an axial region of 6–8 jet diameters, similar to the result shown in figure 3(a) for $kL = 6$ (for $U_c/U = 0.6$ and $St = 0.2$, $\lambda_c = 3D$). This modulation is such that three oscillations are present in the source; i.e. there is significant axial interference in the source, as discussed in § III, leading to the observed superdirectivity in the radiated sound field.

The above estimate of the axial source extent is in agreement with results reported by Hussain & Zaman,⁵ who deduce, using phase-averaged measurements in an excited jet at $St = 0.3$, a flow pattern comprising a train of three coherent structures, characterised by regions of closed vorticity contours, and spanning a region of up to 7 jet diameters from the nozzle exit. This also agrees with the experimental observations of Tinney & Jordan,⁶ who studied the near pressure field of unforced coaxial jets, and found a subsonically convected wave extending up to 8 secondary jet diameters downstream of the nozzle exit. The first two POD modes of

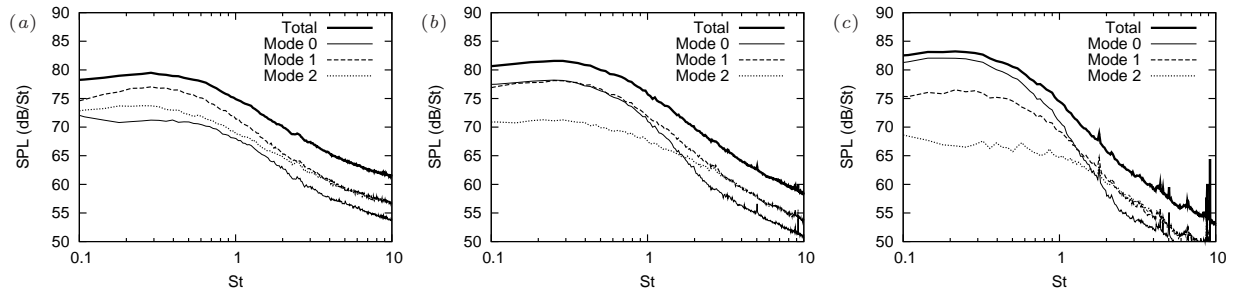


Figure 8. Spectra of individual modes for $M = 0.4$ and (a) $\theta = 40^\circ$, (b) $\theta = 30^\circ$ and (c) $\theta = 20^\circ$

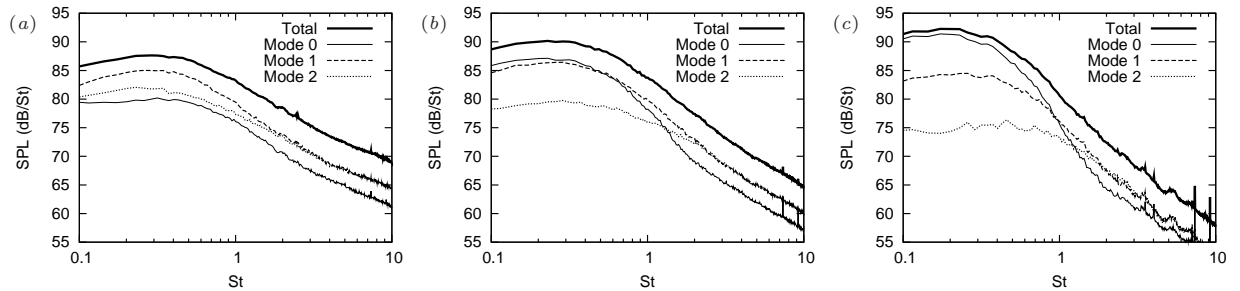


Figure 9. Spectra of individual modes for $M = 0.5$ and (a) $\theta = 40^\circ$, (b) $\theta = 30^\circ$ and (c) $\theta = 20^\circ$

the near field pressure have the shape of a sine and a cosine modulated by an envelope function, and three oscillations are again present in the near field.

The same estimation was performed using the directivities observed at $St = 0.4$, and the results are shown in table 3. For the Mach 0.4 jet the estimated source extent for $St = 0.4$ is roughly half that estimated for $St = 0.2$. Since the wavelength of the convected wave is also halved as the Strouhal number is increased, this means that in this case the source also presents three spatial oscillations. As the Mach number is increased the estimated values of L/D are reduced; however, this does not mean that the source becomes compact, since we are still far from the $kl \rightarrow 0$ limit, as seen in § III.

Table 3. Estimation of source extent using the axisymmetric mode at $St = 0.4$

M	$SPL(\theta = 20^\circ) - SPL(\theta = 45^\circ)$ (dB)	kL	L/D
0.4	11.8	5.94	1.42
0.5	11.8	5.48	1.31
0.6	10.7	4.75	1.13

IV.C. Spectral shape for the different azimuthal modes

We here examine the scaling of the spectra for different Mach numbers. Figure 13 shows the spectra normalised by their maximum value, and plotted versus both Strouhal number, fD/U , and Helmholtz number, fD/c .

