Brief Papers

A Model Predictive Decentralized Control Scheme With Reduced Communication Requirement for Spacecraft Formation

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***Abstract*—This brief investigates the control problem for a number of cooperative spacecraft with communication constraints. It is assumed that a set of cooperative local controllers corresponding to the individual spacecraft is given which satisfies the desired objectives of the formation. It is to be noted that due to the information exchange between the local controllers, the overall control structure can be considered centralized in general. However, communication in flight formation is expensive. Thus, it is desired to have some form of decentralization in control structure, which has a lower communication requirement. This decentralized controller consists of local estimators inherently, so that each local controller estimates the state of the whole formation. Necessary and sufficient conditions for the stability of the formation under the proposed decentralized controller are obtained. It is shown that the decentralized control system, if stable, behaves almost the same as its centralized counterpart. Moreover, robustness of the decentralized controller is studied and compared to that of the corresponding centralized controller. It is finally shown that the proposed decentralized controller comprises most of the features of its centralized counterpart. The efficacy of the proposed method is demonstrated through simulations.**

***Index Terms*—Cooperative systems, distributed control, predictive control, robustness, space vehicle control.**

I. Introduction

FORMATION flying control involving a number of spacecraft in order to accomplish a mission cooperatively has been of special interest in recent years [1]–[11]. The manner of cooperation between the spacecraft determines the architecture of the formation, which has been classified in the literature as five main categories: leader-follower, behavioral, virtual, cyclic, and multiple-input–multiple-output. This classification is, in fact, based on the topology of communication between the spacecraft controllers. In practice, it is desired to have the minimum number of communication links, as it is a grave issue in deep space applications. Lack of sufficient number of communication links, on the other hand, may cause several problems such as deterioration of the overall control performance, inability to avoid collision, inability to detect obstacles, inefficient formation reconfiguration, etc.

In [12], a formation consisting of a number of physically decoupled spacecraft in deep space is defined in terms of the relative positions between the spacecraft as well as the spacecraft attitudes. The main purpose of the formation introduced in [12] is to formulate some of the formation problems such as reorientation and tracking. It is then stated that the attitude of any spacecraft can be controlled by its local outputs without the requirement of knowing the attitude of all of the other spacecraft (i.e., no communication link is required). However, in order to control the formation, each spacecraft should be provided with the information about the relative position between any pair of spacecraft. This information exchange requires communication links. Nevertheless, a method is proposed in [12] to systematically calculate this relative distance in terms of the locally measured information, with no communication requirement. As a result, any given global static controller can be implemented locally with no communication link using the method in [12].

The method proposed in [13] considers a static controller for any spacecraft formation. It is assumed that this controller is designed to satisfy desired specifications. Since this controller takes advantage of all the communication links, it is very difficult to implement it in practice. Hence, a method is introduced in [13] which aims to eliminate some of the communication links from the control structure, and to estimate the corresponding information instead, by means of local observers. The resulting controller behaves closely to the original one, in general, after elapsing the transient time. The controller obtained is much more complicated but has a simple structure, i.e., fewer communication links. Although [13] presents a novel idea, it suffers from the following two practical drawbacks.

- In the control design procedure, certain conditions are required to be satisfied (as will be discussed later). First of all, these requirements do not hold in many practical cases. Furthermore, there is no systematic method for the pole placement via a decentralized *static* controller which is also required in the corresponding control design procedure.
- Some undesirable incidents, such as collision, may happen during the transient time due to the overshoots. As a remedy to avoid these unwanted incidents, one may reduce the transient time by using high gains in the local observers. Nevertheless, this may cause saturation in the actuators.

Since the model of the entire formation is copied in all the local controllers, this can be envisaged as an open-loop control strategy, which is known to be sensitive to parameter variations.

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This method is further developed in [13] and [14] to address this issue by introducing fewer number of communication links.

In this brief, a decentralized control strategy for formation control of a number of spacecraft is introduced. It is assumed that a centralized controller consisting of a set of interacting local controllers for the formation is designed to achieve the desired specifications such as optimal performance. To implement this centralized controller, any local controller should know the information of all other spacecraft, which is undesirable due to the high communication cost. The objective here is to design another controller which performs almost the same as the original controller, while its communication requirement is significantly lower. Throughout this brief, the terms *centralized* and *decentralized* controllers refer to the original multivariable controller (consisting of the interacting local controllers), and the proposed controller with reduced number of communication links, respectively. To this end, the formation is first described by a hierarchical linear time-invariant (LTI) model [15]. A decentralized controller is then derived from any given centralized controller. The idea behind this approach is that each local controller estimates the unavailable states of other spacecraft according to its belief about the model of the formation. Necessary and sufficient conditions for the internal stability of the formation under the proposed decentralized controller are obtained, which are easy to check (these are rather mild conditions which are often satisfied in practice). If the stability conditions hold, it is shown that a more precise knowledge of the initial state and the model of the formation by all of the spacecraft, leads to a smaller discrepancy between the formation under the original centralized controller and its decentralized counterpart, and in the ideal case (perfect knowledge) the two controllers perform identically. It is worth mentioning that if the centralized controller is optimal, its corresponding decentralized counterpart is near-optimal in practice (in presence of inexact knowledge). The near-optimal decentralized controller obtained is superior to the existing control strategies (e.g., the ones proposed in [16]–[25]), as will be shown later. A few practical issues related to the modeling error and the decentralization are then investigated as follows. The nonlinearity of the formation model and its time-varying nature (mainly due to the slowly changing mass) are represented as perturbations in the LTI model, and necessary and sufficient conditions similar to the previous case are obtained subsequently for the internal stability of the perturbed model under the proposed control law. Since different spacecraft may have nonidentical beliefs about the nominal model of the formation, the proposed decentralized control law is modified accordingly to take this practical point into account.

The proposed control law may still suffer from the following drawbacks.

- Most of the time, a linearized model cannot describe the formation accurately for a long period of time.
- Since the local controller of any spacecraft uses the nominal models of other spacecraft, the difference between the nominal model and the real one may cause severe problems (due to the partially open-loop control structure, indeed).
- Unlike the centralized controller, the proposed controller is incapable of detecting certain faults, efficient reconfiguration, etc.

In order to ameliorate the above-mentioned limitations, the proposed controller has been reformulated in the predictive-control framework. More precisely, the communication links required for the implementation of the centralized controller which were eliminated in the proposed decentralized controller are replaced with weak communication links which transmit and receive information in certain time instants only. This implies that instead of removing the communication links perpetually, the communication rate is reduced as a compromise in the tradeoff between the performance and communication cost. It will be shown later how the new model predictive controller takes the above issues into consideration.

This brief is organized as follows. The decentralization of any given centralized controller for spacecraft formation is explained in Section II and its characteristics are investigated accordingly. The proposed work is compared with some of the existing works in Section III. Comprehensive simulation results are given in Section IV. Finally, some concluding remarks are presented in Section V.

