Journal Pre-proof

Tick size, price grids and market performance: Stable matches as a model of market dynamics and equilibrium

Charles Plott, Richard Roll, Han Seo, Hao Zhao

PII: S0899-8256(19)30115-0
DOI: <https://doi.org/10.1016/j.geb.2019.08.004>
Reference: YGAME 3034

To appear in: *Games and Economic Behavior*

Received date: 11 February 2018

Please cite this article as: Plott, C., et al. Tick size, price grids and market performance: Stable matches as a model of market dynamics and equilibrium. *Games Econ. Behav.* (2019), doi: <https://doi.org/10.1016/j.geb.2019.08.004>.

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2019 Published by Elsevier.

DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

Tick Size, Price Grids and Market Performance: Stable Matches as a Model of Market Dynamics and Equilibrium

Charles Plott

Richard Roll

Han Seo

Hao Zhao

California Institute of Technology

SOCIAL SCIENCE WORKING PAPER 1435

August 2019

Tick Size, Price Grids and Market Performance: Stable Matches as a Model of Market Dynamics and Equilibrium

Charles Plott, Richard Roll, Han Seo, Hao Zhao
California Institute of Technology
August 16, 2019

ABSTRACT

The tick size in a financial market is the minimum allowable difference between ask and bid prices. By the rules of each exchange, no transactions can occur within the tick interval. The impact of tick size is an ongoing controversy which we study by experimental methods, whose simplicity helps distinguish among competing models of complex real-world securities markets. We observe patterns predicted by a matching (cooperative game) model. Because a price grid interferes with a competitive equilibrium and restrictions on order flow interfere with information aggregation, the matching model provides predictions when the competitive model cannot, although their predictions are the same when a competitive equilibrium does exist. Our experiments examine stable allocations, average prices, timing of order flow, information flow and price dynamics. Larger tick size invites more speculation, which in turn increases liquidity. However, increased speculation leads to inefficient trades that otherwise would not have occurred.

1. INTRODUCTION: THE POLICY ORIGIN¹

Policy debates on optimal tick size have continued since the decimalization campaign in 1990s. According to Harris (1997)'s review on decimalization, the proponents of small tick sizes argue that they lead to smaller bid-ask spreads, thereby encouraging price competition and decreasing trading costs. The U.S. Securities and Exchange Commission (SEC) shared this viewpoint and since 2000, U.S. equity markets have reduced the tick size from 12.5 cents to a penny.

More recently, however, arguments have emerged to re-increase tick sizes (Weild, Kim and Newport 2012), supported by claims that small tick sizes repress initial public offerings (IPOs) for small cap enterprises and that wider spreads, by increasing profit for market makers, would

¹ The research support of the John Templeton Foundation and the Rising Tide Foundation is gratefully acknowledged. The authors thank Travis Maron and Hsing Yang Lee of the Caltech Laboratory for Experimental Economics and Political Science, for their assistance and advice. We thank Anmol Ratan and Jun Chen for their many helpful comments.

lead to more research and attract investors to small cap stocks, thus benefiting smaller companies. The SEC is skeptical.² Nevertheless, a pilot program of increasing tick size was initiated in 2016, despite the strong suspicions of the Commission.

Empirical studies in the academic literature lack consensus regarding the tick size. A smaller spread and improved price efficiency from small tick size have been reported (Harris 1994, 1997, 1999; Bessembinder 2003; Chakravarty, Wood and Van Ness 2004; Chung, Charoenwong and Ding 2004). However, research suggests that liquidity is harmed since the size of limit orders has become smaller after decimalization (Bacidore, Battalio and Jennings 2003). The effect on informed traders is mixed according to Gibson, Singh and Yerramilli (2003) and subsequent studies.³

Market microstructure theory also has reached no generally accepted conclusion. Liquidity providers and liquidity users, represented by market makers and public traders respectively, have been studied by Demsetz (1968), Garman (1976) and Kyle (1985) among many others. The consensus is simply that small price variation lowers trading cost and leaves more surplus for traders while large tick size provides incentives for market makers to provide liquidity.

Our paper rests on laboratory experiments and a classical model of demand and supply based on fundamentals. Subject suppliers privately buy units from the experimenter and sell the units for a possible profit in a public market to subject buyers. The buyers purchase from the sellers and

² The Jumpstart Our Business Startups Act (the JOBS Act) of 2012 has begun to address the issue. The SEC, according to their report responding to the Act (Report to Congress on Decimalization 2012) and a subsequent report (Recommendation of the Investor Advisory Committee Decimalization and Tick Sizes), demonstrated its belief that decline in smaller company IPOs is caused by exogenous factors other than decimalization and increasing tick size will only benefit market makers at the cost of retail investors.

³ Zhao and Chung (2006) test two contradicting hypotheses: whether decimalization increases the probability of information-based trading due to smaller spread or reduces it because of undercutting, and shows that the first effect dominates. Although a smaller tick size might have an effect on the IPO issuance of small cap companies, the decrease in IPOs is probably due primarily to the increased acquisition of small companies by large corporations (Gao, Ritter and Zhu 2013). In the recent literature of high frequency trading (HFT), Yao and Ye (2018) find that an increase in tick size hinders price competition, generates rents for liquidity provision and encourages speed competition, resulting in higher proportion of liquidity provided by high frequency traders. O'Hara, Saar and Zhong (2018) document the same advantage enjoyed by HFT from large tick size. In addition, their empirical results show that the positive correlation between tick size and market volume only exists when the minimum price variation is binding, and otherwise the opposite relationship is found due to greater adverse selection. Moreover, the literature on stock splitting (Anshuman and Kalay 2002), exchange fees (Chao, Yao and Ye 2015) and dark pools suggest that the market automatically adapts to the optimal tick size itself (Dayri and Rosenbaum 2015), which brings into question the necessity of a uniform standard in tick size.

privately sell the units to the experimenter for a potential profit. All participants are free to seek profits by buying and selling in the public market. The settings allow experimental control over incentives and thus the development of models to predict the outcomes as described in Sections 2 and 3. The competitive equilibrium framework envisions price adjustment toward an equilibrium and conditions that define equilibrium as constructed from experimentally induced parameters. The competitive model also suggests how asymmetric information can be transferred and aggregated through the price formation process. However, in the presence of a price grid, a competitive equilibrium need not exist which prevents the straightforward application of the classical approach.

In their recent paper, Hatfield, Plott and Tanaka (henceforth HPT) (2012, 2016) develop a matching model with a multiple commodity environment and study it experimentally. Their experiments examine cases where the competitive equilibrium exists and cases in which the existence is destroyed by price ceilings and price floors. The model is constructed from the theoretical contributions of Shapley and Shubik (1971), Kelso and Crawford (1982) and Hatfield, Kominers, Nichifor, Ostrovsky and Westkamp (henceforth HKNOW) (2013). In the multiple commodity environment studied by HPT, price controls create non-price competition reflecting different commodity qualities. The stable matches reflect different qualities that emerge in contracts when prices are not allowed to adjust. Existence of the competitive equilibrium is destroyed because a price control prevents contracts at profitable prices. Thus, HPT experiments suggest that the stable match model is a reliable, empirical generalization of the competitive equilibrium that applies when the competitive equilibrium does not exist.

This study rests on the generality of matching principles as suggested by HPT. It extends the investigation to a different environment and a new class of problems. The environment is a single commodity in which prices are restricted to a price grid that does not contain the competitive equilibrium prices. Thus, price grids can destroy the existence of a competitive equilibrium but price grids also impact all prices of a given commodity and thus impact the price discovery process itself. The competition between different permissible price levels of the same commodity is more complex, requiring a new understanding of the role of dynamics and information carried by prices. The analysis also uncovers an incompleteness of the model emerging from a combination of multiple unit incentives and the sequential nature of the trading environment.

2. MARKET ORGANIZATION, PRICE GRIDS AND STABLE OUTCOMES AS MARKET EQUILIBRIA

The paper is structured from two sets of experiments. A first set of experiments (series 1) explores the allocation and dynamic price adjustment consequences of a price grid. A second set of experiments (series 2) explores major implications of a price grid for the information aggregation and transmission predicted by the competitive model and associated order flow. All of the markets we study feature a computerized, continuous, double auction with an order book. Preferences are induced by financial incentives that differ in the two types of experimental settings. In the first setting (series 1), preferences are induced by individual endowments of demand and supply. In series 2, a randomly determined common dividend is paid with each trader having different private information about the dividend.

Series 1 is explained in detail first. The experimental settings, including procedures, parameters and models are explained for series 1 and then with that discussion as background the details of series 2 will be discussed. In both series 1 and series 2, the trading among agents takes place in a public market. However, in series 1, each agent has available a private side-market for interactions with the experimenter in addition to the public market in which trading with other subjects takes place. Subjects classified as “Sellers” in series 1 acquire units at fixed prices from the experimenter in the seller’s private side-market and sell them to other subjects for a profit. For convenience, we sometimes refer to the sellers as “producers” and the act of buying units from the experimenter as “production”. A seller keeps the difference between the price sold to a buyer and the price paid to the experimenter.

Subjects classified as “Buyers” make money by buying in the public market from sellers and selling acquired units at fixed prices to the experimenter in the buyer’s private side-market. The buyer keeps the difference between the price paid to a seller and the price received from the experimenter. Because of the multiple sides of multiple markets, it is sometimes convenient to think about the buy side of the market as “consumers” and the act of selling to the experimenter as “consumption”.