The spectra of the axisymmetric component of the sound field collapse better when plotted as a function of Helmholtz number. Figure 14 shows spectra of azimuthal mode 1, again as a function of Strouhal and Helmholtz number. This time the spectra scale better when plotted as a function of Strouhal number.

The Helmholtz number is related to the source compactness, as $He = D/\lambda$, where λ is the acoustic wavelength. If the source extent is comparable to the acoustic wavelength, the Helmholtz number will play a significant role, for it is a measure of the interference effects from the different parts of the source; discussion on the significance of the Helmholtz number for aeroacoustic applications can be found in the work of Fuchs & Armstrong.²⁴ The scaling of the axisymmetric mode with the Helmholtz number, in addition to its superdirectivity, suggests that the non-compactness of the source plays an important role in the radiation of sound to low axial angles. The scaling of low angle spectra with Helmholtz number, without separation

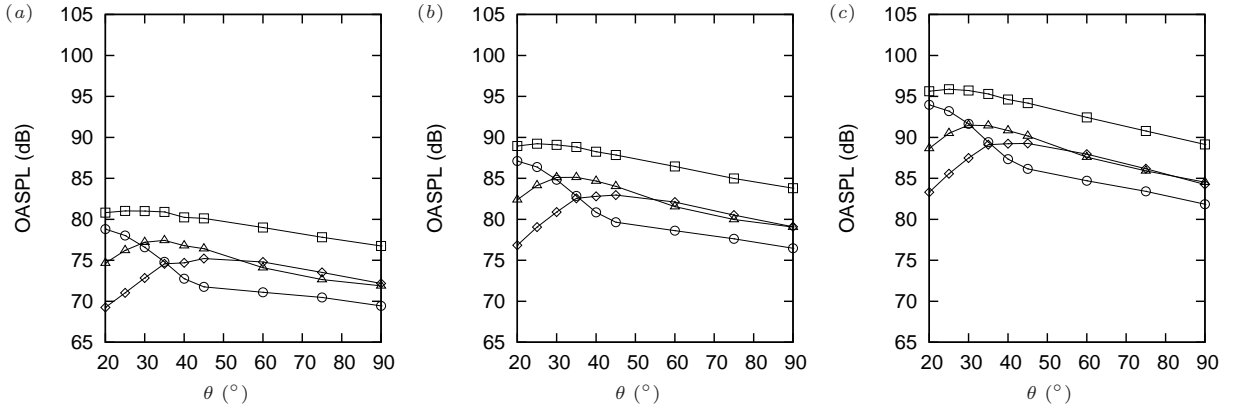


Figure 10. Directivity for (a) $M=0.4$, (b) $M=0.5$ and (c) $M=0.6$: squares, total; circles, mode 0; triangles, mode 1; and diamonds, mode 2.

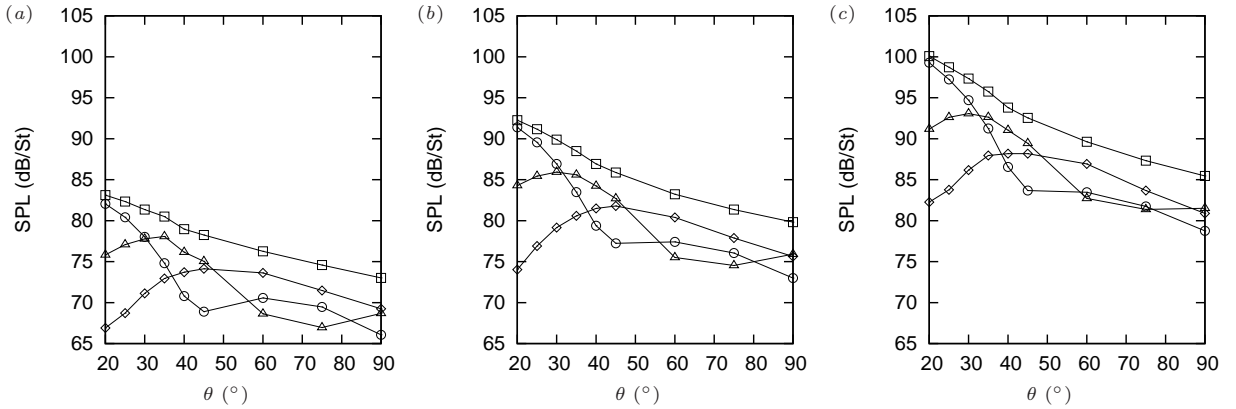


Figure 11. SPL for $St=0.2$ (a) $M=0.4$, (b) $M=0.5$ and (c) $M=0.6$. Same symbols of figure 10.

into azimuthal modes, has been observed previously by Lush,²⁵ Tanna²⁶ and Viswanathan.²⁷ We show here that as the axisymmetric mode accounts for most of the radiation at these angles, the He scaling in the total spectrum is predominantly due to the axisymmetric component. On the other hand, the Strouhal scaling found, for instance at 90° to the jet axis, can be related to the mode-1 scaling with St , seen in figure 14(a), as at higher angles the axisymmetric radiation is no longer dominant.