II. Decentralized Implementation of a Centralized Controller

Consider a formation $\mathcal{F}$ consisting of ν spacecraft. Assume that the model of the formation expressed either in the relative coordinates or in the absolute coordinates has a hierarchical structure with the following state-space equation for the ith spacecraft:

$$\begin{aligned}\dot{x}_i(t) &= A_{ii}x_i(t) + \sum_{j=1}^{i-1} H_{ij}z_{ij}(t) + B_i u_i(t)\\ y_i(t) &= C_i x_i(t)\end{aligned} \tag{1}$$

where $x_i(t) \in \Re^{n_i}$, $u_i(t) \in \Re^{m_i}$, $y_i(t) \in \Re^{r_i}$, $i \in \bar{\nu} := \{1, 2, \ldots, \nu\}$, are the state, the input and the measurable output of the ith spacecraft, respectively, and $z_{ij}(t)$ is a signal representing the effect of the jth spacecraft on the dynamics of the ith spacecraft. The signal $z_{ij}(t)$, $i, j \in \bar{\nu}$, $j < i$, can be regarded as an input for the model of the ith spacecraft coming out of the jth spacecraft as an output, and can be modeled as $z_{ij}(t) = L_{ij}x_j(t)$. Assume now that $z_{ij}(t)$ is measurable for the jth spacecraft, i.e., it can be computed from $y_j(t)$. Define

$$A_{ij} := H_{ij}L_{ij}, \quad i, j \in \bar{\nu}, \ j < i. \tag{2}$$

The formation $\mathcal{F}$ consists of all of the spacecraft in (1), and is represented by the following state-space equation:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \tag{3}$$

where

$$A := \begin{bmatrix} A_{11} & 0 & \ldots & 0\\ A_{21} & A_{22} & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots\\ A_{\nu 1} & A_{\nu 2} & \ldots & A_{\nu\nu}\end{bmatrix}$$

$$B := \begin{bmatrix} B_1 & \ldots & 0\\ \vdots & \ddots & \vdots\\ 0 & \ldots & B_\nu\end{bmatrix}$$

$$C := \begin{bmatrix} C_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C_\nu \end{bmatrix}$$
$$u(t) := [\, u_1(t)^T \quad \dots \quad u_\nu(t)^T \,]^T$$
$$x(t) := [\, x_1(t)^T \quad \dots \quad x_\nu(t)^T \,]^T$$
$$y(t) := [\, y_1(t)^T \quad \dots \quad y_\nu(t)^T \,]^T . \tag{4}$$

Consider a centralized LTI controller K_c with the following state-space representation:

$$\begin{aligned} \dot{\eta}_c(t) &= \Gamma\eta_c(t) + \Omega y(t) \\ u(t) &= M\eta_c(t) + Ny(t) \end{aligned} \tag{5}$$

where $\eta_c \in \Re^\mu$. It is assumed that the controller K_c has been designed by using any proper technique to achieve the control objectives such as optimal energy. The implementation of the centralized controller K_c requires several communication links in general, so that all of the spacecraft can share their outputs with each other. Since this is not pragmatic, it is desired now to implement the centralized control K_c in a decentralized fashion. To this end, define the following vectors:

$$\begin{aligned} x^i &:= [\, x_1^T \quad \dots \quad x_{i-1}^T \quad x_{i+1}^T \quad \dots \quad x_\nu^T \,]^T \\ u^i &:= [\, u_1^T \quad \dots \quad u_{i-1}^T \quad u_{i+1}^T \quad \dots \quad u_\nu^T \,]^T \\ y^i &:= [\, y_1^T \quad \dots \quad y_{i-1}^T \quad y_{i+1}^T \quad \dots \quad y_\nu^T \,]^T \end{aligned} \tag{6}$$

for any $i \in \bar{\nu}$.

Notation 1: The following notations will prove to be convenient in the development of the main results.

- Consider a $\nu\times\nu$ block diagonal matrix T with block entries $I_{r_1\times r_1}, I_{r_2\times r_2}, \dots, I_{r_\nu\times r_\nu}$. Denote the ith block column of T with T_i and the matrix obtained from T by removing T_i with T^i, for any $i \in \bar{\nu}$.
- Similarly, define $\bar{T}$ as a $\nu \times \nu$ block diagonal matrix with the block entries $I_{m_1\times m_1}, I_{m_2\times m_2}, \dots, I_{m_\nu\times m_\nu}$. Denote the ith block column of $\bar{T}$ with $\bar{T}_i$ and the matrix obtained from $\bar{T}$ by removing $\bar{T}_i$ with $\bar{T}^i$, for any $i \in \bar{\nu}$.
- Consider a $\nu\times\nu$ block diagonal matrix $\tilde{T}$ with block entries $I_{n_1\times n_1}, I_{n_2\times n_2}, \dots, I_{n_\nu\times n_\nu}$. Denote the ith block column of $\tilde{T}$ with $\tilde{T}_i$ and the matrix obtained from $\tilde{T}$ by removing $\tilde{T}_i$ with $\tilde{T}^i$, for any $i \in \bar{\nu}$.

One can easily conclude that (for any $i \in \bar{\nu}$)

$$\begin{aligned} y(t) &= [\, T^i \quad T_i \,] \begin{bmatrix} y^i(t) \\ y_i(t) \end{bmatrix} \\ u(t) &= [\, \bar{T}^i \quad \bar{T}_i \,] \begin{bmatrix} u^i(t) \\ u_i(t) \end{bmatrix} \\ x(t) &= [\, \tilde{T}^i \quad \tilde{T}_i \,] \begin{bmatrix} x^i(t) \\ x_i(t) \end{bmatrix}. \end{aligned} \tag{7}$$

By substituting (7) into (5), the controller K_c can be written as follows:

$$\begin{aligned} \dot{\eta}_c(t) &= \Gamma\eta_c(t) + \mathbf{\Omega}^i y^i(t) + \mathbf{\Omega}_i y_i(t) \\ u^i(t) &= \mathbf{M}^i \eta_c(t) + \mathbf{N}^i y^i(t) + \mathbf{N}_i y_i(t) \\ u_i(t) &= \mathbf{M}_i \eta_c(t) + \mathbf{N}^{\bar{i}} y^i(t) + \mathbf{N}_{\bar{i}} y_i(t) \end{aligned} \tag{8}$$

for any $i \in \bar{\nu}$, where

$$\begin{aligned} [\, \mathbf{\Omega}^i \quad \mathbf{\Omega}_i \,] &:= \mathbf{\Omega}\, [\, T^i \quad T_i \,] \\ \begin{bmatrix} \mathbf{M}^i \\ \mathbf{M}_i \end{bmatrix} &:= [\, \bar{T}^i \quad \bar{T}_i \,]^T\, \mathbf{M} \\ \begin{bmatrix} \mathbf{N}^i & \mathbf{N}_i \\ \mathbf{N}^{\bar{i}} & \mathbf{N}_{\bar{i}} \end{bmatrix} &:= [\, \bar{T}^i \quad \bar{T}_i \,]^T\, \mathbf{N}\, [\, T^i \quad T_i \,]. \end{aligned} \tag{9}$$

Likewise, the system $\mathcal{S}$ given in (3) can be decomposed as follows:

$$\begin{aligned} \dot{x}^i(t) &= \mathbf{A}^i x^i(t) + \mathbf{A}_i x_i(t) + \mathbf{B}^i u^i(t) \\ \dot{x}_i(t) &= \mathbf{A}^{\bar{i}} x^i(t) + A_{ii} x_i(t) + B_i u_i(t) \\ y^i(t) &= \mathbf{C}^i x^i(t) \end{aligned} \tag{10}$$

for any $i \in \bar{\nu}$, where

$$\begin{aligned} \begin{bmatrix} \mathbf{A}^i & \mathbf{A}_i \\ \mathbf{A}^{\bar{i}} & A_{ii} \end{bmatrix} &:= [\, \tilde{T}^i \quad \tilde{T}_i \,]^T A\, [\, \tilde{T}^i \quad \tilde{T}_i \,] \\ \begin{bmatrix} \mathbf{B}^i & 0 \\ 0 & B_i \end{bmatrix} &:= [\, \tilde{T}^i \quad \tilde{T}_i \,]^T B\, [\, \bar{T}^i \quad \bar{T}_i \,] \\ \begin{bmatrix} \mathbf{C}^i & 0 \\ 0 & C_i \end{bmatrix} &:= [\, T^i \quad T_i \,]^T C\, [\, \tilde{T}^i \quad \tilde{T}_i \,]. \end{aligned} \tag{11}$$