The values in the private markets for all series 1 experiments are in the appendix. Typically, private markets were limited to five units in series 1 and one unit in series 2. Speculation, buying and selling among the subjects themselves was limited only by potential losses from

“speculation”.⁴ All transactions were managed as purchases and sales through the electronic market that recorded all bids, asks and transactions and thus enforced the economy material balance equations (preventing sales of units or spending of money that does not exist) as well as accounting conventions.

We classify “Fundamentalists” as “Sellers” who never attempt to buy in the public market and “Buyers” who never attempt to sell in the public market.⁵ Since the values in their private market are known, they are not exposed to losses due to their own speculation. By contrast, “speculators” are defined conversely as sellers who initiate a buy order and as buyers who initiate a sell order in the public market. Speculative sellers who buy in the public market make a profit on the transaction only if they can resell at a higher price. Similarly, speculative buyers make a profit only if they buy it back at a lower price. Thus, profits emerging from speculative decisions depend directly on the ability of the speculator to anticipate the decisions of others.

Market supply and demand functions are derived from the induced preferences of individual sellers and buyers in a natural way. For purposes of exposition in the text presented here, the buyers and sellers have incentives for only one unit but the experimental incentives were based on multiple units. The potential costs to the seller of acquiring units from the experimenter, the individual seller’s marginal costs, are limit prices that can be configured into a market supply curve. That is, the marginal cost is the minimum price at which the seller can sell in the public market without making a loss. Let S denote the set of all sellers. Let c_j denote the marginal cost of seller j that the seller j finds in seller j ’s private market. For any price p , there is a subset of sellers $S_p \subseteq S$ such that $S_p = \{j \in S: c_j \leq p\}$. The aggregation of sellers’ limit prices is used to derive the market supply function $x = S(p) = |S_p|$. Namely, the market supply at price p is the number of sellers with marginal cost less than or equal to p . An example is the curve SS in Figure 1.

Similarly, the orders placed in individual buyer’s private markets are the value received from units acquired in the public market (the “utility” so to speak). Buyer buys in the public market and sells to the experimenter at the price found in the buyer’s private market, and keeps the profit.

⁴ Short sales were not allowed in series 1 experiments. A buyer who wants to sell units in the public market must acquire a unit first. A seller needs to purchase a unit in the private market or public market in order to resell in the public market. In series 2 experiments, short sale of 1 unit was allowed. In both series, an agent will start with a loan which must be paid back at the end of each period.

⁵ In series 2 experiments, there is no assignment of buyer and seller by the experimenter. However, as we will show in Section 6, the traders will group as buyers and sellers naturally based on their own private signal.

The marginal values found in the buyer's private markets, can be configured into a market demand curve. Let $\mathbf{B}$ denote the set of all buyers. Let v_i be the marginal value that i finds in buyer i 's private market. For any price p , there is a subset of buyers $D_p \subseteq \mathbf{B}$ such that $D_p = \{i \in \mathbf{B} : v_i \geq p\}$. The aggregation of buyers' preference is used to derive market demand function $x = D(p) = |D_p|$, as shown in Figure 1 as DD. Namely, the market demand at price p is the number of buyers with marginal value higher than or equal to p .

In a computerized double auction market, buyers and sellers tender time stamped offers for units at a stated price (limit orders) that enter the public order book with price priority followed by time priority. A sell order tendered at a price below the highest priced buy order in the book sells at that highest buy order price while a buy order tendered above the lowest priced sell order in the books sells at that lowest sell order price. The classical competitive equilibrium would be the price where the quantity demanded equals the quantity supplied which would correspond to (P^*, Q^*) in Figure 1 if the curves were continuous. The sequential nature of the experimental market creates a gap between the static model illustrated here and the actual underlying economy that will be an issue later.

Figure 1: Market Demand and Supply with a Grid. In the absence of a grid, the competitive equilibrium prices are between ($\text{Min}\{\text{last included demand unit, first excluded supply unit}\}$ and $\text{Max}\{\text{last included supply unit, first excluded demand unit}\}$). As for the case in Figure 1, the upper support of interval is the marginal value of last included unit in the demand curve and the lower support is the marginal cost of the last included unit in the supply curve. The competitive equilibrium quantity is set by the last included demand and supply units. The bold dashed lines signify the equilibrium based on the assumption that the curves are continuous.

When the market has price grids, orders must be tendered at discrete intervals. Examples are the light dashed lines as shown in Figure 1. When the competitive equilibrium price P^* is not on the grid, the competitive equilibrium does not exist. For example, in Figure 1 the competitive market price, P^* , is not on one of the dashed grid lines. The grid prices above and below the competitive equilibrium price are effectively floor and ceiling, bounds on prices, that prevent convergence.

In the classical experimental economics environment organized as a double auction the stable outcome is a natural generalization of competitive equilibrium that can be applied in the presence of price floors and ceilings. A modification can be applied to an analysis of the price grid imposed by a tick.. We adapt the HPT (2012, 2016) approach by modeling trading activity in a continuous double auction market as a matching model.⁶ Under the matching framework, bids and asks are interpreted as traders' attempts to form a match. A match happens when the buyer and seller agree on a price and quantity and sign an exchange contract accordingly. The stable outcome is a situation when no individual wants to leave a current match unilaterally or no pair of agents is willing to break their current match and form a new match with other agents.

3. THEORETICAL FRAMEWORK

A competitive equilibrium does not always exist when a price grid is present and typical convexity conditions are absent. Recent literature used principles from matching theories to construct models of markets without competitive equilibria due to price floors and ceilings.⁷ We inherit that modeling strategy as well as solution concepts to build the theoretical foundation for a market with a price grid.

3.1 Preferences

For purposes of model development, we assume that each buyer and seller deals with only a single unit. At a theoretical level, the assumption of a single unit is more than just an expositional simplification. Multiple unit traders are involved in multiple matches and in the simplified model,

⁶ HPT (2012) focus on single unit trades and use an assignment framework. HPT (2016) generalize the model to multiple unit trades and use a matching model framework.

⁷ The application to floors and ceilings focused on non-price competition in an economy with multiple qualities of a commodity traded in different markets, all of which were constrained by a price ceiling or price floor. The consequence of the control was to influence the pattern of commodity types transacted. The grid studied here has only one type of commodity and the entire range of possible prices for that commodity is impacted.

a value can be attached to each unit. Furthermore, the fact that trading takes place in real time creates theoretical complexities in multiple unit matching models.

Models of stable matching are based on the cooperative game model in which all options are available in a static sense as if the information about all potential contracts arrives simultaneously and before decisions are made, see HKNOW (2013) and HPT (2016). In such a static environment, trading sequence of multiple units is, therefore, not an issue. By contrast, a continuous market operates in real time with no schedule coordinating the timing of decisions. As a result, individual decisions need not conform to the consistency predicted by static theory and might be interpreted as a failure of the rationality conditions of the model. Suppose for example a buyer has utility for two units valued at 100 and 45. Suppose an offer to sell one unit at 60 is tendered and accepted by the buyer (profit $100 - 60 = 40$). Subsequently a sell offer for a unit at 40 is tendered and taken by the buyer (profit $45 - 40 = 5$) giving the buyer a total profit of 45. However, the buyer would prefer to buy only one unit at 40 which would produce a profit of 60 ($100 - 40 = 60$). In the absence of further investigation, the outcome is a violation of the rationality condition of the model. The agent appears to have chosen an option when a preferable option was available.

This issue of “ex post” regret is related to the fact that the agents have multiple items to trade and are forced to trade in a sequence dictated by the trading institution along with the choices of other traders. Since the agent in the example did not have all options available at the time of choice and had no reason to know that the option might become available, the choice does not violate the spirit of a rationality concept. The example illustrates that the matching model is incomplete when applied to sequentially operating processes and the phenomena can occur frequently. Looking ahead at our data, 24% of the trades involve outcomes where an agent would like to change the contracts if switching between his own units were possible. The phenomenon cannot occur if each agent has only one unit.

The multiple unit incentives induced in the experiment can be transformed to the single unit convention, but difficulties come from a combination of two sources. Firstly, the induced demand (supply) values have a declining (increasing) marginal property. Marginal demand (cost) values go down (up) as the number of units goes up, characterizing the declining (increasing) marginal utility (cost) or declining marginal rate of substitution feature of classical preference and production theory. Secondly, which is a fundamental source of theoretical challenges, bids and

asks are tendered in real time with no coordination or structured sequence. As a consequence, an agent might accept a contract that is regretted later when other alternatives become available.

We avoid the issue through the implementation of two conventions but draw attention to the possibility of gaps between the model and the experimental setting. The first convention assigns a value to the units and allows the unit to be assigned a contract if profitable. Thus, we depart from HPT (2016) and the associated model of stable matching by including an experimental fact of "sequence induced preferences" or "order dictated preferences". Values for buyers and sellers are assigned in an experimentally dictated order, meaning that the value of a unit to the buyer or seller is dictated by the number of units consumed or produced by the particular agent prior to the consumption or production of the unit in question. That is, the value of the y th unit to an agent is the marginal value created by the y th unit in an induced value or cost function. Since the marginal value decreases as units increase on the demand side and increases on the cost side the details must be acknowledged.

The second convention is to disallow the transfer of contracts among units. A contract for the y th unit cannot be transferred to the y' th. Substitution of contracts among units or individuals is not allowed. In the continuous double auction, contracts take place in a sequence. The transfer of a contract from one unit to another would be an implicit change of the trading sequence and would separate the market from the real time in which information and contract opportunities take place. It is as if each unit is identified as an agent and the non-transferability of a contract is a natural constraint since it would take place outside the market.

