Negative dynamic conductance from photon-assisted tunneling in superconducting junctions

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We show that a superconductor-insulator-superconductor (SIS) junction may exhibit regions of negative dynamic conductance if it is irradiated by a time-varying signal source which deviates from the conventionally treated constant ac voltage limit. This phenomenon reflects the strong dependence of the junction absorption cross section upon dc bias voltage. Analytic estimates for the magnitude of the negative conductance and its impact upon the frequency down conversion process are obtained in the constant ac current limit.

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The desire for low-noise, high-frequency heterodyne converters has led to the natural consideration of superconducting tunnel junctions for this role. Recently the possibility of frequency conversion with concomitant amplification, termed "conversion gain," using the quasiparticle current in these devices has been indicated by computer simulations and laboratory experiments.[1-3] These results, however, seem to be counterintuitive since the quasiparticle current is predominantly *dissipative* at usual operating conditions.

Necessary criteria for achieving power gain from two-terminal devices have been studied.[4,5] Two general gain processes have been elucidated: parametric amplification (based on nonlinear reactance) and amplification via negative conductance. Because reactive currents are expected to play a minor role in superconductor-insulator-superconductor (SIS) mixers,[3] the first type of gain mechanism appears to be inapplicable. In this letter we demonstrate the possibility of dynamical negative conductance effects in quasiparticle tunneling.

Photon-assisted quasiparticle tunneling in SIS junctions was first observed by Dayem and Martin.[6] The theory of the effect has been developed by Tien and Gordon,[7] Riedel,[8] Werthamer,[9] and Larkin and Ovchinnikov.[10] All of these theories model the interaction of the junction with the time-varying field as a simple adiabatic, time-dependent contribution to the energies of electrons on one side of the tunnel barrier. This interaction simply results in the phase modulation of single particle operators for that side of the junction.

The implicit assumption in such a treatment is that the source of the time-varying field acts to keep the magnitude of the ac voltage across the junction *constant*, irrespective of dynamical effects which may occur. Experimentally, this situation is realized when the characteristic admittance of the signal source, Y_s, is much larger than the junction admittance to the time-varying signal, Y_1. In general Y_1 is composed of static and dynamical pieces, $Y_1 \simeq j\omega_1 C_j + G_1(\omega_1, V_0, V_1)$. The first term on the right represents the capacitive susceptance of the junction (C_j is the junction capacitance). The second term, which can be approximated as purely real for $\hbar\omega_1 \ll \Delta$, is the sum of the normal conductance of the junction, G_{nn}, and dynamic conductance due to absorption of field energy. Here V_0 represents the dc bias voltage and the ac voltage appearing across the junction is assumed sinusoidal, $V_1(t) = V_1 \cos \omega_1 t$.

Recently, however, large conductance and low capacitance junctions have been fabricated.[11] For such devices and common signal source admittances, the constant V_1 assumption of existing theory may not even be approximately correct. For sufficiently small Y_s and large Y_1 the opposite limiting case is approached; that of constant ac current biasing (constant I_1). For that case, the ac voltage across the junction contains all harmonics of ω_1 due to the nonlinearity of the junction admittance. For simplicity we suppose that the first harmonic dominates and the dynamical contribution to Y_1 outweighs the static part; specifically we write $V_1 \simeq I_1 / G_1(\omega_1, V_0)$.

When dc bias and time-varying voltages are simultaneously impressed across an SIS junction the conventional photon-assisted tunneling theory[7] predicts that a dc current will flow, of magnitude

$$I_0(\omega_1, V_0, V_1) = \sum_{n=-\infty}^{+\infty} J_n^2(\alpha) j_1(eV_0 + n\hbar\omega_1). \qquad (1)$$

Here J_n is a Bessel function of the first kind with argument α, where $\alpha = eV_1/\hbar\omega_1$ if V_1 is constant. If I_1 is constant and the first harmonic dominates, α becomes

$$\alpha = eI_1/\hbar\omega_1 G_1(\omega_1, V_0). \qquad (2)$$

The junction response function, as defined by Werthamer,[9] is denoted by $j_1(E)$.

The expression (1) contains the familiar "steps" in the photon-assisted dc tunnel current when V_0 passes through voltages given by $(2\Delta \pm n\hbar\omega_1)/e$, n being an integer.

When the dc bias voltage is in the range of the first photon-assisted current step below the gap voltage, $2\Delta - \hbar\omega_1 < eV_0 < 2\Delta$, and α is small, only the $n = 1$ term in the series (1) is appreciable. If the time-varying source fails to keep V_1 constant, α becomes functionally dependent on the dc bias point, V_0 [cf. (2)]. The low-frequency conductance is then

$$G_0(\omega_1, V_0, V_1) \simeq \left(\frac{\alpha}{2}\right)^2 j_1(eV') \left(\frac{2}{\alpha}\frac{d\alpha}{dV_0} + \frac{1}{j_1(eV')}\frac{dj_1(eV')}{dV_0}\right), \qquad (3)$$

where $eV' \equiv eV_0 + \hbar\omega_1$ and the Bessel function involved has been expanded for small argument.