Using (8) and (10), one can find the following equation relating $x_i(t)$ and $y_i(t)$ to $u_i(t)$:

$$\begin{aligned} \begin{bmatrix} \dot{x}^i(t) \\ \dot{\eta}_c(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}^i + \mathbf{B}^i\mathbf{N}^i\mathbf{C}^i & \mathbf{B}^i\mathbf{M}^i \\ \mathbf{\Omega}^i & \Gamma \end{bmatrix} \begin{bmatrix} x^i(t) \\ \eta_c(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathbf{B}^i\mathbf{N}_i \\ \mathbf{\Omega}_i \end{bmatrix} y_i(t) + \begin{bmatrix} \mathbf{A}_i \\ 0 \end{bmatrix} x_i(t) \\ u_i(t) &= [\, \mathbf{N}^{\bar{i}}\mathbf{C}^i \quad \mathbf{M}_i \,] \begin{bmatrix} x^i(t) \\ \eta_c(t) \end{bmatrix} + \mathbf{N}_{\bar{i}} y_i(t). \end{aligned} \tag{12}$$

Combining the relations $z_{ij}(t) = L_{ij}x_j(t)$ and $A_{ij} = H_{ij}L_{ij}$, $i, j \in \bar{\nu}$, $j < i$, leads to the equation $\mathbf{A}_i x_i(t) = \mathbf{H}^i z^i(t)$, where

$$z^i(t) = [\, 0 \quad \cdots \quad 0 \quad z_{(i+1)i}(t)^T \quad \cdots \quad z_{\nu i}(t)^T \,]^T \tag{13}$$

and $\mathbf{H}^i$ is a block diagonal matrix whose (j, j) block entry is equal to H_{ji} for any $j \in \bar{\nu}$, $i < j$, and 0 otherwise. Define now K_{d_i} as a controller for the ith spacecraft whose state-space representation is given by

$$\begin{aligned} \dot{\eta}_{d_i}(t) &= \begin{bmatrix} \mathbf{A}^i + \mathbf{B}^i\mathbf{N}^i\mathbf{C}^i & \mathbf{B}^i\mathbf{M}^i \\ \mathbf{\Omega}^i & \Gamma \end{bmatrix} \eta_{d_i}(t) \\ &\quad + \begin{bmatrix} \mathbf{B}^i\mathbf{N}_i \\ \mathbf{\Omega}_i \end{bmatrix} y_i(t) + \begin{bmatrix} \mathbf{H}^\mathbf{i} \\ 0 \end{bmatrix} z^i(t) \\ u_i(t) &= [\, \mathbf{N}^{\bar{i}}\mathbf{C}^i \quad \mathbf{M}_i \,]\, \eta_{d_i}(t) + \mathbf{N}_{\bar{i}} y_i(t). \end{aligned} \tag{14}$$

It is to be noted that by assumption $z^i(t)$ is measurable for the ith spacecraft, i.e., it can be computed from $y_i(t)$. Define K_d as a decentralized controller consisting of the local controllers $K_{d_1}, K_{d_2}, \dots, K_{d_\nu}$.

Theorem 1: The formation $\mathcal{F}$ under the centralized controller K_c and the decentralized controller K_d behaves identically in the sense that it has the same state under both controllers, provided the following conditions hold:

$$\eta_{d_i}(0) = \begin{bmatrix} x^i(0) \\ 0 \end{bmatrix}, \quad i \in \bar{\nu}. \tag{15}$$

Proof: As pointed out earlier, the decomposed model of the formation given in (10) under controller K_c given in (8) results in the controller (12) for the ith spacecraft. The proof follows on noting that the controller (14) is the same as (12) due to the relation $\mathbf{A}_i x_i(t) = \mathbf{H}^i z^i(t)$ and (15). ■

Theorem 1 states that the centralized controller K_c for the whole formation can be transformed into an equivalent decentralized controller K_d, if the controller K_{d_i} for the ith spacecraft $i \in \bar{\nu}$ knows exactly the initial states and the modeling parameters of all other spacecraft. It is to be noted that this is not a realistic assumption in practice. To remedy the problem of inaccurate knowledge of the initial state, the following initial state will be deployed:

$$\eta_{d_i}(0) = \begin{bmatrix} \hat{x}^i(0) \\ 0 \end{bmatrix}, \quad i \in \bar{\nu} \tag{16}$$

instead of the one in (15), where $\hat{x}^i(0)$ is the estimate of $x^i(0)$ which is available to the ith spacecraft. Choosing this new initial state can induce some nonzero residues for the unstable modes of the decentralized control system, and consequently make the formation unstable [27]. Hence, the internal stability of the formation $\mathcal{F}$ under the decentralized controller K_d will be investigated in the sequel.

Definition 1: Consider the formation $\mathcal{F}$ given by (3). The modified formation $\mathcal{F}_i$, $i \in \bar{\nu}$, is defined to be a formation obtained from $\mathcal{F}$ by neutralizing the effect of spacecraft $1, 2, \ldots, i-1$ on the ith spacecraft model. The state-space representation of the modified formation $\mathcal{F}_i$ is as follows:

$$\begin{aligned} \dot{x}(t) &= \tilde{A}^i x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{17}$$

where $\tilde{A}^i$ is derived from A by replacing the first $i-1$ block entries of its ith block row with zeros. It is to be noted that $\mathcal{F}^1 = \mathcal{F}$.

Definition 2: Define the decoupled model of the ith spacecraft, $i \in \bar{\nu}$, as

$$\begin{aligned} \dot{x}_i(t) &= A_{ii} x_i(t) + B_i u_i(t) \\ y_i(t) &= C_i x_i(t). \end{aligned} \tag{18}$$

Note that in the decoupled model the effect of any other spacecraft is vanished.

Theorem 2: The formation $\mathcal{F}$ is internally stable under the decentralized controller K_d if and only if the modified formation $\mathcal{F}_i$ is stable under the centralized controller K_c, for all $i \in \bar{\nu}$.

Proof: Since the formation $\mathcal{F}$ is hierarchical, its stability under the decentralized controller K_d is equivalent to the stability of the decoupled spacecraft i under the local controller K_{d_i} for all $i \in \bar{\nu}$. Moreover, it is straightforward to show that the A-matrix of the decoupled spacecraft i given in (18) under the controller K_{d_i} is the same as that of the modified formation $\mathcal{F}_i$ under the centralized controller K_c. This completes the proof. ■

Given the centralized controller K_c, its decentralized counterpart K_d obtained earlier can be applied to the formation $\mathcal{F}$ if and only if the easy-to-check conditions given in Theorem 2 hold. To compare these two controllers, one should note that the centralized controller K_c suffers from the following communication difficulties:

- number of communication links grows with the square of ν;
- communication links should be synchronized;
- controller is vulnerable to the communication links failure in the sense that if one of them fails, the overall controller will not operate normally.

The main advantage of the decentralized controller K_d is that it does not have the previously mentioned difficulties. However, there are a few practical issues regarding the controller K_d which will be addressed in the following subsections.

A. Robust Stability Analysis

In practice, the LTI model (3) cannot precisely describe the formation $\mathcal{F}$. Let the exact model of the formation $\mathcal{F}$, which is a perturbed form of the nominal model, be described by

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ y(t) &= (C + \Delta C)x(t) \end{aligned} \tag{19}$$

where ΔA, ΔB, and ΔC represent the parametric uncertainties accounting for nonlinearity, error in system identification, etc. Applying the controllers K_c and K_d to model (3) of the formation $\mathcal{F}$, two closed-loop systems will be obtained. Denote their A-matrices with A_c and A_d, respectively. Define the matrices $\bar{A}_c$ and $\bar{A}_d$ in a similar way by considering the perturbed model (19) instead of (3). Following the procedure given in [15], a tight bound α_d can be attained such that for any ΔA, ΔB, and ΔC leading to the inequality $\|\bar{A}_d - A_d\| < \alpha_d$, the perturbed model of the formation $\mathcal{F}$ under the decentralized controller K_d is stable, where $\|\cdot\|$ denotes the Frobenius norm. Analogously, a bound α_c can be obtained for the controller K_c, with the same level of conservativeness as α_d.