IV.D. Relationship to similarity spectra

The spectral shape of the axisymmetric mode, with its more narrow frequency band compared to the other azimuthal modes, suggests that the exponential rise in its sound level contributes to the shape of the empirical LSS similarity spectrum of Tam *et al.*²³ Comparisons, for the Mach 0.6 jet, between the LSS and both the total spectrum and that of mode 0 at $\theta = 30^\circ$ and $\theta = 20^\circ$ are shown in figure 15.

We see in figure 15(a) that for $M = 0.6$ the total spectrum does not follow the LSS shape for $\theta = 30^\circ$. On the other hand, for $\theta = 20^\circ$ (figure 15b) there is close agreement between total spectrum and the LSS. For this angle, the total spectrum at low frequencies is dominated by the axisymmetric mode, whose shape is even more narrowband than the LSS shape. We infer that the exponential emergence of the axisymmetric mode at low angles changes the spectral shape as it begins to dominate the total spectrum, passing the spectral shape from a broadband form (empirical FSS similarity spectrum²³) to a more narrowband form (LSS).

However, we note that the characteristic shape of the axisymmetric component of the sound field is narrower than the LSS shape. This suggests, if we consider a linear relationship between the source and the acoustic field (such as is implied by any linearised acoustic analogy), that the acoustic spectral shape related to coherent axisymmetric structures is also more narrowband than the LSS shape. This is in agreement with

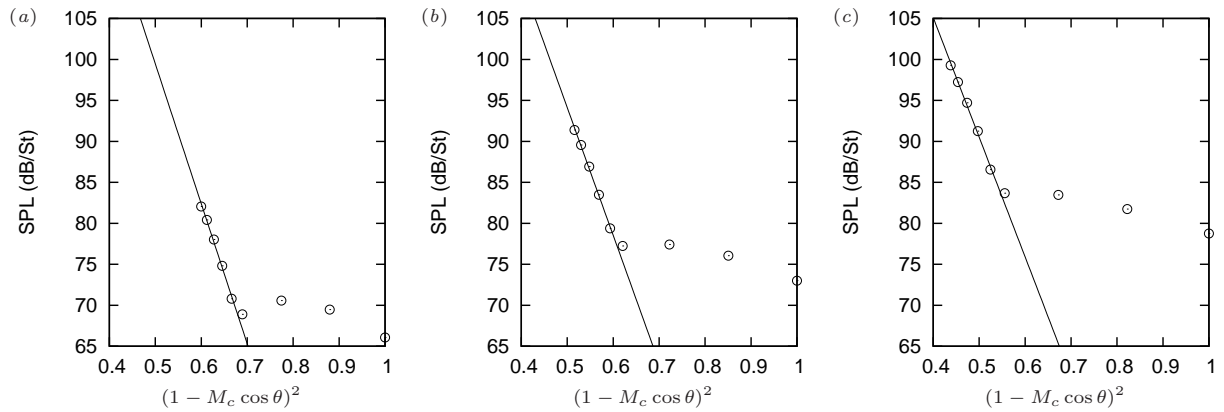


Figure 12. SPL for $St=0.2$ (a) $M=0.4$, (b) $M=0.5$ and (c) $M=0.6$.

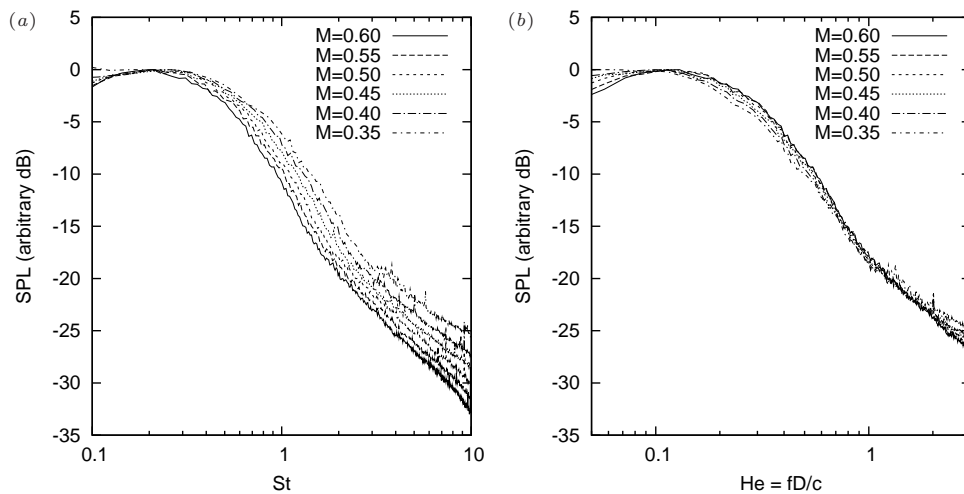


Figure 13. Spectral shapes for azimuthal mode 0 and $\theta = 30^\circ$ as a function of (a) Strouhal number and (b) Helmholtz number.

the analysis of Koenig *et al.*,²⁸ where Proper Orthogonal Decomposition (POD) and a filter based on the continuous wavelet transform were used as metrics to extract the coherent part of the acoustic field of a Mach 0.9 jet. The results show that the coherent spectrum so obtained, both by POD and wavelet transform, is more narrowband than the LSS shape, suggesting that these operations may extract the signature of the axisymmetric mode.