When V_1 is constant, only the second term in (3) contributes to G_0. For SIS junctions with identical superconducting electrodes, $dj_1/dV_0 > 0$ everywhere. Thus, a physical mechanisn to explain negative conductance or conversion gain in the constant V_1 limit is not immediately apparent. Smith *et al.*[12] first pointed out, however, that for junctions driven with a small signal source admittance, V_1 will not be constant and suggested that the (negative) first term of (3) might be sufficient to make G_0 negative.

If the source admittance Y_s is small enough, G_0 assumes a form appropriate for the constant I_1 limit. In this regime (2) applies and indicates that, for small α,

$$G_0(\omega_1,V_0,I_1)\approx I_0(\omega_1,V_0,I_1)\times\left(-2G_1^{-1}\frac{dG_1}{dV_0}+\frac{1}{j_1(eV')}\frac{dj_1(eV')}{dV_0}\right). \tag{4}$$

Below we argue that the first term on the right can be sufficiently large as to make G_0 negative.

To make further progress the functional dependence of the ac junction admittance, $G_1(\omega_1,V_0,I_1)$, upon the dc bias point must be ascertained. That G_1 is an increasing function of V_0 follows from the argument below. The rapid increase in $j_1(eV_0)$ when $eV_0\approx 2\Delta$ gives rise to the photon-assisted current step when $eV' = eV_0 + \hbar\omega_1$ approaches 2Δ. This sharp increase in the photon-assisted current must be accompanied by an equally sharp rise in energy absorption from the time-varying field. The abrupt behavior of both reflects the sudden availability of phase space for tunneling which occurs when the Fermi energy of one electrode is 2Δ above that of the other. As a result, the junction single-photon absorption cross section dramatically increases when $V_0\approx 2\Delta - \hbar\omega_1$, and continues to increase as more quasiparticle states contribute to the tunneling process.

Harris has analyzed the Werthamer–Larkin–Ovchinnikov (WLO) expression for j_1.[13,14] Because the superconducting density of states for each side of the junction is assumed by WLO to have the ideal BCS form, even at finite temperatures the predicted quasiparticle current step is infinitely sharp. Realistically, effects such as gap anisotropy and sample inhomogeneities result in a smearing of the current step over an energy interval δE. If $\hbar\omega_1 \ll \delta E$, no sharp increase in G_1 is to be expected when $V_0\approx 2\Delta - \hbar\omega_1$, in fact the increase in (photon-assisted) tunneling in this region is no longer abrupt and the current step vanishes.

Calculation of G_1 can proceed from the WLO expression for the dissipative part of the photon-assisted quasiparticle current in the form given by Tucker,[15]

$$I(t) = \sum_{n=-\infty}^{+\infty} j_1(eV_0 + n\hbar\omega_1)\times\left(J_n^2(\alpha) + \sum_{m=1}^{\infty}\left[J_n(\alpha)J_{n+m}(\alpha) + J_n(\alpha)J_{n-m}(\alpha)\right]\cos(m\omega_1 t)\right). \tag{5}$$

If smearing effects and leakage currents are small enough such that, for $2\Delta - \hbar\omega_1 < eV_0 < 2\Delta$, $j_1(eV_0)$ is negligible, then (5) may be expanded to lowest order in α to yield the ac current I_1, at frequency ω_1: $I_1 = \frac{1}{2}\alpha j_1(eV_0 + \hbar\omega_1)$. Accordingly we see that $G_1\approx I_1/V_1\approx ej_1(eV_0 + \hbar\omega_1)/2\hbar\omega_1$, which is independent of I_1 to this approximation. This and expression (4) lead directly to the constant I_1 value of the low-frequency conductance at V_0,

$$G_0(\omega_1,V_0,I_1)\approx -I_0(\omega_1,V_0,I_1)\frac{d}{dV_0}[\ln j_1(eV_0 + \hbar\omega_1)], \tag{6a}$$

$$\approx -I_0\frac{d}{dV_0}[\ln G_1] \tag{6b}$$

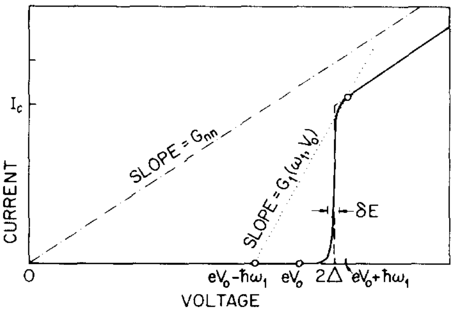

FIG. 1 Connection between finite-difference estimation of the ac junction conductance G_1 and the derivation for small signals, Eq. (7). The value for G_1 calculated at the bias point V_0 corresponds to the slope of the chord connecting points on the *I-V* curve at $eV_0 \pm \hbar\omega_1$. The current step at 2Δ is assumed smeared over a voltage δE.

The possibility of negative conductance arises because, in the small α limit, $I_0\approx\alpha^2 j_1/4\approx I_1^2/j_1$. As the argument of j_1, $eV_0 + \hbar\omega_1$, increases from 2Δ upward, j_1 monotonically increases, resulting in decreasing I_0.