Theorem 3: The bound α_c is less than or equal to the bound α_d.

Proof: The proof is omitted due to its similarity to the proof of Theorem 3 in [27]. ■

Theorem 3 states that the decentralized controller K_d is likely more robust than its centralized counterpart. It is to be noted that the permissible bounds for the perturbation matrices ΔA, ΔB, and ΔC can be found in terms of the bound α_d to guarantee the stability. One can refer to [27] for a detailed discussion.

B. Performance Evaluation

Since the ith spacecraft exploits the initial state $\hat{x}^i(0)$ instead of $x^i(0)$ due to its unavailability, the performance of the formation $\mathcal{F}$ under the decentralized controller K_d will not be identical to that of $\mathcal{F}$ under the centralized controller K_c. In order to evaluate the discrepancy between the performances in the centralized and the decentralized cases, consider the following cost function:

$$J = \int_0^\infty \Delta x(t)^T Q \Delta x(t) dt \quad (20)$$

where Q is a given positive definite matrix and $\Delta x(t)$ denotes the state of the formation given by (3) under the controller K_d minus that under the controller K_c. Define the vector ΔX_0 as shown in (21) at the bottom of the page, where $n := n_1 + n_2 + \cdots + n_\nu$ and $0_{i\times j}$ is a $i \times j$ zero matrix, for any positive integers i and j. The following theorem presents a simple methodology to calculate the cost function J.

Theorem 4: Assume that the formation $\mathcal{F}$ given by (3) is stable under the decentralized controller K_d. The cost function J is equal to $\Delta X_0^T P_d \Delta X_0$, where the matrix P_d is the solution of the following Lyapunov equation:

$$A_d^T P_d + P_d A_d + \Phi^T Q \Phi = 0 \quad (22)$$

where $\Phi = [\, I_{n\times n} \quad 0_{n\times(\nu\mu+(\nu-1)n)}\,]$.

Proof: The proof is omitted due to its similarity to the proof of Theorem 4 in [27]. ■

C. Distributed Model of the Formation

So far, it has been assumed in constructing K_d from the centralized controller K_c that any two different spacecraft consider the same model for the formation $\mathcal{F}$. This assumption is not pragmatic in general, and hence the controller K_d will be modified now to account for this type of modeling mismatch. Define $\mathcal{F}^i$ as a virtual formation whose model is the belief of the ith spacecraft about the formation $\mathcal{F}$, for any $i \in \bar{\nu}$ (note that the model of the formation $\mathcal{F}$ is, in fact, distributed among the spacecraft as $\mathcal{F}^1, \ldots, \mathcal{F}^\nu$). In the ideal case, all the formations $\mathcal{F}^1, \mathcal{F}^2, \ldots, \mathcal{F}^\nu$ are identical to $\mathcal{F}$. Consider the centralized controller K_c, and analogously to the procedure given for designing the local controllers $K_{d_1}, \ldots, K_{d_\nu}$ for the formation $\mathcal{F}$, design the local controllers $K_{d_{i1}}, \ldots, K_{d_{i\nu}}$ for the formation $\mathcal{F}^i$, for any $i \in \bar{\nu}$. Define now the decentralized controller $\bar{K}_d$ as the union of the local controllers $K_{d_{11}}, K_{d_{22}}, \ldots, K_{d_{\nu\nu}}$. At this point, it is desired to check the stability of the formation $\mathcal{F}$ given in (3) under the decentralized controller $\bar{K}_d$, which is, indeed, a modified form of K_d.

Definition 3: Consider the formation $\mathcal{F}$ given by (3). The modified formation $\mathcal{F}_i^i$, $i \in \bar{\nu}$, is defined to be a formation obtained from $\mathcal{F}^i$ by neutralizing the effect of spacecraft $1, 2, \ldots, i-1$ on the model of the ith spacecraft.

Theorem 5: The formation $\mathcal{F}$ is internally stable under the decentralized controller $\bar{K}_d$ if and only if the modified formation $\mathcal{F}_i^i$ is stable under the centralized controller K_c, for all $i \in \bar{\nu}$.

Proof: The proof is omitted due to its similarity to the proof of Theorem 2. ■

Theorem 5 presents a simple test to check the stability of the formation $\mathcal{F}$ under the controller $\bar{K}_d$. The other properties of this controller such as robust stability and performance can be obtained in line with those given for the controller K_d.

D. Predictive-Control-Based Approach

Regarding the decentralized controller K_d, there are some practical issues as follows.

1) The model of each spacecraft is nonlinear and time-varying, while it is assumed here to be LTI. However, as pointed out in [28], this assumption is valid only in the vicinity of the operating point and for a short period of time.
2) The controller K_d is not capable of accounting for the effect of time-varying perturbation in the model of the formation.
3) The structure of the local controller for each spacecraft is contingent upon the modeling matrices and initial states of other spacecraft in an open-loop manner. Although this will not affect the stability of the formation provided the aforementioned conditions hold, it can degrade the control performance.
4) The centralized controller K_c can be designed in such a way that it is capable of avoiding a possible collision, detecting a fault, or passing a barrier without hitting it. In contrast, the decentralized controller K_d does not necessarily have these capabilities, which are of great importance in the real-world applications.

In order to ameliorate the applicability of the controller and address the above issues to some extent, a pseudo decentralized controller $\tilde{K}_d$ will be proposed now based on the decentralized controller K_d. Consider a sampling period h, and assume for now that any spacecraft can measure the states of any other spacecraft at the sampling instants $0, h, 2h, \ldots$. For any $i \in \bar{\nu}$, apply the controller given by (14) to the ith spacecraft in the time interval $[0, h)$, with the initial state $\eta_{d_i}(0) = [\hat{x}^i(0)^T \quad 0]^T$, where $\hat{x}^i(0)$ denotes the states of the other spacecraft measured at time $t = 0$ (as discussed earlier). At the instant $t = h$, measure the states of the other spacecraft to obtain $\hat{x}^i(h)$. For the time interval $[h, 2h)$, apply the controller given by (14) (as before) to the ith spacecraft, with the new initial state $\eta_{d_i}(h) = [\hat{x}^i(h)^T \quad 0]^T$. Following the same strategy, the state of the controller (14) at the time instants $2h, 3h, \ldots$ should be updated. The union of these local controllers will be referred to as the pseudo decentralized controller $\tilde{K}_d$. The controller $\tilde{K}_d$ has the following advantages.

- The linear model considered in (3) can describe the formation in the intervals of duration h (for a sufficiently small h) with a high precision.
- Any controller observes the states of the other spacecraft with a relatively low rate, to compensate for the negative effects discussed in 1) and 3) above.
- For any positive integer τ, the controller of the ith spacecraft, $i \in \bar{\nu}$, observes the states of the other spacecraft at $t = \tau h$. Then, it can predict the trajectory of the whole formation in the interval $[\tau h, (\tau+1)h)$ from the state of

$$\left[0_{1\times n} \quad \left(\hat{x}^1(0) - x^1(0)\right)^T \quad 0_{1\times\mu} \quad \left(\hat{x}^2(0) - x^2(0)\right)^T \quad 0_{1\times\mu} \quad \ldots \quad \left(\hat{x}^\nu(0) - x^\nu(0)\right)^T \quad 0_{1\times\mu} \right]^T \quad (21)$$

its controller [see (12) and (14)]. If it is known that no collision for the ith spacecraft will occur in the interval $[\tau h, (\tau+1)h)$, the ith local controller of $\tilde{K}_d$ proposed before would be applied to the ith spacecraft in this interval; otherwise, an emergency local controller should be applied to the ith spacecraft in this interval.