IV.E. Velocity dependence of the sound radiation for each azimuthal mode

Close examination of figures 10(a), (b) and (c) shows that the velocity dependence of the OASPL at each angle is not the same for the different azimuthal modes. Such variations are also observed in the SPL for $St=0.2$, shown in figures 11(a), (b) and (c). In order to evaluate this velocity dependence as a function of both θ and azimuthal mode, we performed fits of both OASPL and SPL for $St = 0.2$ as

$$\text{OASPL(dB)} = a + 10n \log_{10}(U), \quad (4)$$

$$\text{SPL(dB/St)} = a + 10n \log_{10}(U), \quad (5)$$

respectively. This was done for the total values of OASPL and SPL, and also for the individual contributions of azimuthal modes 0, 1 and 2. Results are shown in figure 16.

The velocity exponents shown in figure 16(a) do not show clear trends among the different azimuthal modes for higher angles. However, we note that for low angles the mode-0 exponent is higher than both

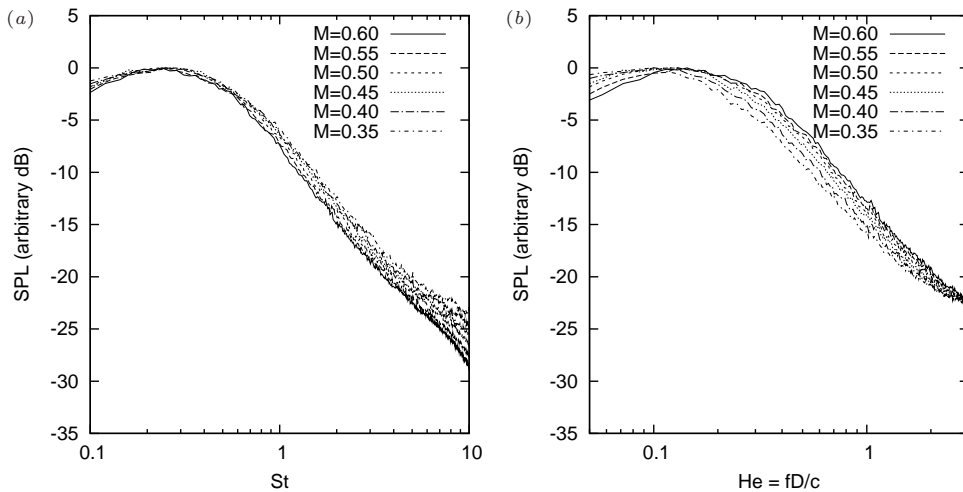


Figure 14. Spectral shapes for azimuthal mode 1 and $\theta = 30^\circ$ as a function of (a) Strouhal number and (b) Helmholtz number.

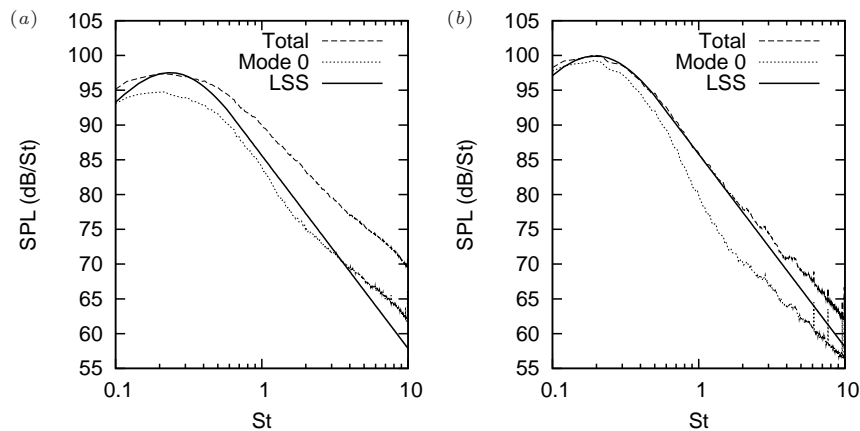


Figure 15. Comparison with the LSS similarity spectrum for (a) $\theta = 30^\circ$ and (b) $\theta = 20^\circ$ for the Mach 0.6 jet.

that of the other azimuthal modes, and that of the total spectrum. If we extrapolate these trends for higher Mach numbers, we can expect that for low angles the mode-0 dominance in OASPL, observed for $M = 0.6$, will be even more pronounced at higher subsonic Mach numbers.

Considering the velocity dependence of the sound power level for $St=0.2$ at low angles, shown in figure 16(b), these effects are even more marked, the velocity exponent of the axisymmetric mode for low angles being considerably higher than that of the other modes. This, as shown in section III, is an indication of non-compactness of the source.