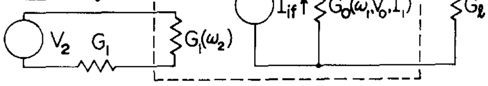

FIG. 2. Simple conceptual model for an SIS junction down converter. A large (pump) signal at ω_1 from current generator I_1 with shunt admittance Y_s current biases the junction at this frequency. A small signal voltage, $V_2\cos(\omega_2 t)$, is also imposed by a second source which is impedance matched to the junction at ω_2. The junction, represented by components within the dashed box, delivers a converted signal current at ω_{if} with output conductance $G_0(\omega_1,V_0,I_1)$ to a load G_l. For clarity, circuitry to effect a separation of signal and pump sources is not shown. (Analysis for the case of matched pump and signal source is presented elsewhere.[17])

It is instructive to utilize the form of the junction response function obtained for identical BCS superconductors at zero temperature.[9] In this limit j_1 vanishes below the gap voltage and $j_1(2\Delta) = (\pi/4)(G_{nn}2\Delta/e)$. In the region just above 2Δ, j_1 increases approximately linearly with slope G_{nn}. In this limit, G_1 near the edge of the first photon-assisted current step has the value

$$G_1(\omega_1, 2\Delta - \hbar\omega_1, V_1)\approx\frac{e}{2\hbar\omega_1}\frac{\pi}{4}\frac{G_{nn}2\Delta}{e}. \tag{7}$$

This is precisely the value obtained by considering not the local derivative of I_0 at the dc bias point $V_0 = 2\Delta - \hbar\omega_1$, but its finite difference value. This is depicted in Fig. 1, which shows that G_1 may be much larger than G_0. Although (7) indicates G_1 grows without bound as ω_1 is reduced, smearing of the quasiparticle current step acts to impose a ceiling on the ac conductance. Practically, one expects an upper bound of order $G_{nn}(2\Delta/\delta E)$, where δE is the energy scale of the step smearing.

Use of (6) and (7) provides an estimate of the magnitude of the negative conductance in the limit of a sharp current step. The result obtained is $G_0(\omega_1, V_0, I_1) \approx -\frac{1}{4}\pi G_{nn}(I_1/I_c)^2$, where I_c is the zero temperature critical current, $(\pi/4)(G_{nn}2\Delta/e)$. The assumption of small α imposes the constraint $I_1 \ll I_c$ on these approximations, therefore, high current density (large G_{nn}) junctions are most favorable for observation of this effect.

To qualitatively understand heterodyne conversion in the constant I_1 limit, two effects arising from the application of the time-varying field at ω_1 are to be noted. First, the shape of the *I-V* curve is changed; $G_0(V_0)$ becomes $G_0(\omega_1, V_0, I_1)$. Second, the application of another time-varying signal, $V_2(t) = V_2 \cos(\omega_2 t)$, where $V_2 \ll V_1$ and $\omega_2 \approx \omega_1$, produces a quasi-dc current component at the *i*ntermediate (difference) *f*requency $\omega_{\text{if}} \equiv |\omega_2 - \omega_1|$. For $\omega_1 \approx \omega_2$, a small V_2 will not appreciably change the low-frequency *I-V* curve described by $G_0(\omega_1, V_0, I_1)$, and the magnitude of the tunnel current at ω_{if} is, for constant I_1, approximately $I_{\text{if}} \approx eI_1V_2/2\hbar\omega_1$. A simple conceptual model for down conversion consistent with these two effects is shown in Fig. 2. G_1, I_{if}, and G_0 are strongly dependent upon the photon-assisted tunneling process.

Conversion gain $\mathscr{G}$ is defined as the ratio of the if power dissipated in the load to the available signal power at ω_2. For the simple model of Fig. 2 conversion gain becomes possible when G_0 is negative.[16] At the dc bias point $V_0 = 2\Delta - \hbar\omega_1$,

$$\mathscr{G} = \left(\frac{\Delta}{\hbar\omega_1}\right)\left[\frac{2}{\pi}\left(\frac{I_c}{I_1}\right)^2\frac{G_l}{G_{nn}} - 1\right]^{-1}. \tag{8}$$

Large gain is available as G_l approaches $\frac{1}{2}\pi G_{nn}(I_1/I_c)^2$, a small quantity since the assumption $I_1 \ll I_c$ is implicit in Eq. (8). Gain results from the fact that negative G_0 effectively reduces G_l.

Practical application of the negative conductance phenomenon requires understanding the effect of a smeared quasiparticle current step, finite junction capacitance, and ac biasing not at the constant I_1 limit.[17] The least studied and potentially most serious of these nonidealities is the first; however, low capacitance junctions exist which manifest extremely sharp *I-V* characteristics.[18] For these a range of ω_1 should exist where $\delta E < \hbar\omega_1 < 2\Delta$ and negative conductance may occur.

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Note added in proof. Since the submission of this paper an amplification of Ref. 12 has appeared [A. D. Smith and P. L. Richards, J. Appl. Phys. **53**, 3806 (1982)].

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