Precise Determination of the Nucleon Radius in ^{3}He

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(Received 26 December 1985; revised manuscript received 27 February 1986)

A new analysis of inclusive quasielastic electron-scattering data for ^{3}He is presented. Integrated momentum distributions are derived from the data for the first time and the Q^2 dependence is shown to exhibit a slight deviation from the perfect scaling. If this is interpreted as a change in the nucleon radius, a result for the ratio of the nucleon radius in ^{3}He to the free-nucleon radius of 1.025 ± 0.011 is obtained, corresponding to an upper limit of 3.6% for the radius increase.

PACS numbers: 25.30.Fj, 13.40.Fn, 21.40.+d, 27.10.+h

The possibility that the electromagnetic form factors of the nucleon may change in the nuclear medium has been a subject of much recent debate. The discussion has been motivated by some interpretations[1] of deep-inelastic scattering data[2] on nuclei ("European Muon Collaboration effect") which involve an increase in the confinement volume for quarks in the nuclear medium. Since the nucleon size is determined by the Q^2 behavior of form factors in elastic electron scattering, one would expect that an analogous study of the corresponding process (quasielastic scattering) in nuclei would yield direct information on the size of the nucleon in nuclei. This approach was suggested by Sick[3] in a recent paper in which he used inclusive quasielastic electron-scattering data on ^{3}He to obtain a limit of 6% for the increase in the radius of the nucleon in ^{3}He. Others have attempted analyses of quasielastic-scattering data on ^{12}C at lower Q^2.[4] In this Letter, I perform a new analysis of the ^{3}He data that improves the precision of the determination of the nucleon radius by an order of magnitude and leads to an upper bound which is about half as large as previous results for the increase of the nucleon radius compared to the free nucleon.

The property of y scaling in inclusive electron scattering was first suggested by West[5] and applied to the data of Day *et al.*[6] by Sick, Day, and McCarthy.[7] The basic idea is that the quasielastic cross section should be given by that of nucleons in motion in the nucleus with an initial momentum distribution $n(p)$. Using only the impulse approximation and one-photon exchange, one can obtain the expression

$$d^2\sigma/d\Omega\, d\nu = \int d^3p\, n(p)[Z\sigma_p + N\sigma_n], \qquad (1)$$

in which σ_p and σ_n are the free-proton and -neutron cross sections evaluated at the appropriate kinematic points and contain a δ function to ensure that the final-state nucleon struck by the virtual photon the is on mass shell. By integration and use of the delta function, the above expression becomes[8]

$$d^2\sigma/d\Omega\, d\nu = \sigma_N(m_N/|\mathbf{q}|)\int_{p_{\min}}^{p_{\max}} n(p)p\, dp, \qquad (2)$$

where $p_{\min}$ and $p_{\max}$ are functions of the variables Q^2 and ν (electron energy loss), and σ_N is the elastic nucleon cross section at Q^2 summed, without the recoil factor, over protons and neutrons. In the limit $Q^2 \to \infty$, the integral reduces to $F(y)$, a function of the variable y only (independent of Q^2), where y is the solution of the equation for ν,

$$\nu = [y^2 + 2y|\mathbf{q}| + |\mathbf{q}|^2 + m_N^2]^{1/2} + [y^2 + M_{A-1}^2]^{1/2} - M_A. \qquad (3)$$

Here, M_A and M_{A-1} are the invariant masses of the initial nucleus and residual nucleus, respectively, and y is the component of initial nucleon momentum parallel to $\mathbf{q}$. The scaling function $F(y)$ is related to the initial-state momentum distribution by

$$F(y) = 2\pi\int_{|y|}^{\infty} n(p)p\, dp. \qquad (4)$$

The function $F(y)$ derived from experimental data may not be independent of Q^2 for several reasons. If Q^2 is too low, then we might not yet have reached the asymptotic scaling region, or final-state interactions might be important. Generally, it has been found that, for $\nu - \nu_{\text{elastic}} > 0.1$ GeV, $Q^2 > 0.5(\text{GeV}/c)^2$, and $y < 0$, $F(y)$ is independent of Q^2.[7] Another reason for a breakdown of scaling behavior would be that the nucleon form factors might be modified in a nucleus, thus changing the Q^2 dependence of σ_N. One would then expect to find that if the form factors were appropriately modified in σ_N then scaling behavior would be improved.