Use of the values obtained for n to estimate the source length based on the wave-packet model of section III leads to $kL \approx 3$, and therefore to a source extent of 3–4 jet diameters, which, although still being a significant axial extent for the source, is roughly half that estimated in section IV.C based on the directivities for $M = 0.4, 0.5$ and 0.6 . The derivation of the velocity exponent with Crow's¹³ wave-packet model assumes that the source extent and maximum amplitude do not change with increasing Mach number, and it also assumes a constant ratio between convection and jet speeds. However, the development of the velocity fluctuations do change as a function of Mach number. There is a compressibility effect in turbulence, and higher Mach number flows exhibit lower turbulent fluctuations (see Lele²⁹ and references therein for studies on compressible mixing layers). Linear stability theory also predicts lower growth rates as the Mach number is increased.^{30, 31}

Comparison of velocity spectra on the jet centerline is shown in fig. 17 for $x = 2D$ and $x = 4D$. The centerline spectrum is chosen to evaluate the axisymmetric mode of the velocity fluctuations. In stability theory the boundary conditions on the jet centerline are of zero transverse velocity and arbitrary finite

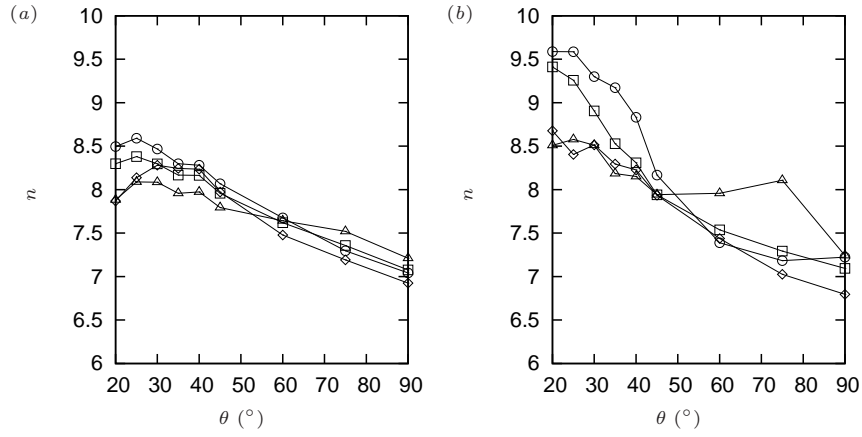


Figure 16. Velocity exponent n obtained by fits of the (a) OASPL and (b) SPL for $St = 0.2$: squares, total; circles, mode 0; triangles, mode 1; and diamonds, mode 2.

streamwise velocity for $m = 0$, and zero streamwise velocity for all higher order azimuthal modes;³² therefore, we expect the centerline spectrum to be representative of the axisymmetric mode; indeed, such measurements have been used in the past for comparison with stability results (Crow & Champagne,³ Michalke,³³ Crighton & Gaster³⁴). We see in fig. 17 that for both positions the amplitude of the velocity fluctuations, when normalised by the jet velocity, decreases as the Mach number is increased. This can be attributed to the lower growth rate predicted by stability theory for higher Mach numbers. This suggests that to account for the appropriate velocity dependence in the wave-packet model of section III, one should account for the reduction of the normalised amplitude \hat{u}/U as the Mach number is increased, leading to lower velocity exponents than the results of fig. 3(c).

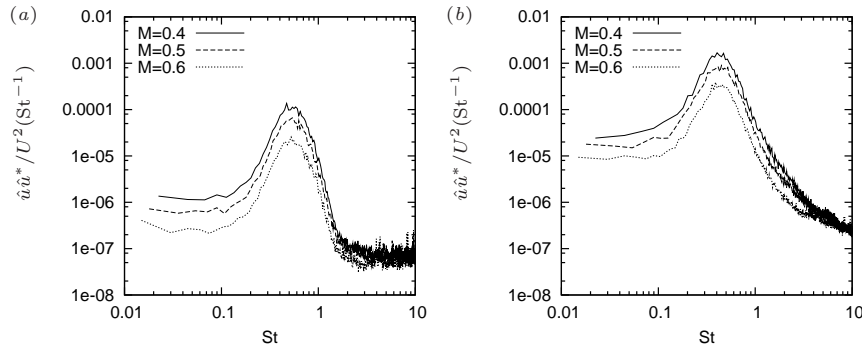


Figure 17. Velocity spectra at the jet centerline for (a) $x = 2D$ and (b) $x = 4D$.

Together with the observations of the previous sections, the trends in figures 16(a) and (b) allow the following scenario to be postulated with regard to the effect of increasing jet Mach number on the radiated sound:

1. As the jet Mach number is increased, the sound radiation of the axisymmetric mode grows faster than the sound field of the higher order modes (figure 16a);
2. The increase in the axisymmetric radiation is even more pronounced near the spectral peak (figure 16b);
3. The velocity increase therefore causes the sound radiation at low angles to be dominated by the axisymmetric mode, especially at the peak Strouhal number;
4. The narrowband character of the axisymmetric mode causes the appearance of the peaky LSS similarity spectrum at low axial angles (figure 15).