With the exception of speculation, the combined impact of the conventions is that each unit can be associated with its own value as determined by its order in the consumption/acquisition.⁸ Since the units have their own value there is no need to carry the identity of the (human) agent associated with the value. For theoretical purposes, we can index the demand side units from high to low and the supply side units from low to high. Therefore, the unit index becomes the agent. We continue by letting the unit be the agent, or equivalently assuming that each agent has only one unit.

⁸ Let the form of redemption value and cost functions be $H(T)$ where T the total number of contracts. Now let $h(y) = H(y) - H(y-1)$ so $H(T) = \sum_{y \in \{1, 2, \dots, T\}} h(y)$ and the value of unit y is $f(y)$. If $H(T)$ is interpreted as a utility function, then $h(y)$ is the marginal utility and if $H(T)$ is the cost then $h(y)$ is the marginal cost of the y th unit. The marginal value of a unit becomes the unit's own value and thus the unit can be treated as an individual with a value for a single unit.

A concern remains with this modeling strategy. Units from the same agent have an implicit order among themselves that the unit with higher redemption value must be matched earlier. This order may be disrupted if we simply treat the units as individual agents. As a result, our model may predict a larger set of stable outcomes than there should be as we ignore the implicit order of units from the same agents. However, such contracts cannot occur in the experimental data because by system design the multiple unit agents must redeem the value or cost along the ranking list. Therefore, our testing results of the convergence of empirical contracts into the stable outcome will not be affected.

As an example, assume a subject buyer has two units valued respectively at 100 and 65. Since each unit has its own value and acts as an agent, they can trade separately. Suppose an offer to sell at 60 is tendered and it is taken by the agent with a 65 value leaving the 100 value unmatched. The ex post regret problem would seem to occur if the match is interpreted as involving the single multiple unit agent or alternatively, it could be interpreted as an inefficient match if each unit is viewed as an agent. Interestingly, while the example occurs in theory it can never occur in practice because the unit purchased at 60 is automatically matched to the 100 value if the two values are held by a single person.

Given the assumption of single unit incentives, the market demand and supply functions can be derived. There is a finite set of buyers $\mathbf{B}$ and a finite set of sellers $\mathbf{S}$ trading the same good in a market. Let the value for the one unit of buyer agent i be $V(i)$ and the cost of the one unit of seller agent j be $C(j)$. Let $p(i,j)$ be the price paid by i to j for the one unit supplied to i by j . A trade results in the utility for the buyer equal to $u_i(p(i,j)) = V(i) - p(i,j)$ and the utility for the seller equal to $u_j(p(i,j)) = p(i,j) - C(j)$. The notation $p(i,i)$ means that agent i does not engage in a trade (trades with self) and thus make no gains from the match, namely $u_i(p(i,i)) = 0$.

Recall $D(p)$ is the number of buyers who have an induced demand values no lower than p and $S(p)$ is the number of sellers who have an induced cost no higher than p . For a given price p , the market demand and supply functions are:

$$D(p) = |\{i \in \mathbf{B} : V(i) \geq p\}| \text{ and}$$

$$S(p) = |\{i \in \mathbf{S} : C(i) \leq p\}|$$

The classical definition of competitive equilibrium (CE) is a mathematical property of the parameters. The CE price is a price p^* and a quantity q^* , such that $D(p^*) = q^* = S(p^*)$.

Given that integer values can be part of the analysis, more detailed definitions are needed for precision. The CE price is an interval⁹. This interval and its corresponding quantity will be called the CE if it is not constrained away by the price grid and the virtual competitive equilibrium, the VCE, if the price grid makes attainment impossible.

3.2 Matches, Contracts and Outcomes

A “match” results in a “contract” that specifies the parties and the terms of the trade. A “matching” refers to a process of proposing or seeking trading partners while contracts are the result. Denote a contract as $\langle (i,j), p(i,j) \rangle$, where (i,j) denotes the two agents, buyer i and seller j , and $p(i,j)$ is the trading price paid by i to j . A contract satisfies conditions of individual rationality, $u_i(p(i,j)) = V(i) - p(i,j) > 0$ and $u_j(p(i,j)) = p(i,j) - C(j) > 0$. An outcome in the market is a set of contacts that satisfy the resource constraint of agents (one unit per person) and are contracted at feasible prices (prices permitted by the conditions of a grid). Denote an outcome as G . The buyer i 's utility from the outcome G is $u_i(p(i,j))$ and the seller j 's utility from the outcome G is $u_j(p(i,j))$ where $\langle (i,j), p(i,j) \rangle \in G$.

3.3 Price Grid and Virtual Competitive Equilibria

Given price grids, the feasible prices take the form $p = m \cdot g$ where m is a natural number and g is the size of the grid. If $\mathbf{P}$ is the set of admissible prices, then $\mathbf{P} = \{p: p = m \cdot g \text{ for some natural number } m\}$. Suppose p^* is the equilibrium price of a model. If there exists integer natural number m^* such that $p^* = m^* \cdot g$, then, abstracting from the dynamics, the situation is the same with the no-grid case in the sense that the CE price is feasible. However, if such m^* does not exist then the CE price is not feasible and from the point of view of a model, the CE does not exist.

⁹ The definition of equilibrium in the presence of integer units is a bit more complex. Let z be an index of buy orders (bids) ordered from high to low and sell orders (asks), ordered from low to high. Thus, z is an index of ordered pairs $(b(z), a(z))$, where $b(z)$ is the bid, and $a(z)$ is the ask of the z^{th} pair. Let z^* be the smallest z for which $b(z+1) < a(z+1)$. Thus z^* is the index of the “last trade”, the last accepted bid and the last accepted ask. The competitive equilibrium price is any $p^* > 0$ such that:

- (i) For $z \leq z^*$, $b(z) \geq p^*$ and $a(z) \leq p^*$; and
- (ii) $p^* \in [\max\{b(z^*+1), a(z^*)\}, \min\{b(z^*), a(z^*+1)\}]$.

When the CE price is not feasible because of the grid, we identify the VCE price as p^* and we also identify two prices that are important, p^U and p^L . According to the definition of demand and supply set, we can find two price levels $p^U \in \mathbf{P}$ and $p^L \in \mathbf{P}$ as the lowest price and highest price, respectively such that:

$$p^U - p^L = g, D(p^U) < S(p^U) \text{ and } S(p^L) < D(p^L) \text{ and } p^U > p^* > p^L.$$

Namely at two feasible neighbor grids p^U and p^L , there is excess demand at p^L and excess supply at p^U . p^U and p^L are called the upper and lower bounding prices respectively. Notice that if the grid is size 0 the p^U and p^L can be interpreted as the upper and lower supports of the interval of CE prices that define the CE.

3.4 Stable Outcomes

When there exists a price grid such that the CE price is not feasible, we apply stable outcome to describe the equilibrium of the market. The stable outcome must satisfy two conditions:

S1: Non-blocking condition: An outcome G' blocks an outcome G if there is a buyer-seller pair (i', j') such that $u_{i'}(p'(i', j')) > u_{i'}(p(i, j))$ and $u_{j'}(p'(i', j')) > u_{j'}(p(i, j))$, where (i', j) and (i, j') are buyer-seller pairs matched in G or unmatched individuals if $i' = j$ or $i = j'$, and $\langle (i', j'), p'(i', j') \rangle \in G'$. An outcome G is not blocked if there is no other outcome G' that blocks G .

S2: Individual rationality condition: An outcome G is individually rational if for all $\langle (i, j), p(i, j) \rangle \in G$, $u_i(p(i, j)) > 0$ and $u_j(p(i, j)) > 0$.

Note that in a multiple unit environment, we treat each unit from the same person as an independent agent, therefore the individual rationality condition is different from that used in HPT (2016). We impose the individual rationality constraint for each unit instead of individual subjects who typically trade multiple units. By treating each unit separately, we are able to rule out ex post regret by subjects.

3.5 Characterization of Stable Outcomes

Our discussion will be confined to the conditions that are relevant to the limited conditions of our experiments. The demands and supply functions are linear. Figure 2 illustrates how the stable outcome will appear in series 1 experiments. Shown there is a VCE, P^* , bounded by the lower bounding grid, P^L , and the upper bounding grid, P^H .

Figure 2: Matching Model Allocation: The Stable Outcome

As can be seen, if a grid prevents the use of price levels between these two bounds, the CE price does not exist from the point of view of a market model. That is, the CE can be defined as a mathematical property of the parameters but it does not exist as a property of a model of market behavior. While the analysis above reflects incentives and values induced with certainty used in series 1, it also applies to series 2 in which individual agents receive different signals on the common value of the good.

The following propositions identify key features of a stable outcome in an environment structurally similar to the experimental environment. Proofs are contained in the appendix.

Proposition 1. The Competitive Equilibrium is a stable outcome

Proposition 2. Stable outcomes cannot involve contracts above p^U or below p^L .

Proposition 3. The stable outcomes always include buyers with values at or above the upper bounding grid or sellers with costs that are at or below the lower bounding grid or both. Thus, the limitation of values at or outside the lower and upper price bounds influences the proportion of trades at the two bounding prices.

Proposition 4: Based on induced demands and supplies alone (assuming there is no speculative buying and reselling), the theoretical equilibrium trading volume is in a range defined by intervals $[\text{Max}\{D(p^U), S(p^L)\}, \text{Min}\{D(p^L), S(p^U)\}]$.