- For any positive integer τ, the controller of the ith spacecraft measures the states of other spacecraft at $t = (\tau+1)h$, and compares them with their predictions obtained in terms of the measurements at $t = \tau h$. If there is a sizable discrepancy between a measurement and the corresponding prediction, it implies that a fault has occurred in the formation, and a proper action (e.g., reconfiguration) should be taken.

There are some important issues which need to be considered in the design of $\tilde{K}_d$. First, one can use an emergency controller for collision avoidance if certain stability conditions are satisfied [29], [30]. A rigorous collision avoidance analysis is required in a practical setup with several vehicles, which is suggested for future research. Second, the formation under the controller $\tilde{K}_d$ is envisaged as a closed-loop system, but there are jumps in some of the control states at the instants $h, 2h, \ldots$ (note that there is no jump in the state of the formation). If the closed-loop system does not satisfy a number of conditions, these jumps might destabilize the closed-loop system for sufficiently small values of h. The relevant conditions and a lower bound on h for avoiding instability are intensively investigated in the literature [31].

So far, it is assumed that any spacecraft can measure the states of the other spacecraft at the sampling instants $0, h, 2h, \ldots$. However, if some of the states cannot be measured at either all instants or even some instants (due to the shadow phenomenon [12]), the update in the controller for these specific states should inevitably be ignored at those instants.

III. Comparison With the Existing Methods

Unlike the present work which considers K_c as a dynamic controller, the work [13] considers a centralized controller of the form $u(t) = \mathbf{M}x(t)$. The objective of [13] is to implement this controller with the minimum number of communication links, which means that the local controller of any spacecraft should estimate the state of the whole formation $\mathcal{F}$ (this work is further developed in [14]). This is carried out by designing a decentralized observer, or more precisely, by obtaining the matrices $L_1, L_2, \ldots, L_\nu$ so that the eigenvalues of the matrix

$$I \otimes (A + B\mathbf{M}) - \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} [\mathbf{B}_1\mathbf{M} \quad \ldots \quad \mathbf{B}_\nu\mathbf{M}] + \begin{bmatrix} L_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_\nu \end{bmatrix} C \quad (23)$$

lie in some desirable locations in the left-half s-plane, where $\otimes$ represents the Kronecker product and $\mathbf{B}_i$ denotes the matrix obtained from B by setting all of its block entries to zero, except for $B_i (i \in \bar{\nu})$. Now, the problem reduces to placing the modes of a system [corresponding to the equation (1) and the observer] whose dimension is ν times greater than that of the formation $\mathcal{F}$, in desirable locations in the complex plane by means of a decentralized *static* state feedback. This is more or less equivalent to the pole placement for a system via a centralized *static* output feedback. The pole placement conditions are, however, very restrictive when a static output feedback is to be utilized. As a matter of fact, the conditions for stabilizability with respect to a static output feedback are not satisfied in most cases. This brief is superior to [13] and [14] in terms of the order of the controller. More precisely, [13] and [14] duplicate the model of the whole formation into the ith local controller, for any $i \in \bar{\nu}$. However, the information of the ith spacecraft is always accessible for the ith local controller, and hence the corresponding model need not be provided as part of the whole formation. This introduces some kind of redundancy which not only will increase the order of the controller, but also may cause a problem in the case when there is a mismatch between the nominal and the real models of the formation. Apart from this issue, [13] and [14] suffer from the high-gain observer problem, as discussed earlier in the introduction.

As an application of the method proposed here, a decentralized near-optimal control law K_d with respect to a LQ performance index can be designed and subsequently be implemented as a predictive controller $\tilde{K}_d$. Several works have been presented in the literature, which mainly aim to design a decentralized static state (output) feedback, due to the complexity of designing a dynamic one [23], [24]. In other words, they intend to compute the matrices $L_1, L_2, \ldots, L_\nu$, such that the decentralized controller K_d with the ith local control law $u_i(t) = L_i x_i(t)$ acts as a near-optimal controller. Some relevant works are as follows.

1) The methods proposed in [19], [20], and [21] aim to design a gain L_i, for any $i \in \bar{\nu}$, in terms of the decoupled model of the ith spacecraft, rather than the whole formation. Since the interaction between different spacecraft is neglected in this design procedure, the overall performance of the controller may be poor in general, as discussed in detail in [15].
2) The approach given in [25] employs a decentralized form of the results given in [26] to find the control parameters $L_1, \ldots, L_\nu$ sequentially. In other words, the gain L_1 is constructed based on the model of the first subsystem. Then, the gain L_2 is obtained in terms of the models of subsystem 1 with the controller L_1, and subsystem 2. All other gains are obtained following the same procedure. The main advantage of this approach is its reduced offline computation requirements [25].
3) Another approach is to obtain the gains $L_1, \ldots, L_\nu$ concurrently by using iterative numerical algorithms [16]–[18], [22]. This type of design technique is, in fact, the extended version of the algorithms for designing optimal centralized static output feedback gain, such as Levine-Athans and Anderson-Moore methods. Although this technique always results in a better performance compared to the preceding methods, it has several shortcomings. First of all, it only presents necessary conditions, which are mainly in the form of complicated coupled nonlinear matrix equations. Second, these iterative algorithms require an initial stabilizing static gain, which should satisfy certain requirements. Finally, by using a dynamic feedback law instead of a static one, the overall performance of the system can be improved significantly.

In contrast with the aforementioned approaches, the controller designed in this brief is a dynamic one which normally performs quite closely to the optimal centralized controller, as discussed in [15]. This issue will be demonstrated in the simulation results.

IV. Simulation Results

While the main focus of this brief is on practical application of spacecraft formation, the proposed algorithm can be applied to the formation flying of unmanned aerial vehicles (UAV) too. In other words, the application domain of the brief can be extended to the cooperative control of any group of vehicles, regardless of their dynamic behavior. To clarify this issue, the results obtained in this work are applied to the formation flying problem of a group of UAVs in [2].

Consider a leader-follower formation $\mathcal{F}$ consisting of three UAVs. Label the leader as UAV 1, and the followers as UAV 2 and UAV 3. It is desired to control the planar motion of the formation. The lateral kinematic model with the dynamic extension for the ith UAV, i =1, 2, 3, is as follows:

$$\begin{bmatrix} \dot{X}_i(t) \\ \dot{Y}_i(t) \\ \dot{\psi}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t)\cos(\psi_i(t)) \\ v_i(t)\sin(\psi_i(t)) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_i(t) \\ \omega_i(t) \end{bmatrix} \tag{24}$$

where $X_i(t), Y_i(t), \psi_i(t), v_i(t), a_i(t)$, and $\omega_i(t)$ denote the horizontal coordinates, the vertical coordinates, the heading angle, the speed, the acceleration, and the angular velocity of the ith UAV, respectively. By changing the variables as mentioned in [2], the following linear state-space representation for the ith UAV will be obtained

$$\dot{z}_i(t) = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} z_i(t) + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u_i(t) \tag{25}$$

where I_2 and 0_2 represent the 2×2 identity matrix and the 2×2 zero matrix, respectively, and

$$\begin{aligned} z_i(t) &= [\,X_i(t) \quad Y_i(t) \quad v_i(t)\cos(\psi_i(t)) \quad v_i(t)\sin(\psi_i(t))\,]^T \\ u_i(t) &= \begin{bmatrix} a_i(t)\cos(\psi_i(t)) - v_i(t)\omega_i(t)\sin(\psi_i(t)) \\ a_i(t)\cos(\psi_i(t)) + v_i(t)\omega_i(t)\cos(\psi_i(t)) \end{bmatrix}. \end{aligned} \tag{26}$$