These observations suggest that at higher subsonic Mach numbers the observed axisymmetric radiation will have increased importance.

Although the use of a velocity exponent n is useful to scale jet data at different Mach numbers and predict the increase of sound level as the jet velocity increases, it should be noted that non-compact sources, such as are described by the wave-packet model of eq. (2), lead to a velocity dependence for the sound intensity that departs from a U^n form. In eq. (2) this effect is due to the convective Mach number M_c in the argument of the first exponential function. Nonetheless, for the present experiments no significant departure from the fitted n exponents in figure 16 was observed. However, the Mach number range of the present tests is not comprehensive, and so a conclusive answer is not presently available regarding the precise form of the velocity dependence for the different azimuthal modes. Deviations from a U^n law can be seen in the results of Lush,²⁵ which spanned Mach numbers from 0.3 to 1. These questions are to be addressed in the near future via experiments performed at higher Mach number

V. Conclusion

An experimental investigation of the azimuthal components of the sound radiated by subsonic jets in the Mach number range $0.35 \leq M \leq 0.6$ has been carried out using a ring array comprising six microphones. For this Mach number range the axisymmetric mode is seen to be highly directive, large increases in intensity being observed as the angle to the downstream jet axis is decreased. This trend is more marked for the peak frequencies. The observed increase is such that the axisymmetric mode dominates the sound radiation for low polar angles. The spectral shape of the axisymmetric mode presents a narrow band form, and this causes the formation of the empirical LSS shape as the mode 0 begins to dominate the total spectrum.

An exponential change of SPL with the parameter $(1 - M_c \cos \theta)^2$ is predicted by wave-packet models, using an axially non-compact source distribution. The non-compactness leads to interference between different regions of the source; the sound radiation is, as a result, concentrated at low angles, and, for subsonic convection velocities, decreases exponentially as $(1 - M_c \cos \theta)^2$ is increased. This effect has been observed for the axisymmetric mode, a decay of 15.4dB being seen for the peak frequency. With this value, and a wave-packet *Ansatz*, the axial extent of the source has been estimated to be of the order of 6-8 jet diameters for the $M = 0.4, 0.5$ and 0.6 jets. Further evidence of the importance of the non-compactness of the source for the axisymmetric mode is observed in a Helmholtz scaling of mode 0 and in a velocity dependence with an exponent of 9.6 for low angles.

The present results suggest that the axisymmetric radiation, which is seen to dominate at low angles, can be appropriately modelled if, instead of considering the turbulent field to be formed by stochastic eddies with random phase,¹⁴ the axial interference over a non-compact source region is taken into account (see for instance Michalke³⁵ or Michel³⁶). The problem is not in the formulation of an acoustic analogy, but in the way the source is modelled.

For modelling purposes, we can think of the axial source interference in two ways, which are not mutually exclusive. The first is in an average sense: we look for an averaged mutual interference between the different positions of a jet, and particularly for its average effect in the sound field. For this evaluation, correlations and interspectra are appropriate measures, and, especially in the near field, as shown by Tinney & Jordan⁶ and Reba *et al.*,³⁷ these prove to be significant over a region extending several jet diameters from the nozzle exit. Furthermore, since for many practical applications determination of the radiated spectra is sufficient, this can be accomplished by coupling such correlation data with an acoustic analogy, as done, for example, by Karabasov *et al.*,³⁸ among others, or with a Kirchhoff surface, as shown by Reba *et al.*³⁷ For such an approach, stability calculations may constitute an appropriate dynamic model, and indeed it has been shown that reasonable predictions can be obtained for the radiated sound at low angles.³⁹ The mean velocity field of the present jets has been used for the solution parabolised stability equations (PSE),^{40, 41} and good agreement found for the growth of velocity fluctuation amplitudes and for the radiated sound field for $0.3 \leq St \leq 0.9$.

A second approach for studying such source interference effects involves taking things from an instantaneous standpoint. Since a turbulent jet is not a periodic flow, these interferences are expected to change with time. This leads to periods when the interference is destructive, during which we have periods of “relative quiet”;⁴² or, periods during which the destructive interference may be less significant, producing thus high-energy temporally-localised bursts in the acoustic field. Such behaviour has been observed, experimentally, in the sound field by Hileman *et al.*⁴² and Koenig *et al.*²⁸ at low polar angles. Evaluation of the instantaneous interference between coherent structures in a flow is not an easy task experimentally, but such endeavours

appear worthwhile considering the additional physical insight to be gained in terms of the dynamic law of jet noise source mechanisms. Furthermore, as seen by Cavalieri *et al.*,⁴³ the details of the mutual interference in the source region can be crucial for the understanding of differences between uncontrolled, noisy flows and their controlled, quieter counterparts. In that study, intermittent sound production in the noisy flow was prevented by an optimal controller, which led to small changes in the instantaneous interference between neighbouring vortices; we can infer from this study that appropriate control strategies, acting in real time in order to manipulate such temporally-localised interference, may allow significant reductions in the noise radiated by jets. It is clear that averages of this instantaneous interference will lead to the same values obtained using metrics such as correlations and interspectra, but in the instantaneous scenario one can more easily pinpoint specific ‘events’ in the flow underpinning significant sound radiation, because the cloudiness of averaging has been removed.