Proposition 3 addresses the case of asymmetry between market demand and supply as well as relative to the VCE. As is illustrated in the figure, when there is more demanded at p^U than is supplied at p^L , not all buyers able to buy at p^L will find a seller who can sell at p^L . That is, there is limited supply at p^L which forces buyers to contract at p^U . The number of contracts that occur at p^U and p^L is dictated by relative slopes of the demand and supply functions. The matching model suggests that prices will bounce between p^L and p^U in proportions dictated by the demand and supply slopes as well as the range of potential volumes. Proposition 4 provides a partial characterization. The support is easily inferred from Figure 2.

4. EXPERIMENTAL SETUP

The general structure of the experiments is contained in Table 1. A total number of 13 experiments were conducted. The number of periods, size of grid, preference inducing method and

number of subjects all vary to allow a robustness check on the experimental parameters. We use Exp1.1 to refer experiment session 1.1, etc. for the others. Series 1 experiments included three sessions conducted with subjects from Purdue University. All series 2 experiments were conducted on site in the Caltech Laboratory for Experimental Economics and Political Science (EEPS)¹⁰.

Table 1: Experiments and Conditions

Series	Experiment	Date	Subjects	Environment	Periods	Grid size	Preference	Number of subjects
1	1.1	20160212	Caltech	Remote	[2,6] with grid [7-11] no grid	250	Private value	4B4S
1	1.2	20160224	Caltech	Remote	[3,6] with grid [7-11] no grid	250	Private value	6B3S
1	1.3	20160307	Caltech	Remote	[2,7] with grid [8-12] no grid	250	Private value	4B5S
1	1.4	20160325	Purdue	Remote	[2,6] with grid [7-10] no grid	20	Private value	6B6S
1	1.5	20160329	Purdue	Remote	[2,7] with grid [8-10] no grid	20	Private value	12B11S
1	1.6	20160331	Purdue	Remote	[2,5] with grid [6-10] no grid	20	Private value	10B10S
1	1.7	20160507	Caltech	Remote	[8,12] with grid [2-6] no grid	20	Private value	7B8S
2	2.1	20170804	Caltech	On site	11	10	Common value; Limited information	8
2	2.2	20170807	Caltech	On site	11	10	Common value; Limited information	8
2	2.3	20170808	Caltech	On site	11	10	Common value; Limited information	8
2	2.4	20170809	Caltech	On site	11	no grid	Common value; Limited information	8
2	2.5	20170810	Caltech	On site	11	no grid	Common value; Limited information	8
2	2.6	20170811	Caltech	On site	11	no grid	Common value; Limited information	8

Note: Period 1 is a trial period. Period 2 in Exp1.2 and period 7 in Exp 1.7 are dropped due to operational mistakes.

A grid can induce surprising behavior. It is similar to price ceilings and floors that are known to induce prices bounded away from constraints (HPT 2016, Isaac and Plott 1981, Smith and Williams 2008). Removal of binding constraints can cause explosive behavior.

¹⁰ Differences in laboratory facilities, subtle difference in instruction, procedures, software and subjects environment make no significant difference to the substantive conclusions.

All markets were conducted through Caltech's electronic market system, Marketscape. This program meets standard conditions for market experiment through a double auction market with an open book. All experiments here were conducted with an artificial currency called "francs". The parameters were set such that a subject would earn about \$4.50 per period if the market was 100% efficient.

Two different experimental series are reported. The first series consists of seven experiments, all of which use very similar demand and supply functions and differ primarily according to the scaling of units, the size of the grid and when and if the grid is imposed. The second series consists of six experiments with common values, limited private information about the common value and whether or not a grid is imposed

5: SERIES 1: INDUCED PRIVATE VALUE PREFERENCES

In this series, sellers acquire units for sale at fixed prices from the experimenter in their private side market. The seller keeps the difference between the price sold in the public market and the price purchased from the experimenter. Buyers make money by buying in the market from sellers and selling acquired units at fixed prices to the experimenter.

Exp1.1, Exp1.2, Exp1.3 and Exp1.7 were performed with Caltech undergraduates as subjects. Exp1.4, Exp1.5 and Exp1.6 were conducted with Purdue undergraduates as subjects.

The number of participants had an impact on volume and on the time required for market adjustments. The length of a period was 8 minutes for an experiment with about 8 subjects and it was 14 minutes when the number of subjects was around 20.

Preferences were not public. The only difference among periods within an experiment was the existence and size of the grid; the grid was 250 for some experiments and 20 for others. In some cases, different grid sizes are used in different periods.

Table 1 lists the size of the grid for each period of each experiment. As can be seen, some experiments start with a "large" grid for several periods after which the grid is reduced to 1. In other experiments, the experiment starts with a grid equal to 1 for several periods after which a larger grid is imposed. These manipulations are intended to provide insights about the impact of a grid and about grid size; they also open the possibility of observing any changes in equilibrium or equilibration due to experience with and without a grid, should such be an impact of the grid.

Individual preferences are always one of eight “types”. The letter B indicates one of four buyer types and the S indicates one of four sellers’ types. The numbers associated with a “type”, e.g. B1 or S2, identify parameters that are linear transformations of a common base. Thus, the parameters across experiments in the same series differ only by linear transformations (origin and scale) in the sense that all buyers and sellers in an experiment are related to the base by a common linear transformation. The scale is used for the selection of different grids across experiments. Period 1, always a practice period, has a special set of instructional parameters. The complete list of all parameters in all experiments is in the Appendix.

All parameters for experiments in series 1 are similar and as shown in Figure 1. Market demands and supplies were piecewise linear. There is a slight difference in demand and supply slopes beyond the competitive equilibrium in selected experiments. This was done to increase the excess demand and supplies at grid prices. The competitive equilibrium price was always near the middle between two grid prices. Series 1 experiments shed light on two questions. First, what is the impact of a price grid? Second, which model best captures market behavior in the presence of a price grid? A third question about the influence of a grid on the use of information is studied with the series 2 experiments.

The data pattern from a typical experiment is shown in Figure 3 and illustrates the patterns observed in all experiments. In the first several periods, bids and asks are distributed sparsely across a wide range of prices and on the grids (prices in units of 20) when grids are in place. As time goes by the orders (buy and sell orders) are more concentrated at the bounding grids around the CE price of 210. Actual contracts first appear over a wide range of prices and then move toward the bounding grids (200,220) as predicted by the matching model, with trading prices bouncing between the two bounding grids. After the grid is removed in period 8, prices soon move to inside the bounding grids and a convergence to the CE price is observed.

Figure 3: Market Data from Experiment 1.5. This figure illustrates typical pattern and how data are displayed for series 1 experiments. Price is on the vertical axis and time is on the horizontal; vertical dashed lines separate trading periods. The competitive equilibrium is approximately 210. The first period is a practicing period without a grid. After the first practice period a price grid of 20 is imposed and kept in place until it is removed in period 8. The small solid circles are orders (black are bids to buy and red are asks to sell), The larger open circles represent contracts. The black open circles are buyers accepting sellers' offers to sell (take asks). The open red circles are sellers accepting buyers' offers to buy (take bids).

Our first result reports on the convergence of market price when there is a price grid. The equilibration involves the dynamic properties of individual contracts in contrast to equilibrium, which is based on the static concept of a stable outcome. The distinction is subtle. While an outcome is a collection of contracts, the contracts themselves have the capacity to dissolve and reform and in an empirical sense that can be interpreted as stable or unstable. In that sense a contract is a concept based on terms (i.e. price) as opposed to the parties involved. Thus, we create the concept of empirical stability to characterize the property of contracts that endure over time. A contract is *empirically stable* if the price of this contract emerges again and becomes implemented after previously formed contract at that price has ended.¹¹ At the end of a period, all contracts are automatically broken. Empirically stable contracts involve prices that become part of reformulated contracts and are implemented in subsequent periods but might involve different pairing of people.

Empirically unstable contracts are those with prices that do not become part of new contracts. Instability of a contract occurs if at least one side of the contracting pair does not offer or declines the terms of the previous contract when offered. In operational terms, empirical instability of a contract is when only one side or neither side of a previously successful (but terminated) contract returns to the contract price (a buy offer or a sell offer of the terms found in previous contracts). Usually an offer is made but the other side does not respond by taking the offer. Instability therefore can be detected in terms of unaccepted bids and asks at previous contract prices.

Participants who find better partners do not return to previous contract levels and are not part of attempts to form contracts on previous terms. On the other hand, those who cannot find better opportunities return to the previous contract prices and try to establish contracts with offers (bids and asks) of previous terms. As offers fail to be accepted those making the offers initiate new offers that are worse for themselves but better for a possible counter-party.

The identification of empirical stability of contracts in the data follows the discussion above. Since only the contract price is relevant, we classify the contracts according to whether contracts reform at the same price in next period or unravel. Specifically, if we find no bids or asks at the contract price in next period then the contract is denoted as a *two-side unraveling contract*. If we find only one side returns, namely only bids or asks at the price in the next period, the contract is

¹¹ The dynamics are similar to the process of “recontracting” postulated by Edgeworth where the recontracting takes place at designated times and need not involve the same agents.

denoted as a *one-side unraveling contract*. If both bids and asks occur in the next period, the contract is reformulated and denoted as an *empirically stable contract*.