Assume that all UAVs are desired to fly at the same velocity $(v_{\mathbf{x}}, v_{\mathbf{y}})$, with the distance vector $(d_{\mathbf{x}_j}, d_{\mathbf{y}_j})$ between the jth UAV and the $(j+1)$th UAV, for j =1, 2. Define now the following vectors:

$$\begin{aligned} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} &:= \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} - \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} - \begin{bmatrix} d_{\mathbf{x}_1} \\ d_{\mathbf{y}_1} \end{bmatrix} \\ \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} &:= \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} - \begin{bmatrix} z_{31} \\ z_{32} \end{bmatrix} - \begin{bmatrix} d_{\mathbf{x}_2} \\ d_{\mathbf{y}_2} \end{bmatrix} \\ \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} &:= \begin{bmatrix} z_{13} \\ z_{14} \end{bmatrix} - \begin{bmatrix} v_{\mathbf{x}} \\ v_{\mathbf{y}} \end{bmatrix} \\ \begin{bmatrix} x_{23} \\ x_{24} \end{bmatrix} &:= \begin{bmatrix} z_{23} \\ z_{24} \end{bmatrix} - \begin{bmatrix} v_{\mathbf{x}} \\ v_{\mathbf{y}} \end{bmatrix} \\ \begin{bmatrix} x_{33} \\ x_{34} \end{bmatrix} &:= \begin{bmatrix} z_{33} \\ z_{34} \end{bmatrix} - \begin{bmatrix} v_{\mathbf{x}} \\ v_{\mathbf{y}} \end{bmatrix} \end{aligned} \tag{27}$$

where z_{ij} represents the jth entry of the vector z_i, for any $i \in \{1,2,3\}, j \in \{1,2,3,4\}$. Therefore, the model of the formation in the relative coordinates frame can be obtained as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ I_2 & 0_2 & -I_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & I_2 & 0_2 & -I_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} I_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 \\ 0_2 & I_2 & 0_2 \\ 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & I_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{28}$$

where

$$\begin{aligned} x_1 &= [\,x_{11} \quad x_{12}\,]^T \\ x_2 &= [\,x_{21} \quad x_{22} \quad x_{23} \quad x_{24}\,]^T \\ x_3 &= [\,x_{31} \quad x_{32} \quad x_{33} \quad x_{34}\,]^T. \end{aligned} \tag{29}$$

It is known that each UAV can measure its relative position (and consequently its relative velocity) with respect to each of the other UAVs by using a GPS-based architecture [2]. Therefore, it is assumed in this example that each UAV is equipped with this measuring device. Various decentralized controllers will be presented in the following.

A. Decentralized Near-Optimal Control Law

It is desired to design three local controllers for the UAVs such that the performance index J_d given by

$$J_d = \int_0^\infty \left(x^T(t)x(t) + u^T(t)u(t)\right)\, dt \tag{30}$$

is satisfactorily small. Two different design techniques will be used and the results will be compared here: the iterative numerical procedure given in [17], and the method proposed in this brief. Suppose that each initial state is uniformly distributed in the intervals [200,400], and that any two distinct initial state variables are statistically independent. It is to be noted that the units used for distance and velocity in the state vectors are feet and feet per second, respectively. Assume that any two different UAVs consider the same expected value for the initial state of the remaining UAVs, and that the nominal model of each UAV is exactly known by the other UAVs (because the UAVs are identical). Assume also that the real initial state variables are all equal to 400, which correspond, in fact, to the worst case scenario (maximum discrepancy between the real initial state variables, i.e., 400, and the corresponding expected values, i.e., 300, which are used with the proposed controller). Now, obtain the optimal centralized controller $u(t) = Nx(t)$ derived from the Riccati equation, and denote it by K_c. Since the conditions of Theorem 2 are satisfied for the formation $\mathcal{F}$, the decentralized controller K_d corresponding to the centralized controller K_c can be obtained, as mentioned earlier. The iterative numerical procedure of [17] gives a static decentralized state feedback law which results in a performance index equal to 2 257 085. The performance index obtained by applying the method proposed

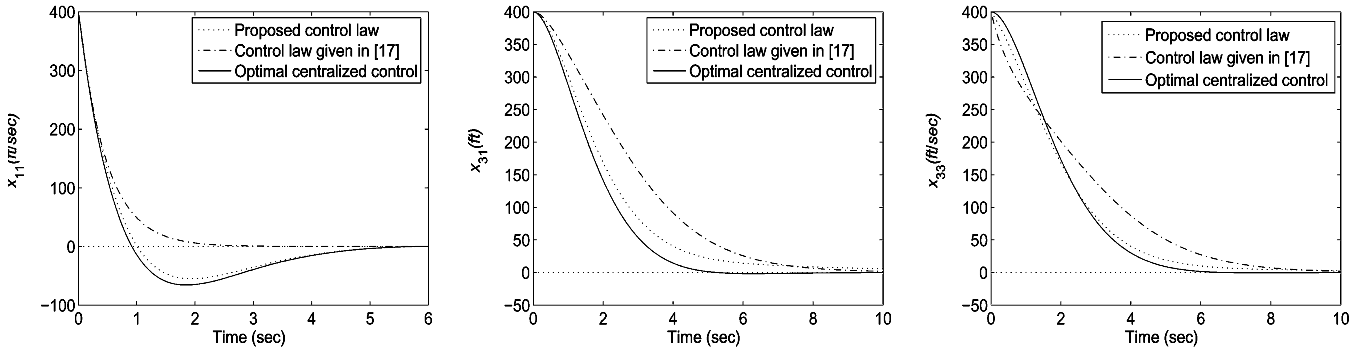

Fig. 1. State variables x_{11}, x_{31}, and x_{33} using three different design techniques.

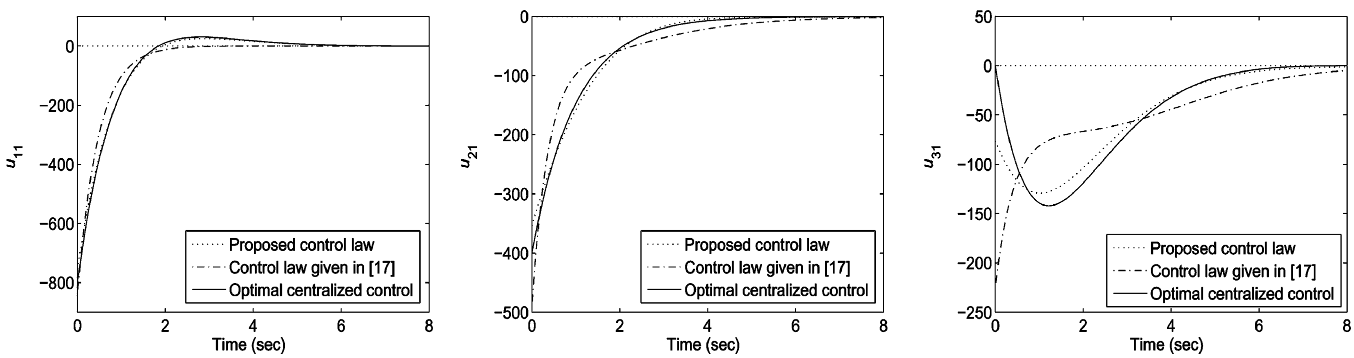

Fig. 2. Control signals u_{11}, u_{21}, and u_{31} using three different design techniques.

in this brief, on the other hand, is equal to 2 090 939, while the best achievable performance index corresponding to the centralized LQR controller is equal to 2 068 513. This means that the relative errors of the performance indices obtained by using the methods given here and in [17], with respect to the optimal centralized performance index are 1.08% and 9.12%, respectively. This shows clearly that the controller proposed in this brief outperforms the one presented in [17], significantly.

