# Three-dimensional pictorial transmission in optical fibers

Amnon Yariv

*California Institute of Technology, Pasadena, California 91125*
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Modal phase dispersion limits image transmission in optical fibers to distances too short to be of general interest. A technique based on nonlinear optical mixing is described for modal phase equalization and recovery of a transmitted image.

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When we view a scene, say a tree in bloom, through a window, we are not usually aware of the seemingly obvious fact that the electromagnetic field containing the pictorial information has passed with minimal distortion through the glass window. The question then arises as to why we cannot view the same scene through a very long glass fiber, since the latter may be considered as merely a window of extremely small area and very large thickness.

A detailed theoretical analysis[1] of the transmission of pictorial information through transparent fibers shows that the image is invariably "smeared" in propagation through finite-diameter fibers. This smearing can be shown to result from the fact that, due to its finite cross-sectional dimensions, the fiber is not a spatially invariant system. An equivalent explanation is in terms of the phase velocity dispersion of the fiber modes which are excited by the picture field. This second point of view provides an especially simple description of the picture propagation and affords a means for calculating the typical "smearing" distance of a given fiber. We find that the distance depends on the fiber dimensions and index profile, on the wavelength, and on the number of resolution elements contained in the picture field. In a near-optimal case of a parabolic-index fiber with, say, 100×100 resolution elements, this distance does not exceed a few meters.

The modal description of the propagation of a picture field shows that fundamentally the picture is "smeared" due to the buildup (with distance) of phase differences between the many fiber modes excited by the picture. The problem of image restoration is thus reduced to one of *modal phase compensation*. One approach based on nonlinear optical mixing is described below. A second approach in which the compensation is accomplished holographically will be described elsewhere.[2]

Consider a multimode waveguide (fiber) which is capable of supporting $N\times N$ confined modes of propagation. A *coherent* picture field $f_0(x, y, t)$ limited to $N\times N$ resolution elements can be used to excite at some input plane ($z=0$) the modes of the waveguide. The excited field can be obtained by expanding the incident field (or more precisely, its spatially band-limited form) in terms of the waveguide modes $E_{mn}(x, y)$ as

$$f_0(x, y, t)=f_0(x, y)\exp(i\omega t)=\sum_{m,n=0}^{N} A_{mn}E_{mn}(x, y)\exp(i\omega t), \tag{1}$$

where $\omega$ is the radian frequency of the laser used to obtain the object field $f_0$.

The output field at $z=L$ is

$$f_1(x, y, t)=\sum_{m,n} A_{mn}E_{mn}(x, y)\exp[i(\omega t-\beta_{mn}L)], \tag{2}$$

where differential mode attenuation and intermode scattering are neglected.

In order that the transmitted field $f_1$ at $z=L$ be proportional to the input field $f_0$, it is necessary that the modal phase shifts $\beta_{mn}L$ differ by an integral multiple $2\pi$. This can happen only when

$$\beta_{mn}=\beta_{00}+I(m, n)a, \tag{3}$$

where $I(m, n)$ is an integer which depends on $m$ and $n$, and $a$ is any constant. The original picture field will be repeated (to within a complex proportionality constant) at planes $L_s$, where

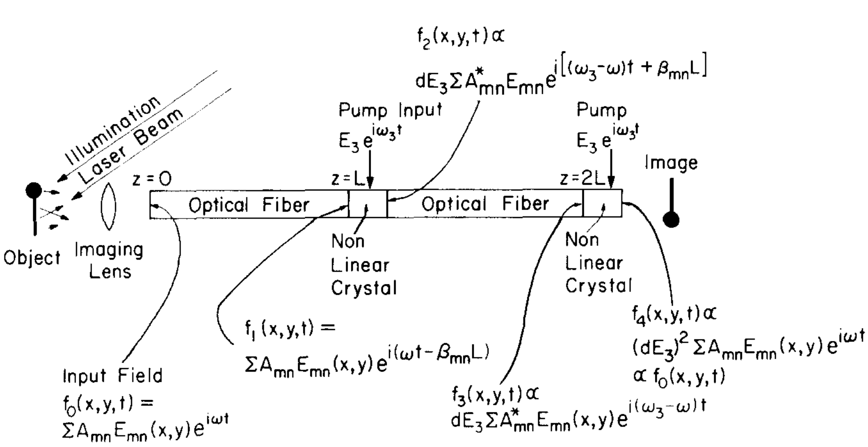


FIG. 1. Optical image transmission system using two segments of an optical fiber and two nonlinear crystals for modal phase equalization.