According to the prediction of Proposition 2, the stable outcome only contains contracts at the two bounding grids. Consider sellers and buyers who have contracts above the upper bounding grid. The next period the seller (or some seller) would return to that price with an ask, but buyers, having observed contracts at lower prices, would not respond with an accepting bid. The contract at that price would not be stable (empirically) because one of the parties would like to return but the other does not. Similarly, a contract below the lower bounding grid would unravel because the seller can find a better deal. Contracts will unravel at non-bounding grids and appear at the bounding grids as a convergence process.

These phenomena are summarized in Result 1. As indicated by the example above, the one-sided nature of the dynamics of an unstable contract helps provide an intuition about price movement. Instability is mostly one-side unraveling rather than two sided. Typically, one side finds a better contract and does not want to return while the other side is not able to find a favorable alternative and wants to return to the old contract. Figure 4 shows that this is most often the case. Two-side unraveling contracts account for only 4% of all the contracts across all experiments, compared to 23% for one-side unraveling contracts.

Figure 4: Distribution of Empirically Stable and Unstable Contracts at Different Price Levels

Result 1. (i) The prices that are not equal to bounding grid prices are empirically unstable and (ii) empirically stable contract prices converge to the bounding grids.

Support. To show that contracts at the non-bounding prices are unstable, we consider three different price groups (i) contracts at prices above the upper bounding grid, (ii) contracts at prices below the lower bounding grid and (iii) contracts at the bounding price grids. In addition, to be complete, we check the quantity of each type of contract at the upper bounding grid, at the lower bounding grid, one or two grids above the upper bounding grid and one or two grids below the lower bounding grid. Very few contracts exist at farther grids.

Figure 4 shows that there are fewer empirically stable contracts at non-bounding grids than at the two bounding grids. For contracts at the bounding grids, the percentage of empirically stable contracts is 84%, much greater than 56% for contracts at prices higher than the bounding grids and 16% for the contracts at prices lower than the bounding grids. In addition, the absolute number of empirically stable contracts is larger at bounding grids than non-bounding grids (459 vs. 94), despite the fact that the bounding grids only contain two prices while the non-bounding prices could be at any other price level.

Contracts at non-bounding prices are unstable since they fail to reappear in next period. As a convergence process, empirically stable contract prices tend to be at the two bounding grids in later periods. To show the trend statistically, we apply the Ashenfelter-El-Gamal (AEIG) test¹² to analyze the effect of time:

$$Y_{kt} = \rho_{11}d_1\left(\frac{1}{t}\right) + \rho_{12}d_2\left(\frac{1}{t}\right) + \dots + \rho_{1k}d_k\left(\frac{1}{t}\right) + \dots + \rho_{1K}d_K\left(\frac{1}{t}\right) + \rho_2\left(\frac{t-1}{t}\right) + \varepsilon$$

where k indicates the particular experiment, t represents time as measured by market periods in the experiment and K is the total number of experiments. d_k is a dummy variable that takes 1 for experiment k and 0 otherwise and ρ_{1k} is the origin of a possible convergence process. Notice that if $t = 1$ then the value of the dependent variable is equal to ρ_{1k} for experiment k . ρ_2 is the asymptote of the dependent variable. As t gets large the weight of ρ_{1k} is small because $\frac{1}{t}$ approaches zero while the weight of ρ_2 approaches 1. Notice that ρ_2 is common to all experiments. Finally, ε is the random error term that is distributed normally with mean zero.

¹² Noussair, Plott and Riezman (1995) first employ this method to study convergence of market data.

To make comparison across experiments and pool the data, we look at contracts in different price groups (above, below and at the bounding grids), without referring to the exact contract price, whose magnitude varies in different experiments. We measure the empirical stability of a set of contracts observed each period by the proportion of empirically stable contracts over all contracts in that set. We want to show that the empirical stability of contracts at non-bounding prices decreases over time, while the empirical stability of contracts at bounding grids increases or remains high.

Table 2 presents the estimated coefficients from the AEIG test. As indicated by the estimation of coefficients (ρ_{11} to ρ_{17}), stability at the bounding grids stays at very high level. In all experiment, it starts at a minimum of 61.9% and converges to 90.8% (ρ_2), with all the estimated coefficients significant at 99% level. Note that the estimated empirical stability, the ratio, can exceed one or fall below zero because we use a linear fit. For the group of contracts with prices above the bounding grids, there is a decreasing trend of stability for 5 out of 7 experiments. For example, the estimated coefficient starts at 87.5% in experiment 2 and converges to 42.4% in the end. For the group of contracts with prices below the bounding grids, there is no obvious trend since the coefficients are not statistically significant, but stability stays low for all the experiments (the highest level is 35.8% in Exp1.1).

Table 2: Convergence of Empirical Stability at Different Price Levels

	ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}	ρ_{17}	ρ_2	n	R ²
Empirical stability at the bounding grids	0.880*** (0.14)	0.826*** (0.14)	1.005*** (0.19)	0.891*** (0.14)	0.964*** (0.19)	0.747*** (0.14)	0.619*** (0.12)	0.908*** (0.06)	54	0.942
Empirical stability above the upper bounding grid	0.83 (0.79)	0.875** (0.38)	1.165*** (0.39)	0 (0.32)	0.559*** (0.19)	0.567 (0.34)	-0.6 (0.88)	0.424** (0.19)	33	0.666
Empirical stability below the lower bounding grid	0.358 (0.26)	-0.386 (0.85)	0.241 (0.27)	-0.0236 (0.23)	-1.157 (1.81)	0 (0.40)		0.386 (0.28)	16	0.429

Standard errors in parentheses. *** pr<0.01, ** pr<0.05, * pr<0.1. n is the number of observations (periods) included in each regression.

We now establish the link between empirically stable contracts and the stable outcome defined by theory. Result 2 illustrates that the repeated contracts we observe until the end of the experiment (empirically stable contracts) are the stable matches predicted by theory.

Empirically unstable matches are revealed by unsuccessful attempts to reestablish them while empirically stable matches are visibly successful. The empirical stability defined in Result 1 is based on whether a contract reforms or not. In fact, the theory developed in previous section also makes predictions about the stable outcomes. According to Section 3, contracts in a stable outcome must have two properties. (1) Stable contracts must be priced at two bounding grids (no blocking pair) and (2) stable contracts must be profitable to both sides (individual rationality). Therefore, we can identify whether a contract is in the stable outcome using the same criteria.

The first criterion is easily checked, since we can directly compare the price of each contract with the bounding grids. The second criterion requires supplementary information, since the traders' profits are not specified in the contract. As buyers' values are determined by the prices, they sell to the experiments in the private market and sellers' costs are determined by the prices they buy from the experiments in the private market, we can track the redemption value and cost of each unit and associate them with each contract based on the time of trade execution. In the end, a contract is deemed to be in the stable set if it is priced at the bounding grids, the redemption value is higher than the contract price, and the cost is lower than the contract price.

Besides the stable outcome, during the converging process, there will also be unstable matches that will eventually disappear. In addition, since we don't restrict the direction of trading in the public market, participants could trade on both sides of the market in an attempt to speculate. When the trading prices are bouncing between two bounding grids, speculators in theory could make a profit if they buy at the lower bounding grid and sell at the higher bounding grid. Therefore, we don't necessarily anticipate that speculative contracts will completely disappear with time.

The primary research interest is to test the explanatory power of matching model in a market with price grids. The major prediction of the model is a stable outcome. If the theory correctly describes a market with grids, the empirically stable contracts observed should be elements in the stable outcome. Hence, we want to test whether the empirically stable contracts are consistent with the theoretical prediction. Notice that, the empirically stable contract is only identified by its price. Therefore, an empirically stable contract observed over two periods could come from two different stable outcomes since the associated buyers and sellers may be different.

However, that does not affect our conclusion that the empirically stable contracts are consistent with contracts that form a stable outcome. The finding is summarized in Result 2.

Result 2. Empirically stable contracts converge to contracts predicted by predicted by stable outcome.

Support: In the experiments, we observe two types of contracts, empirically stable and unraveling contracts, defined by whether they reform in next period or not. On the theoretical side, we define three sets of contracts, the *stable set* (stable outcome), *unstable set* and *speculative set*. For contracts involving only fundamentalists, we divide them into stable and unstable sets. The stable set is the set of contracts that belong to a stable outcome; the unstable set is the set of contracts that are not in the stable set.. We define the set of contracts associated with speculative trading (at least one agent in the contract is not a fundamentalist) as speculative set. Speculation creates contracts that are not stable however speculative contracts might repeat over time and appear as empirically stable contracts. Each contract in the data should have both an empirical identification and a theoretical identification. Result 2 states the prediction that the empirically stable contracts fall into the stable set as the outcome of convergence.

Table 3 is a cross tabulation that summarizes number of each type of contract pooling all experiments. Putting aside the speculative set, we find that more unraveling contracts fall into the unstable set than the stable set (88 vs. 32) while more empirically stable contracts fall into the stable set (229 vs. 134). Alternatively, stated in the opposite way, the contracts predicted stable by the theory are more likely to reform in the experiment, while those predicted to be unstable are more likely to disappear.