Fig. 1 depicts the time response of the system under the controller proposed in this brief (dotted curve), the controller proposed in [17] (dashed curve), and the optimal centralized controller (solid curve) for three state variables $x_{11}(t)$, $x_{31}(t)$, and $x_{33}(t)$. Moreover, the control signals $u_{11}(t)$, $u_{21}(t)$, and $u_{31}(t)$ obtained by using the three methods mentioned previously are depicted in Fig. 2 in a similar way. It is to be noted that despite the relatively big differences between the real initial variables (400 ft for distance errors and 400 ft/s for speed errors) and the corresponding expected values which are used to construct the proposed controller, the results obtained are reasonably close to the time response of the system under the centralized LQR controller.

The results obtained show that the method introduced in this brief outperforms the one in [17]. On the other hand, as pointed out earlier, the controller obtained by the method proposed in [17] has a better performance compared to the ones given in [19]–[21], and [25], and arrives at the same solution as [16], [18], and [22]. This exhibits superiority of the proposed design technique over the existing ones.

B. Predictive Control Applied to the Perturbed Model

It is desired to show that using the controller $\tilde{K}_d$ instead of K_d is vital, when the linear model of the formation is not exact. In other words, in the case when the LTI model of the formation is subject to uncertainties, the velocities and positions of all UAVs should be measured at instants $0, h, 2h, \ldots$ by each of the local controllers, where h is the sampling time which needs to be chosen sufficiently small. Consider now the initial states $x_1 = [400{,}1200]$, $x_2 = [2000{,}2400{,}1200{,}1600]$, $x_3 = [2800{,}3200{,}800{,}1200]$, and the controller K_c as $u(t) = 2Nx(t)$, where N is introduced in the preceding subsection. Assume that the exact values of the A-matrix and B-matrix of the formation are equal to $1.1A$ and $0.9B$, respectively, and that each UAV knows the states of the other UAVs with 10% error at $t = 0$, and can measure them accurately at the subsequent sampling instants $h, 2h, \ldots$. The difference between the relative position of UAV 2 with respect to UAV 3 along the $\mathbf{x}$-axis under centralized and decentralized controllers is depicted in Fig. 3 for

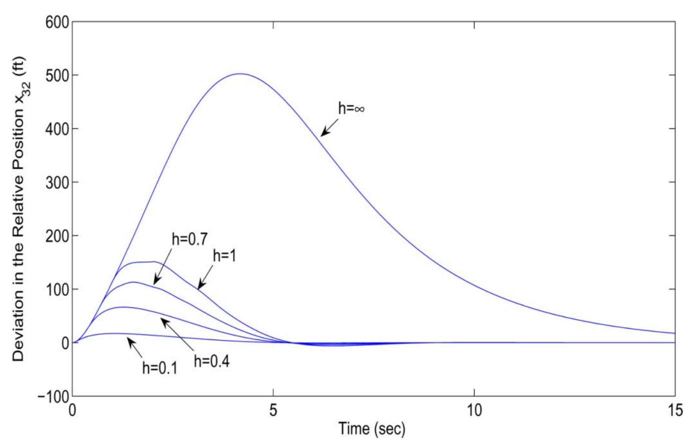

Fig. 3. Difference between the relative position of UAVs 2 and 3 in both cases of centralized and decentralized controllers for different values of h.

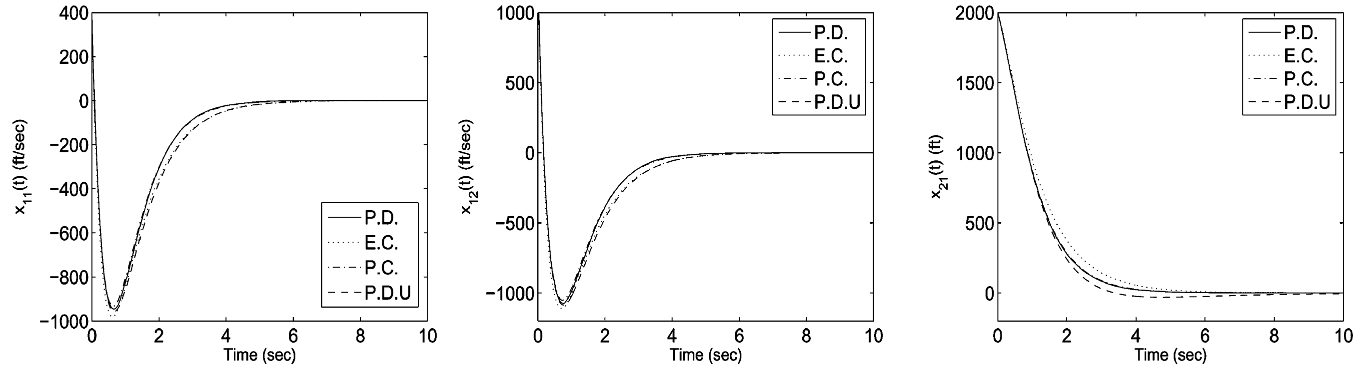

Fig. 4. State variables x_{11}, x_{12}, and x_{21} using four different design techniques.

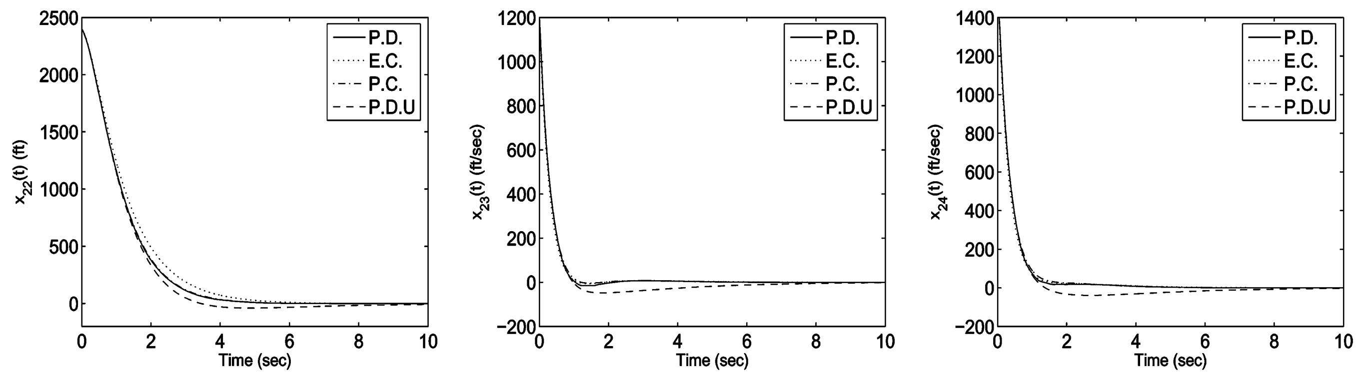

Fig. 5. State variables x_{22}, x_{23}, and x_{24} using three different design techniques.