Acknowledgments

The present work is partially supported by CNPq, National Council of Scientific and Technological Development – Brazil, and by the European-Russian program ORINOCO (FP7-AAT-2010-RTD-Russia; project number 266103).

References

- ¹Michalke, A. and Fuchs, H. V., “On turbulence and noise of an axisymmetric shear flow,” *Journal of Fluid Mechanics*, Vol. 70, 1975, pp. 179–205.
- ²Fuchs, H. and Michel, U., “Experimental evidence of turbulent source coherence affecting jet noise,” *AIAA J*, Vol. 16, No. 9, 1978, pp. 871–872.
- ³Crow, S. C. and Champagne, F. H., “Orderly structure in jet turbulence,” *Journal of Fluid Mechanics*, Vol. 48, No. 3, 1971, pp. 547–591.
- ⁴Moore, C. J., “The role of shear-layer instability waves in jet exhaust noise,” *Journal of Fluid Mechanics*, Vol. 80, No. 2, 1977, pp. 321–367.
- ⁵Hussain, A. K. M. F. and Zaman, K. B. M. Q., “The ‘preferred mode’ of the axisymmetric jet,” *Journal of Fluid Mechanics*, Vol. 110, 1981, pp. 39–71.
- ⁶Tinney, C. E. and Jordan, P., “The near pressure field of co-axial subsonic jets,” *Journal of Fluid Mechanics*, Vol. 611, 2008, pp. 175–204.
- ⁷Maestrello, L., “Statistical properties of the sound and source fields of an axisymmetric jet,” *American Institute of Aeronautics and Astronautics Conference*, Vol. 1, 1977.
- ⁸Juvé, D., Sunyach, M., and Comte-Bellot, G., “Filtered azimuthal correlations in the acoustic far field of a subsonic jet,” *AIAA Journal*, Vol. 17, 1979, pp. 112.
- ⁹Brown, C. and Bridges, J., “Acoustic efficiency of azimuthal modes in jet noise using chevron nozzles,” Tech. rep., National Aeronautics and Space Administration, 2006.
- ¹⁰Kopiev, V., Chernyshev, S., Faranosov, G., Zaitsev, M., and Belyaev, I., “Correlations of jet noise azimuthal components and their role in source identification,” *16th AIAA/CEAS Aeroacoustics Conference and Exhibit*, Stockholm, Sweden, June 7-9 2010.
- ¹¹Mollo-Christensen, E., “Measurements of near field pressure of subsonic jets,” Tech. rep., Advisory Group for Aeronautical Research and Development, Paris, France, 1963.
- ¹²Mollo-Christensen, E., “Jet noise and shear flow instability seen from an experimenter’s viewpoint (Similarity laws for jet noise and shear flow instability as suggested by experiments),” *Journal of Applied Mechanics*, Vol. 34, 1967, pp. 1–7.
- ¹³Crow, S. C., “Acoustic gain of a turbulent jet,” *Phys. Soc. Meeting, Univ. Colorado, Boulder, paper IE*, Vol. 6, 1972.
- ¹⁴Crighton, D. G., “Basic principles of aerodynamic noise generation,” *Progress in Aerospace Sciences*, Vol. 16, No. 1, 1975, pp. 31–96.
- ¹⁵Lighthill, M. J., “On sound generated aerodynamically. I. General theory,” *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 1952, pp. 564–587.
- ¹⁶Crighton, D. G. and Huerre, P., “Shear-layer pressure fluctuations and superdirective acoustic sources,” *Journal of Fluid Mechanics*, Vol. 220, 1990, pp. 355–368.
- ¹⁷Sandham, N. D., Morfey, C. L., and Hu, Z. W., “Sound radiation from exponentially growing and decaying surface waves,” *Journal of Sound and Vibration*, Vol. 294, No. 1, 2006, pp. 355–361.
- ¹⁸Ffowcs Williams, J. E. and Kempton, A. J., “The noise from the large-scale structure of a jet,” *Journal of Fluid Mechanics*, Vol. 84, No. 4, 1978, pp. 673–694.
- ¹⁹Cavalieri, A. V. G., Jordan, P., Agarwal, A., and Gervais, Y., “Jittering wave-packet models for subsonic jet noise,” *to appear in Journal of Sound and Vibration*, 2011.
- ²⁰Laufer, J. and Yen, T.-C., “Noise generation by a low-Mach-number jet,” *Journal of Fluid Mechanics*, Vol. 134, 1983, pp. 1–31.
- ²¹Bridges, J. E. and Hussain, A. K. M. F., “Roles of initial condition and vortex pairing in jet noise,” *Journal of Sound and Vibration*, Vol. 117, No. 2, 1987, pp. 289–311.