Table 3: Empirically and Theoretically Stable Contracts

		Theoretical			
		Unstable set	Stable set	Speculative set	Total
Empirical	Unraveling	88	32	87	207
	Stable	134	229	194	557
	Total	222	261	281	764

All contracts belong to one of six categories. Our primary interest is to ascertain among all the empirically stable contracts how many fall into the stable set. Hence, we calculate the percentage of empirically stable contracts in the stable set over all the empirically stable contracts. The AEIG test is applied to test the trend. To make a comparison, we also test the trend of changes of stable contracts in other sets. As showed by Table 4, the proportion of empirically stable contracts in the stable set displays an increasing trend in all experiments except Exp1.2. It starts at 26.5% in Exp1.1 and converges to 65.7% by the final. In Exp1.2, it starts at 70.8%, a very high level and stays high until the end. Contrary to the stable set, empirically stable contracts in the unstable set experience an obvious declining trend. It starts at 67.5% over all empirically stable contracts in experiment 1 and ends up at 17.1%. The empirically stable contracts in speculative set show a mixed trend. The percentage decreases over time in Exp1.4, Exp1.5 and Exp1.6, and increases but stays low in all other experiments. In conclusion, the contracts repeatedly observed in the experiments are consistent with the contracts predicted by the stable outcome.

Table 4: Trend of Empirically Stable Contracts in Theoretical Sets

	ρ_{-11}	ρ_{-12}	ρ_{-13}	ρ_{-14}	ρ_{-15}	ρ_{-16}	ρ_{-17}	ρ_{-2}	n	R ²
Stable contracts in the stable set	0.265 (0.21)	0.708*** (0.21)	0.363* (0.21)	0.0941 (0.21)	-0.132 (0.21)	0.138 (0.21)	0.0831** (0.04)	0.657*** (0.10)	29	0.86
Stable contracts in the unstable set	0.675*** (0.10)	0.311*** (0.10)	0.733*** (0.10)	0.147 (0.10)	0.308*** (0.10)	0.175 (0.10)	0.00176 (0.02)	0.171*** (0.05)	29	0.9
Stable contracts in the speculative set	0.0673 (0.12)	-0.0135 (0.12)	-0.0877 (0.12)	0.766*** (0.12)	0.833*** (0.12)	0.693*** (0.12)	0.00884 (0.12)	0.157** (0.06)	29	0.887

Standard errors in parentheses. *** pr<0.01, ** pr<0.05, * pr<0.1. n is the number of observations (periods) included in each regression.

The next two results compare our contract prices with CE prices. Due to the discontinuity of the value functions, CE prices form a range. When there is a price grid, contract prices cannot equal CE prices unless the prices are on the grid, but the CE prices can be computed from the parameters and referenced as the VCE, virtual competitive equilibria. For each period, we calculate the average price of completed trades, denoted the Average Transaction Price (ATP). We

compare the average contract prices in a period with the CE when it exists and VCE otherwise. Then we ascertain whether the ATP lies within the interval of prices defined by the VCE.

Result 3 reveals that prices approach the interval (range) of CE prices. This property emerges when the grids do not exist, which is sometimes imposed when the market opens and when the grids are removed after having been in place for a number of periods. The second result, Result 4, tells us that the ATP is close to the VCE when grids are in place.

Result 3. When the CE, the Competitive Equilibrium, exists the CE prices emerge as contract prices. The emergence occurs when grids do not exist at the market opening and when previously existing grids are removed.

Support. The grids and units differ across experiments so the patterns are most easily seen in terms of the pricing. If the average price is within the interval defined by the CE price range, then the price converges perfectly and we denote the error as being zero. If the average price is not in the interval, then we calculate the difference between the average price and the competitive equilibrium price range and scale it across experiments by dividing it by the average price.

An AEIG test demonstrates that the difference between the ATP and the CE price range shrinks with time. As shown in Table 5, coefficients for the starting period in Exp1.1 to Exp1.6 are not statistically different from 0, meaning that when the grids are removed the ATP falls into the set of competitive equilibrium prices immediately. An exception is Exp1.7, which began with no information about the bounding grids. In that experiment, the AEIG test shows a clear declining trend of deviation ending at 1.12% in the last period, which is statistically indistinguishable from zero.

In addition to the deviation from the ATP, we also examine the variation of price changes over time by measuring the relative standard deviation of price. As the AEIG test shows, for Exp1.1, Exp1.2, Exp1.3 and Exp1.7, there is a clear trend that the variation of prices decreases over time. Combining this with the fact that ATP is close to the CE, we conclude that contract prices converge to CE in general.

Table 5: Deviation of ATP from CE

	ρ_{-11}	ρ_{-12}	ρ_{-13}	ρ_{-14}	ρ_{-15}	ρ_{-16}	ρ_{-17}	ρ_{-2}	n	R ²
ATP-VCE /ATP	0.0323 (0.02)	0.0198 (0.02)	0.0241 (0.02)	0.00167 (0.02)	-0.0026 (0.02)	0.00689 (0.02)	0.253*** (0.02)	0.0112 (0.01)	33	0.847
Relative standard deviation of price	0.0645* (0.03)	0.131*** (0.03)	0.0764** (0.03)	0.02 (0.03)	0.01 (0.03)	0.03 (0.03)	0.0859** (0.03)	0.0337** (0.01)	33	0.721

Standard errors in parentheses. *** pr<0.01, ** pr<0.05, * pr<0.1. n is the number of observations (periods) included in each regression.

The next result, Result 4, involves the information content of the average transaction price focused in periods with a grid. Although a competitive equilibrium does not exist, the VCE () is positioned where the CE would exist if there were no grid. Result 4 has two parts. The first part states that the Average Transaction Price (ATP) converges close to the VCE. The second part demonstrates that the VCE is a better predictor of the ATP than is the midpoint of the bounding grid prices, the average boundary price (ABP).

Result 4. When a price grid is in place (i) average transaction prices (ATP) converge to the VCE and (ii) the VCE is a better predictor of the ATP than is the midpoint of the bounding grid prices (ABP).

Support. Figure 5 depicts the convergence of average price by pooling the data across different experiments. We calculate |ATP-VCE|/ATP and |ATP-ABP|/ATP, which is the value on the vertical axis. With the exception of period 2, the ATP is always closer to VCE than it is to the ABP. The difference between the ATP and VCE disappears by period 6, implying that even with a price grid, the average transaction prices in the market converge to the CE price.

Figure 5: Convergence of Average Contract Price with a Price Grid. The value on the vertical axis is the percentage deviation from the average trading price. Black line is the deviation of VCE and the dashed line is the deviation of ABP.

Results 1 through 4 establish that the major pricing features of the stable outcome emerge. Stable outcome also involves specific predictions about the actions of agents. However, all predictions are complicated by speculation, liquidity provision and other seemingly extraneous behavior. The next result presents a clearer view of the relationships. The grid does increase speculation and liquidity consistent with arguments often advanced. However, the mechanism appears to be through exposure to risk and associated losses. The next result summarizes the relationships observed in the markets.

Result 5. Trade volume increases with the existence of a price grid as compared with the situation without the grid. The existence of a grid increases speculation and the associated losses due to speculation serves as a subsidy to trading and increased volume.

Support: A brief review of the data is needed. The parameters change across experiments. For each separate experiment, since the number of participants and preference parameters may differ, the predicted volume of trades and other activities (i.e. speculation) can be different. However,

within each experiment, the parameters do not change whether or not a grid exists. This allows us to make a comparison of trading volume and intensity of speculation within an experiment by studying the periods with a grid and those without a grid.

The average measures are constructed in the following way. For each period, we divide contracts with multiple units into multiple contracts with a single unit. We then identify the type of a contract as f if both sides of that contract are fundamentalists, fs if one side is a fundamentalist and the other side is a speculator and s if both sides are speculators. We let the variables f^t , fs^t and s^t denote the number of contracts of each type in period k . We let T^t be the total number of contracts in period t so $f^t + fs^t + s^t = T^t$. Denote the total number of traders in period t as N^t .

The contracts per trader in period t (the average number of contracts made by each trader) is $2T^t/N^t$. The number of speculative contracts per trader in period t is $(fs^t + 2s^t)/N^t$. For each speculative contract, we calculate the speculator's profitability in US currency (dollars). We calculate the net profits from speculation in period t , denoted as speculation payoff, $s\pi^t$. The overall surplus of the fundamentalists are enhanced by the subsidy from speculators. We let the total social surplus be π^t in period t . The fundamentalists' total surplus is $\pi^t - s\pi^t$. The ratio $(\pi^t - s\pi^t)/\pi^t$ measures the level of subsidization that exacerbates the fundamentalists' surplus in period t . Since speculators usually suffer losses from speculation, they subsidize the fundamentalists and possibly attract more units in the market.

We compare the subsample of periods with price grids (37 periods) and the subsample of periods without price grids (32 periods). Note the sample size of periods when speculation exists is smaller because not all periods have speculation.

Table 6 presents t-tests on volume in terms of contracts per trader, speculative contracts per trader, speculation payoff and level of speculators' subsidization in the periods with and without a price grid. Both the number of contracts per trader and speculative contracts per trader are larger in periods with a grid (3.59 and 0.60) than those without a grid (3.36 and 0.48). Based on the speculation payoff, speculators in a period with a grid lose 5.82 USD in total, which is more than three times their loss in a period without a grid (1.60 USD). The speculators' loss subsidizes the fundamentalists. As shown by the level of speculators' subsidization, in a period with a grid, the ratio of fundamentalists' aggregate profits over social surplus is 1.12, which is larger than the ratio in a period without a grid (1.03). The difference is statistically significant at 95% level as 1.12 does not fall into 1.03's 95% confidence interval.