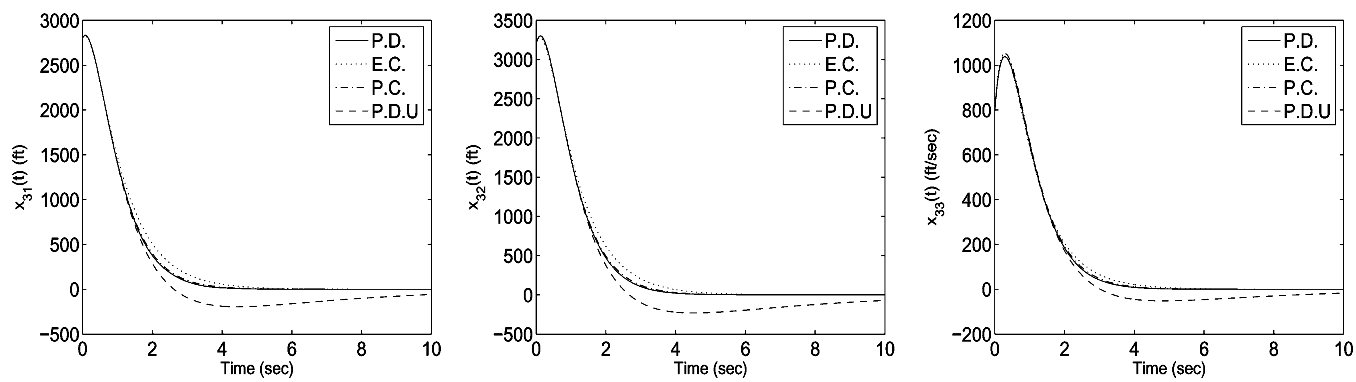

Fig. 6. State variables x_{31}, x_{32}, and x_{33} using three different design techniques.

different values of h. It can be easily observed from Fig. 3 that the difference between the relative position under centralized and decentralized controllers vanishes as h becomes smaller. This implies that for small values of h, the formation with the perturbed model behaves almost identically under the centralized controller K_c and the decentralized controller K_d. Moreover, in the case of a large h, i.e., when the decentralized controller measures the states of the formation less frequently, there can be a huge difference between the formation under centralized and decentralized controllers. This, in turn, may lead to a collision in the formation under the decentralized control law. It is to be noted that between the sampling instants the decentralized controller operates in an open-loop fashion when it comes to processing the nonidentical information, and hence this time interval should ideally be short.

Suppose now that any UAV can measure the states of the other UAVs accurately at the sampling instants $0, h, 2h, \ldots$ (with a possible exclusion of 0), while the model of the formation is perturbed. The states of the formation corresponding to four different cases are depicted in Figs. 4–7(a). The abbreviations P.D., P.D.U, P.C., and E.C. in these figures represent the decentralized controller applied to the perturbed model of the formation with $h = 0.4$, the decentralized controller applied to the perturbed model of the formation with $h = \infty$, the centralized controller applied to the perturbed model of the formation, and the centralized controller applied to the exact model of the formation, respectively.

C. Formation Trajectory in Planar Coordination

Consider the decentralized controller given in Section IV-B, and assume that the state-space matrices A and B of the formation are subject to 10% error as the previous case. Assume also that each UAV considers the zero initial states for the other UAVs at $t = 0$ (because they may not be measurable initially), but it can measure the states at the sampling instants

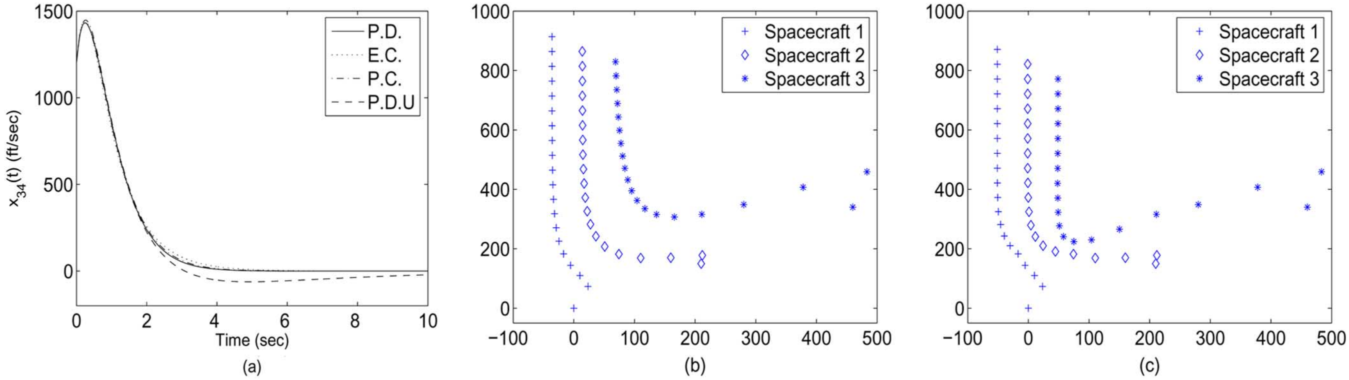

Fig. 7. (a) State variable x_{34} using three different design techniques. (b) The trajectory of the formation under the decentralized controller K_d for $h = \infty$. (c) The trajectory of the formation under the decentralized controller K_d for $h = 2$.

$t = h, 2h, \ldots$ precisely. Consider now the following specifications for the formation:

- initial positions of UAVs 1–3 are $(0, 0)$, $(210, 150)$, and $(460, 340)$, respectively;
- initial velocity vectors of UAVs 1–3 are $(500, 500)$, $(500, 580)$, and $(660, 500)$, respectively;
- desired velocity vector for all UAVs is $(0, 100)$;
- desired relative distance of the first UAV with respect to the second one is $(50, -50)$;
- desired relative distance of the second UAV with respect to the third one is $(50, -50)$.

The trajectory of the formation under the decentralized controller K_d is sketched in Fig. 7(b) and (c) for $h = \infty$ and $h = 2$, respectively. It is to be noted that UAVs 1–3 in these figures are shown by the symbols $+$, $\diamond$, and $*$, as indicated in the legend. It can be observed from Fig. 7(b) and (c) that the formation converges to its desired trajectory faster for $h = 2$ (in general, the transient response is longer for a larger h).

This example provides a thorough comparison with the algorithm given in [17], which is not capable of efficiently handling the formation problems with more than three agents. The method proposed in this brief, however, can be applied to the formations with higher number of spacecraft. This claim is based on the observation that the decentralized stability condition given in Theorem 2 holds for a wide range of values of ν, examined by the authors.

V. Conclusion

This brief deals with the decentralized implementation of a centralized controller designed for spacecraft formation. The objective is to meet the design specifications with a reduced communication cost. A decentralized control law is first derived from a given centralized controller, where the communication links between the local controllers of different spacecraft are eliminated. Easy-to-check necessary and sufficient conditions are given to verify whether the stability of the original centralized controller guarantees that of its decentralized counterpart. Since each local controller of the proposed decentralized control law is contingent upon its belief about the model of the other spacecraft, the stability and robust performance of the decentralized controller are investigated in terms of the uncertainties in the beliefs. It is shown that the more precise the beliefs are, the closer the decentralized controller to its centralized counterpart is, and in the ideal case when there are no uncertainties in the models, the two controllers perform identically. It is then assumed that the model of the formation is subject to perturbation, which is mainly due to the unmodeled dynamics and the nonlinearities ignored in the LTI model. In this case, the stability of the formation under the proposed decentralized controller is studied in a similar manner as the ideal model case. The main advantage of the proposed decentralized controller is the elimination of the communication links between the local controllers of different spacecraft. However, this can potentially have a negative impact on the output performance in the presence of uncertainties, mismatch of the beliefs, etc. To address this tradeoff between the communication cost and the robust performance, a predictive control scheme is proposed to implement the controller. The resultant decentralized model predictive control strategy constitutes rather weak communication links between the local controllers as the information exchange can be carried out periodically with a low rate. The effectiveness of the proposed method is demonstrated through simulations.

The authors are presently investigating the effect of time-delay in the communication between the spacecraft. Furthermore, for future research a more rigorous analysis is required to investigate whether the stability conditions could be satisfied with the proposed construction for the existing collision avoidance controllers.

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