  

$$aL_s = 2\pi s, \qquad s = 1, 2, 3, \ldots$$

so that from (3)

$$\exp[i(\beta_{mn}L_s - \beta_{00}L_s)] = \exp[il(m, n)aL_s] = 1.$$

Unfortunately, condition (3) is not satisfied by real finite-diameter waveguides. As an example, let us consider a parabolic-index fiber. The dispersion relation[3] is

$$\beta_{mn} = k[1 - (2/k)(n_2/n)^{1/2}(m + n + 1)]^{1/2},$$

where $n_2$ is the quadratic index coefficient. $\beta_{mn}$ reduces to the form of (3) only if we limit ourselves to the first two terms in the Taylor expansion (which is equivalent to a paraxial ray treatment). The inclusion of the next term can be used to obtain the "smearing" distance

$$L_{\text{smearing}} \simeq \frac{2\pi^2}{\lambda(n_2/n)(m_{\max} + n_{\max} + 1)^2}.$$

The phase compensation scheme is illustrated by Fig. 1. The transmitted field $f_1(x, y)$ at $z = L$ enters a short section of a nonlinear optical crystal in which it "mixes" with an intense "pump" field $E_3(x, t) = E_3 \times \exp(i\omega_3 t)$ oscillating at $\omega_3$ to yield a field proportional to the product of the individual fields

$$f_2(x, y, t) \propto d[E_3 \exp(i\omega_3 t) + E_3^* \exp(-i\omega_3 t)] \times \{\sum_{m,n} A_{mn}E_{mn} \exp[i(\omega t - \beta_{mn}L)] + A_{mn}^* E_{mn} \exp[-i(\omega t - \beta_{mn}L)]\},$$

where $d$ is an appropriate nonlinear coefficient.[4] Restricting our attention to the difference frequency $(\omega_3 - \omega)$ term, we have

$$f_2(x, y, t) \propto dE_3 \sum_{m,n} A_{mn}^* E_{mn} \exp\{i[(\omega_3 - \omega)t + \beta_{mn}L]\} + \text{c.c.}; \qquad (4)$$

since $\omega_3 - \omega \simeq \omega$ we note that $f_2(x, y)$ is the complex conjugate of $f_1(x, y)$. This generation of a complex conjugate field, accomplished in holography by interfering a "signal" field and a "reference" field on a photosensitive surface,[5] is accomplished here *in real time* by nonlinear optical mixing.

A detailed derivation and justification of (4) in the case of multimode fields is contained in Ref. 1. The conjugate field $f_2(x, y)$ is next launched into the second leg of the propagation path. If the second segment is identical to the first, then after an additional distance $L$ we have, according to (2),

$$f_3(x, y, t) \propto dE_3 \sum_{m,n} A_{mn}^* E_{mn} \exp\{i[(\omega_3 - \omega)t + \beta_{mn}L]\} \times \exp(-i\beta_{mn}L) = dE_3 \sum_{m,n} A_{mn}^* E_{mn} \exp[i(\omega_3 - \omega)t]. \qquad (5)$$

A second stage of nonlinear mixing now yields the complex conjugate of (5):

$$f_4(x, y, t) \propto (dE_3)^2 \sum_{m,n} A_{mn}E_{mn} \exp(i\omega t) = d^2E_3^2 f_0(x, y, t),$$

so that the original three-dimensional object field is restored. The proportionality constant relating $f_4$ to $f_0$ may exceed unity since the nonlinear process is one of parametric amplification[4] so that the intermediate mixing steps may yield image intensification as well as phase compensation.

Two comments of practical interest should be made here: (i) It is implied in Eq. (4) that the various fiber modes undergo equal parametric amplification in the nonlinear crystal. Since the parametric amplification depends on phase velocity mismatch, which in this case depends on the modal dispersion, the condition of equal mode gain is approached in the limit of strong pumping where the parametric gain[3] $g_{\text{param}}$ exceeds the phase mismatch $\Delta\beta$. (ii) The process of parametric mixing leads to an intensification of the original picture field at $\omega$ as well as to the generation of the conjugate field at $\omega_3 - \omega$. The original field needs to be removed since its mode phases are wrong. This can be accomplished by orienting the crystal in such a way that the two fields are orthogonal to each other. The field at $\omega$ can be eliminated by a polarizer.

The proposed method of phase compensation can be viewed formally as a form of real-time holography. Beyond the specific implementation described above, the discussion is meant to draw attention to the inherent fundamental role of nonlinear optical operations in the transmission and processing of pictorial information.


[1] A. Yariv (unpublished).
[2] A. Gover, C.P. Lee, and A. Yariv (unpublished).
[3] A. Yariv, *Quantum Electronics*, 2nd ed. (Wiley, New York, 1975).
[4] J.A. Armstron, N. Bloembergen, J. Ducuing, and P.S. Pershan, Phys. Rev. **127**, 1918 (1962).
[5] See, for example, J.W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).