Table 6: Relationship between Grid and Volume

	Without Grid			With Grid		
	Number of Periods	Mean	Confidence Interval	Number of Periods	Mean	Confidence Interval
Contracts per trader	32	3.36 (0.13)	[3.10, 3.62]	37	3.59 (0.15)	[3.29, 3.89]
Speculative contracts per trader	32	0.48 (0.11)	[0.25, 0.71]	37	0.60 (0.13)	[0.33, 0.86]
Speculation payoff	19	-1.60 (0.84)	[-3.36, 0.17]	24	-5.82 (1.75)	[-9.45, -2.19]
Level of speculators' subsidization	19	1.03 (0.01)	[1.00, 1.06]	24	1.12 (0.04)	[1.04, 1.19]

Mean reports the average of the variables across periods. Standard errors in parentheses. The confidence interval is at 95% level.

Contracts per trader is the average number of contracts per trader in each period: $2T/N$.

Speculative contracts per trader is the average number of speculative contracts per trader in each period: $(fs+2s)/N$.

Speculation payoff is the total payoff of speculators in each period in terms of USD: $s\pi$.

Level of speculators' subsidization is the ratio of fundamentalists' aggregate profits over social surplus in each period: $(v-s\pi)/v$.

The result is understandable in terms of the overall model. The fact that grid invites more speculation means that the grid serves as a central mechanism to attract excess demand and supply. As we defined in Section 2, “fundamentalists” are sellers who never attempt to buy in the public market and buyers who never attempt to sell in the public market. By contrast, “speculators” are sellers who occasionally buy or buyers who sell. When a grid exists, both more fundamentalists and speculators trade more often. Thus, there is increased overall trade volume. In general, a larger tick size seems to attract speculators by suggesting more profit opportunities. Speculators bring more trade volume not only because they trade with both sides of the market, but also because they tend to hang onto inventory by the end of the experiment (due to bad management). This introduces additional demand and supply, which arises despite speculators' losses.

According to Proposition 4, the maximal volume predicted by the matching model is larger than the theoretical volume under CE. The extra volume induced by grid comes from inefficient supply and demand, which would not be traded without the price grid. The matching model hence predicts possibly decreased social welfare attributable to a grid.

The next result connects the properties as features of market dynamics.

Result 6. Efficiency increases with period (learning) and decreases with a grid.

Support. The underlying economic parameters are the same whether or not a grid is in place. Thus, from a technical point of view, total achievable surplus is identical with or without the grid. Similarly, the efficient allocation is included as a stable outcome when the grid is in place. However, because the efficient allocation is only one of the many equilibria when the grid is in place and there is no mechanism that naturally guides exchanges between specific partners, maximal surplus is achieved with a grid only due to sheer luck.

Sometimes subjects carry a leftover inventory at the end of the experiment, which clearly decreases their profit. Therefore, when calculating the social surplus, we adjust the data assuming that any purchases from the experimenter that resulted in a leftover inventory have no effect on social surplus,

We perform a regression on adjusted surplus by the existence of grid and different periods, with experiment fixed effect using the model:

$$\text{Adjusted Surplus} = \alpha + \beta_1 * \text{Grid} + \beta_2 * \text{Period} + \text{FE}_{\text{Exp}}$$

where FE_{Exp} are experiment specific effects.

The existence of grid is associated with loss in surplus at a significance level of 0.05 as can be seen from the first line of Table 7. Furthermore, each period is also associated with a gain in surplus at a significance level of 0.0432 as can be seen from the second line of Table 7; This suggests that subjects are learning in repeated episodes of trading.

Table 7: Factors Affecting Social Surplus

Coefficients	Estimate	Std. Error	t value	Pr(> t)
Grid	-307.25	154.75	-1.986	0.0517
Period	55.95	27.09	2.066	0.0432 *
Constant(α)	13094.92	282.58	46.34	<2e-16 ***

Other data that supports the idea that a grid leads to increased unwelcome liquidity is that the number of fundamental trades is positively correlated with speculative trades. When we perform a regression on the number of fundamental trades to speculations with the model:

$$\text{Fundamental trades} = \alpha + \beta_1 * \text{Speculation} + \beta_2 * \text{Number of subjects}$$

We observe a significant positive correlation between the two at a significance level of 0.02 as can be seen from the first row of the Table 8. In other words, the price grid invites more

speculation which may subsidize the fundamental traders and attract inefficient units into the market. This as a result lowers social surplus.

Table 8: Speculative Contracts Increase Fundamental Trades

Coefficients	Estimate	Std. Error	t value	Pr(> t)
Speculative contracts	0.10829	0.04526	2.393	0.0196 *
Subjects	1.2637	0.06852	18.443	<2e-16 ***
Constant(α)	1.83665	0.77188	2.379	0.0203 *

6. SERIES 2: TRADING WITH UNCERTAIN PAYOFFS AND ASYMMETRIC INFORMATION

The previous sections highlight the importance of matching principles in the dynamics of price adjustment. This section expands the scope by allowing limited and heterogeneous information across agents. The asset now pays a common dividend about which each individual has only a small amount of information; indeed, information about the dividend is quite limited even after it is aggregated across agents.

It is well known that individuals with asymmetric information can improve their knowledge through market interactions. It is also thought that the information is carried by the bids and asks in the order flow. If and how that phenomenon might survive the imposition of a grid that restricts order flow are central questions. Thus, we now study information aggregation in the presence of a grid and if matching theory helps to understand the process.

In these experiments limited information, six experimental sessions were conducted, each with eight Caltech undergraduates and graduate students. Each experimental session had twelve independent periods including a first practice period. Three experimental sessions were conducted with a grid and three without. In experiments with a grid, the minimum price difference is 10.

Prior to the opening of a period, each subject was endowed with four units of the asset and was given private information about the probability of the dividend that period. Negative inventories, short sales, were allowed only up to -1. In addition to the initial asset endowment, each subject started with a 4,000 franc loan, which was repaid at the end of each period. Each unit of the asset held at the end of the period paid a single dividend the holder. The dividend was randomly determined each period from a uniform distribution between 100 and 400 and paid at

the end of each period. For comparison across experiments, the dividend and private signals were identical in pairs of sessions, one with and one without a grid, while they differed across pairs, (Exp2.1, Exp2.6), (Exp2.2, Exp2.5) and (Exp2.3, Exp2.4) .

At the beginning of a period, each subject received a private signal drawn randomly from a truncated normal distribution ($\sigma=60$) centered on the actual future dividend (known only to experimenter at the period's beginning.) Each subject was given a personalized table with the Bayesian posterior probabilities calculated for each bin of value. Subjects' ability to perform probability updating is not a variable. The draws and the information are contained in the Appendix. Figure 6 illustrates the data and order flow produced in a market with a grid. Sell orders are red and buy orders are blue. Contracts are the large circles. Periods are separated by the vertical lines. The tick size is 10, as revealed by the horizontal lines where every order and trade is located.

As the figure reveals, contracts are signed at more widely scattered prices in the early stage of each period. In addition, the ratio of the number of orders to the number of contracts changes within a period. In the early stage when subjects have limited knowledge from the activities of other traders, there appears to be more hesitation to hit a bid or ask and thus sign a contract. In the later stages, if an equilibrium emerges, traders issue bids and asks that are more likely to be taken. As a result, if an equilibration emerges there is a higher offer-to-trade ratio in the early stages of a period and a decline in the ratio later. The tighter patterns of contracts near the end of the periods give an impression of market equilibration.

Figure 6: Market Data from Experiment 2.2. This figure illustrates a typical pattern and how data are displayed for series 2 experiments. Price is on the vertical axis and time is on the horizontal; vertical solid lines separate trading periods. Each black dot is a bid, each red dot is an ask, each empty black circle indicates that a ask is taken and each empty red circle indicates that a bid is taken. Grid size is 10.

6.1 Stable Outcome as the Equilibrium

With known values (as in series 1), stable outcomes are related to the CE when no grid exists and to the nearest bounding grids on both sides of the VCE when grid exists.

Under conditions of uncertainty, candidates for the CE and VCE are unknown but theory suggests they might be inferred from private information held by participants. If participants have no other information, the expected dividend can be used as a measure of limit prices. Those with relatively high signals should be the demanders with the signal magnitude ranking buyers from high to low, thereby creating a demand function. Those with relatively low signals ranked from low to high create a supply function. The candidate CE or VCE based on private information would be at the intersection of these two functions. Of course, market participants also observe the flow of bids and asks and presumably modify their prior beliefs (and the demand and supply functions) in response.

The next result demonstrates that subjects self-divide into demanders and suppliers based on the private information they receive. That is, high private signals create demanders and low private signals create suppliers and the median private signals are held by traders at the intersection of the implicit supply and demand functions. These latter traders are the agents whose private signals and thus expected values place their limit prices closest to the CE or the VCE.

Result 7: Private signal values separate agents into demanders and suppliers that create a stable match. Receivers of high signals as buyers match up with receivers of low signals as sellers. The median signals serve as the CE of VCE of the private information equilibrium model.

Support. We calculate each subject's net purchase in each period and we rank the subjects according to their signal with the highest signal ranked 1, second highest 2 and so on. We run two linear regressions of net purchases on signal rank for experiments with and without grids. The coefficients are significant and negative in both cases.

Static theory suggests that bounding prices should be close to the CE of VCE. However, evidently due to uncertainty, prices do not converge to the same bounds in all periods. If the traders' valuations are correlated with their signals, receivers of high signals will bid at the upper bounding grid and receivers of low signals will ask at the lower bounding grid.

Subjects with high signals are more likely to buy at the upper grid (Table 9, column 2) and those with low signals are more likely to sell at the lower grid (column 3). The coefficients are significant and have the expected sign. In a word, the equilibrium bounding grids divide subjects into buyers and sellers based on their private signals.