In Ref. 3 the effect of modifying the form factors was explored. A systematic change in σ_N was introduced by mulitplication by a ratio of dipole functions,

$$G(Q^2) = (1 + Q^2R_0^2)^4/(1 + Q^2R^2)^4, \qquad (5)$$

where $R_0^2 = 1/[0.71\ (\text{GeV}/c)^2]$ is a dipole parametrization of the nucleon elastic scattering data.[9] It was determined in Ref. 3 that a 6% change in the radius parameter R (from R_0) was sufficient to significantly break the observed scaling behavior. Furthermore, if one were also simultaneously to vary the nucleon mass as $1/R$ as suggested by bag models, the tolerable change in R was reduced to 3%.

The approach used in this work is similar to that of Ref. 3, but I improve the precision and reliability of the analysis by taking it one step further. In Ref. 6, there are three data sets at $Q^2 = 0.95$, 2.2, and 3.7 $(\mathrm{GeV}/c)^2$ where the condition $\nu - \nu_{\mathrm{elastic}} > 0.1$ GeV is satisfied and data are available for a continuous range of $-0.3\ \mathrm{GeV}/c < y < 0.1\ \mathrm{GeV}/c$. For these three sets of data, I computed the values of $F(y)$ and then integrated to obtain

$$\gamma(Q^2) = 2\int_{-\infty}^{0} F_{\mathrm{expt}}(y)\,dy. \tag{6}$$

Only data for $y < 0$ were used, since the data do not scale for $y > 0$. The results were checked for stability and the effect of finite y cutoffs; the dominant uncertainty in γ was found to be the statistical uncertainty of the quoted cross sections (an overall 3% normalization uncertainty[6] is included in each γ). The values of γ obtained are listed in Table I, and plotted in Fig. 1.

This new procedure of integrating the scaling functions has two advantages. One is that the statistical precisions of the results are improved, as will be evident in the analysis presented below. The second is that there is a "sum rule" due to the normalization of the momentum distributions,

$$\int_{-\infty}^{0} F(y)\,dy = \tfrac{1}{2}. \tag{7}$$

This means that we have a model-independent result for γ in the scaling limit when $R = R_0$. Since the form factors at $Q^2 = 0$ are independent of the radius parameter in the dipole formula, we still know $\gamma(Q^2 = 0)$ when $R \neq R_0$. This is a big advantage over the procedure used in Ref. 3 where only the consistent scaling of $F(y)$ at different values of Q^2 was used to infer the limit on the radius increase.

Defining the quantity $\beta = 1/\gamma^{1/4}$, one can compute the radius ratio at each Q^2 using the relation

$$R/R_0 = [(1 + 1/Q^2R_0^2)\beta - 1/Q^2R_0^2]^{1/2}. \tag{8}$$

This equation assumes that the σ_N are modified as in Eq. (5). These results are listed in Table I and are seen to be mutually consistent, as is expected from scaling. The weighted average is computed to be

$$\rho = R/R_0 = 1.031 \pm 0.005, \tag{9}$$

from which one can conclude that the radius increase must be less than 3.6%. The expected $\gamma(Q^2)$ are plotted in Fig. 1 for $\rho = 1.031$ and $\rho = 1.06$ and it is clearly evident that the experimental values limit ρ to be much less than 1.06.

I have investigated the validity of this very precise result in various ways. The effect of not reaching the scaling limit would have been to yield values of γ that were lower than expected in the scaling limit, and so would cause an overestimate of the radius increase. [A consistency check can be made by use of a parametrized function fit to the experimental $F(y)$ and computation of a γ integral with the Q^2-dependent limits $p_{\min}$ and $p_{\max}$ to obtain the expected violation of scaling. No significant effect was evident for $Q^2 > 0.75\ (\mathrm{GeV}/c)^2$ with $y > -0.4\ \mathrm{GeV}/c$.]