- ²²Cavaleri, A. V. G., Daviller, G., Comte, P., Jordan, P., Tadmor, G., and Gervais, Y., “Using large eddy simulation to explore sound-source mechanisms in jets,” *to appear in Journal of Sound and Vibration*, 2011.
- ²³Tam, C., Golebiowski, M., and Seiner, J. M., “On the Two Components of Turbulent Mixing Noise from Supersonic Jets,” *2th AIAA/CEAS Aeroacoustics Conference and Exhibit*, State College, PA, May 6-8 1996.
- ²⁴Fuchs, H. and Armstrong, R., “Turbulent source coherence and Helmholtz number as aerodynamic noise parameters,” *Structure and Mechanisms of Turbulence II*, 1978, pp. 189–201.
- ²⁵Lush, P. A., “Measurements of subsonic jet noise and comparison with theory,” *Journal of Fluid Mechanics*, Vol. 46, No. 03, 1971, pp. 477–500.
- ²⁶Tanna, H., “An experimental study of jet noise Part I: Turbulent mixing noise,” *Journal of Sound and Vibration*, Vol. 50, No. 3, 1977, pp. 405–428.
- ²⁷Viswanathan, K., “Aeroacoustics of hot jets,” *Journal of Fluid Mechanics*, Vol. 516, 2004, pp. 39–82.
- ²⁸Koenig, M., Cavaleri, A. V. G., Jordan, P., Delville, J., Gervais, Y., Papamoschou, D., Samimy, M., and Lele, S. K., “Farfield pre-filtering and source-imaging for the study of jet noise,” *16th AIAA/CEAS Aeroacoustics Conference and Exhibit*, Stockholm, Sweden, June 7-9 2010.
- ²⁹Lele, S., “Compressibility effects on turbulence,” *Annual Review of Fluid Mechanics*, Vol. 26, No. 1, 1994, pp. 211–254.
- ³⁰Michalke, A., “Survey on jet instability theory,” *Progress in Aerospace Sciences*, Vol. 21, 1984, pp. 159–199.
- ³¹Morris, P., “The instability of high speed jets,” *International Journal of Aeroacoustics*, Vol. 9, No. 1, 2010, pp. 1–50.
- ³²Batchelor, G. K. and Gill, A. E., “Analysis of the stability of axisymmetric jets,” *Journal of Fluid Mechanics*, Vol. 14, No. 04, 1962, pp. 529–551.
- ³³Michalke, A., “Instabilität eines Kompressiblen Runden Freistrahls unter Berücksichtigung des Einflusses der Strahlgrenzschichtdicke,” *Z. Flugwiss.*, Vol. 19, 1971, pp. 319–328; English translation: NASA TM 75190, 1977.
- ³⁴Crighton, D. G. and Gaster, M., “Stability of slowly diverging jet flow,” *Journal of Fluid Mechanics*, Vol. 77, No. 2, 1976, pp. 397–413.
- ³⁵Michalke, A., “A wave model for sound generation in circular jets,” Tech. rep., Deutsche Luft- und Raumfahrt, November 1970.
- ³⁶Michel, U., “The role of source interference in jet noise,” *15th AIAA/CEAS Aeroacoustics Conference(30th Aeroacoustics Conference)*, 2009, pp. 1–15.
- ³⁷Reba, R., Narayanan, S., and Colonius, T., “Wave-packet models for large-scale mixing noise,” *International Journal of Aeroacoustics*, Vol. 9, No. 4, 2010, pp. 533–558.
- ³⁸Karabasov, S., Afsar, M., Hynes, T., Dowling, A., McMullan, W., Pokora, C., Page, G., and McGuiirk, J., “Jet Noise: Acoustic Analogy Informed by Large Eddy Simulation,” *AIAA journal*, Vol. 48, No. 7, 2010, pp. 1312–1325.
- ³⁹Colonius, T., Samanta, A., and Gudmundsson, K., “Parabolized stability equation models of large-scale jet mixing noise,” *Procedia Engineering*, Vol. 6, 2010, pp. 64–73.
- ⁴⁰Rodríguez, D., Samanta, A., Cavaleri, A. V. G., Colonius, T., and Jordan, P., “Parabolized stability equation models for predicting large-scale mixing noise of turbulent round jets,” *17th AIAA/CEAS Aeroacoustics Conference and Exhibit*, Portland, OR, USA, June 5-8 2011.
- ⁴¹Cavaleri, A. V. G., Rodríguez, D., Jordan, P., Colonius, T., and Gervais, Y., “Inlet conditions and wave-packets in subsonic jet noise,” *Turbulence and Shear Flow Phenomena*, Ottawa, Canada, July 28-31 2011.
- ⁴²Hileman, J. I., Thurow, B. S., Caraballo, E. J., and Samimy, M., “Large-scale structure evolution and sound emission in high-speed jets: real-time visualization with simultaneous acoustic measurements,” *Journal of Fluid Mechanics*, Vol. 544, 2005, pp. 277–307.
- ⁴³Cavaleri, A. V. G., Jordan, P., Gervais, Y., Wei, M., and Freund, J. B., “Intermittent sound generation and its control in a free-shear flow,” *Physics of Fluids*, Vol. 22, No. 11, 2010, pp. 115113.