The *only uncertainty* in the definition of the kinematic variable y is the degree of average excitation of the $A-1$ system. I have assumed that M_{A-1} is twice the nucleon mass (no excitation). The effect of adding more energy to the recoiling system (higher M_{A-1}) is to increase the values of γ obtained, which leads to lower values of ρ. If we allow an extra 10 MeV of energy for internal motion of the residual pair, the average value of ρ is reduced to 1.02. Further excitation of the recoiling system causes problems because the $F(y)$ peak shifts into the $y < 0$ region. In principle, the effect of exciting the residual two-body system could be estimated by spectral functions derived by Faddeev techniques.

If we were to assume that the normalization uncertainty was correlated among the three data sets, it would increase the uncertainty in the average value of

TABLE I. Numerical results for $\gamma(Q^2)$ and ρ (defined in the text).

Q^2	$\gamma(Q^2)$	ρ
0.95	0.886 ± 0.045	1.027 ± 0.010
2.2	0.808 ± 0.045	1.036 ± 0.008
3.7	0.828 ± 0.045	1.028 ± 0.007

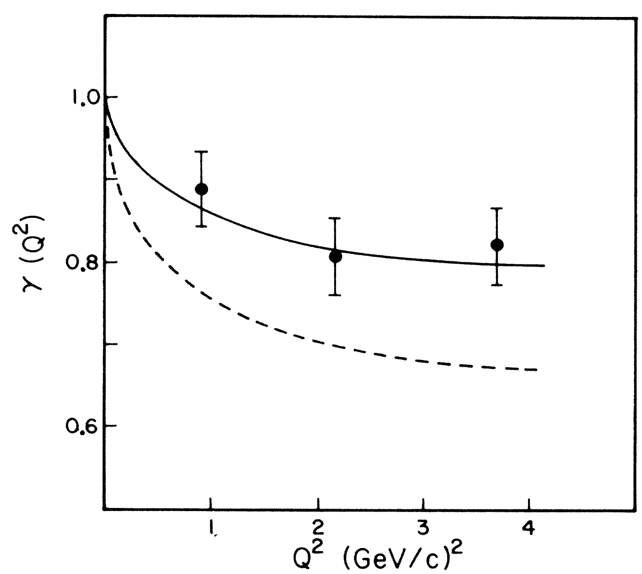

FIG. 1. Experimental values for the parameter γ as a function of Q^2 plotted with curves corresponding to $\rho = 1.031$ (solid) and $\rho = 1.06$ (dashed). See text for definition of symbols.

ρ to 0.008. Note that the normalization uncertainty allows for uncertainty in the free-nucleon cross sections[9] used in the analysis.

One point of concern is that the scaling function $F(y)$ at $y=0$ shows a significant 15% increase over the analyzed range of Q^2, which is not evident at lower y ($y < -0.02$ GeV/c). Thus there is some evidence that perhaps some other reaction mechanism (such as meson exchange or isobar effects) might be playing a role near $y=0$. The apparent extra strength in this region is estimated (from the amount of scaling violation) to contribute less than 3.6% to γ and so is not very significant at the 5% level which is of concern in this context. If one were to decrease γ at 3.7 $(\mathrm{GeV}/c)^2$ by 3.6%, the average value of ρ would increase by 0.2%.

I have also performed the analysis by simultaneously changing the nucleon mass ($\propto 1/\rho$) and the magnetic moment ($\propto \rho$) with the radius parameter as would be indicated by bag models. The result is $\rho = 1.017 \pm 0.003$, which suggests that such models could be used to infer lower values of ρ from these data.

In conclusion, it appears that the treatment using minimal assumptions gives the largest radius increase, while several modifications are possible to bring the radius parameter down as low as $\rho = 1.014$. The parameter ρ is thus constrained to the range $\rho = 1.025 \pm 0.011$ under a broad range of assumptions by the data considered in this work. In particular, the upper limit indicated by this treatment ($\rho < 1.036$) is considerably lower than previously obtained limits. Finally, forthcoming data of this type on heavier nuclei should yield information of similar quality with the same techniques, leading to a definitive determination of the properties of the nucleon electromagnetic form factors in nuclear matter.

I thank B. W. Filippone and S. E. Koonin for useful comments. This work was supported by National Science Foundation Grant No. PHY85-05682.

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