Table 9: Value Separating Property of Private Signal

	(1)	(2)	(3)	(4)
	With Grid	With Grid	With Grid	No Grid
VARIABLES	Net Purchase	Total purchase at upper bounding grid	Total sell at lower bounding grid	Net purchase
Rank of Expected Value	-0.950*** (0.066)	-0.200*** (0.041)	0.211*** (0.035)	-0.411*** (0.097)
Constant	4.276*** (0.335)	1.540*** (0.208)	-0.256 (0.177)	1.847*** (0.491)
Observations	264	192	192	264
R-squared	44.0%	11.1%	15.9%	6.4%

We examine the evidence for whether contracts settle at a stable outcome as in series 1 experiments. Two features are prominently suggested by the matching model among the results reported in Section 5: the emergence of contracts only at adjacent grids and a convergence in bids and asks. Are these features preserved in the presence of asymmetric information?

The following theory provides predictions on where the adjacent grids should be if they are the equilibrium prices. A private information equilibrium in this context is a situation wherein trades are made near the median of private signals. A rational expectations equilibrium corresponds to a hypothetical scenario whereby trading evolves as if all private signals were public; this corresponds to the maximum likelihood estimate from aggregated signals. If a grid is in place, the equilibrium would be a VCE and the contract data would be at the bounding grids as suggested by the results in series 1.

Result 8: In the common value environment with asymmetric information, market activity exhibits equilibration over time.

- (i) Contract prices display a reduction in variance over time.
- (ii) The ratio of order flow (bids plus asks) relative to trade falls and approaches 1 (bids and asks trade quickly).
- (iii) When grids exist, then contract near the end of a period occupy only two, adjacent prices.

Support. We take every four neighboring prices and calculate their standard deviation. The results in Table 10 show that contract prices generally exhibit a reduction in variance using AEIG test for all six experiments, with and without grids.

Fifteen out of 33 periods in experiments with price grids (Exp2.1 to Exp2.3) and 16 out of 33 periods in experiments without grids (Exp2.4 to Exp2.6) exhibit a significant declining trend in

the volatility of trading prices. Because of the price grid, the ending level of moving standard

Table 10: Trend of Moving S.D.

Period	2	3	4	5	6	7	8	9	10	11	12	ρ_{-2}	n	R ²
Exp2.1	15.76*** (5.80)	4.363 (5.76)	-2.898 (5.76)	21.13*** (5.72)	26.86*** (5.72)	4.217 (5.69)	4.114 (5.70)	22.27*** (5.80)	0 (7.05)	5.363 (5.84)	-0.0701 (5.84)			
Exp2.2	-0.166 (5.80)	26.79*** (5.80)	28.39*** (5.80)	27.36*** (5.74)	2.819 (5.71)	15.65*** (5.92)	18.51*** (5.84)	7.963 (6.05)	0.409 (5.76)	-0.661 (5.74)	7.264 (5.92)	7.048*** (0.77)	224	0.802
Exp2.3	89.97*** (5.71)	7.572 (5.76)	31.24*** (5.84)	2.455 (5.80)	1.57 (5.80)	20.08*** (5.72)	1.815 (5.80)	24.64*** (5.71)	5.901 (5.74)	37.69*** (5.84)	13.51** (5.76)			
Exp2.4	31.18*** (6.30)	4.167 (6.35)	7.976 (6.32)	-1.679 (6.36)	3.445 (6.35)	41.68*** (6.39)	4.284 (6.60)	10.4 (6.46)	8.517 (6.52)	17.65*** (6.35)	0.0216 (6.35)			
Exp2.5	-3.357 (6.29)	3.902 (6.46)	39.70*** (6.75)	12.12* (6.42)	1.404 (6.39)	4.996 (6.60)	10.11 (6.75)	20.99*** (6.46)	30.21*** (6.39)	13.81** (6.75)	-1.018 (6.46)	6.196*** (0.64)	324	0.697
Exp2.6	43.26*** (6.31)	4.173 (6.27)	10.33 (6.27)	21.56*** (6.32)	22.86*** (6.33)	17.29*** (6.39)	17.64*** (6.33)	38.06*** (6.35)	8.152 (6.31)	59.39*** (6.29)	34.24*** (6.36)			

Standard errors in parentheses. *** $pr < 0.01$, ** $pr < 0.05$, * $pr < 0.1$ The upper 3 rows are experiments with grid, in which each period is a trend. The lower 3 rows are experiments without grid, in which each period is a trend.

deviation is higher when there is a grid (7.048 vs. 6.196). Nearly half of the periods display a significant declining trend whether or not there is a grid. Moreover, for the remaining periods,

even though the AEIG test does not reveal a significant trend, the starting level of volatility is so low that it cannot be distinguished from the ending level. In those periods, price variation is small from the beginning, which is also consistent with existence of equilibrium.

The process of price discovery and equilibration is also exposed by a declining offer-trade ratio property of the order flow. The ratio measures the number of unsuccessful bids and asks prior to a trade. Since it decreases as markets equilibrate and the number of periods advance, it suggests the amount of searching and haggling in the markets decrease together with price volatility. The process is true for both the grid and no grid case.

Next, we check whether bounding grids emerge as the market prices in later stage of each period. We divide all the trades within a period into two halves according to the time of trade. The two halves have the same amount of trades (the first half has one more if the total number is odd). Trades in the second half of each period are considered “late” whose prices are compared to the bounding grids. As shown by the number of equilibrium prices in Table 12, 19 out of 33 periods

Table 11: Trend of Offer Trade Ratio

Period	2	3	4	5	6	7	8	9	10	11	12	ρ_2	n	R ²
Exp2.1	9.650*** (2.58)	11.68*** (2.57)	5.466** (2.58)	4.663* (2.56)	5.073** (2.56)	5.185** (2.56)	3.963 (2.56)	4.677* (2.59)	8.045*** (2.67)	6.379** (2.58)	14.26*** (2.59)			
Exp2.2	11.07*** (2.57)	4.990* (2.57)	3.627 (2.57)	5.258** (2.57)	1.098 (2.56)	7.240*** (2.59)	4.122 (2.58)	3.765 (2.63)	9.219*** (2.57)	8.089*** (2.57)	5.736** (2.59)	1.207*** (0.29)	293	0.577
Exp2.3	3.491 (2.56)	15.83*** (2.57)	7.287*** (2.58)	13.30*** (2.61)	2.323 (2.58)	4.649* (2.56)	6.393** (2.58)	5.162** (2.57)	3.299 (2.57)	2.847 (2.59)	4.692* (2.57)			
Exp2.4	3.783* (2.29)	6.553*** (2.31)	6.017*** (2.30)	5.514** (2.31)	5.217** (2.32)	4.198* (2.31)	6.424*** (2.38)	6.933*** (2.32)	3.075 (2.33)	4.297* (2.31)	3.175 (2.31)			
Exp2.5	5.431** (2.29)	6.102*** (2.32)	6.864*** (2.36)	6.426*** (2.32)	5.779** (2.32)	6.581*** (2.34)	10.91*** (2.36)	6.172*** (2.32)	7.692*** (2.31)	7.010*** (2.36)	9.787*** (2.32)	1.908*** (0.20)	401	0.555
Exp2.6	4.743** (2.30)	8.919*** (2.29)	2.967 (2.29)	3.259 (2.30)	3.565 (2.30)	6.477*** (2.31)	4.611** (2.31)	4.624** (2.31)	1.445 (2.30)	4.221* (2.29)	8.105*** (2.31)			

Standard errors in parentheses. *** $pr < 0.01$, ** $pr < 0.05$, * $pr < 0.1$ The upper 3 rows are experiments with grid, in which each period is a trend. The lower 3 rows are experiments without grid, in which each period is a trend.

have two prices in their second half of trades and 4 out of 33 periods have only one price in the group of second half of trades. For all 19 periods with two prices at the end, the two prices are adjacent. This support the view not only that information aggregation is occurring, but also that

the matching model does a good job of predicting the pattern of equilibrium prices. The bounding grids are similar to those we uncovered in the first series of experiments. Due to price grid, agents are unable to contract at CE prices. The equilibrium outcome is actually a set of matches and the two adjacent contract prices ensure that the matching is stable.

Table 12: Equilibrium Prices and Bounding Grids

Experiment	Period	2	3	4	5	6	7	8	9	10	11	12
Exp2.1	Number of equilibrium prices	2	2	2	2	2	2	2	3	1	2	1
	Adjacent price	Y	Y	Y	Y	Y	Y	Y		Y	Y	Y
Exp2.2	Number of equilibrium prices	2	4	2	2	4	3	1	2	2	2	2
	Adjacent price	Y		Y	Y			Y	Y	Y	Y	Y
Exp2.3	Number of equilibrium prices	3	3	2	1	3	2	2	2	3	3	3
	Adjacent price			Y	Y		Y	Y	Y			

6.2 The Information Contained in Prices

Two models of information are now examined. The rational expectations equilibrium holds that all information available to the market is reflected in market price. In this case, information is summarized by the maximum likelihood estimator (MLE) based on the pooling of all signals. By contrast, the private market equilibrium holds that demands and supplies depend only on private information. In this case, the price reflects the median of market signals.

Figure 7 contains the time series for all trades in Exp2.2 with a grid, and an analogous experiment without a grid, Exp2.5. The states and information were identical for respective periods in the two experiments. The equilibrium predicted by the private information model assumes each individual applies the expected utility model and the information contained in his or her own signal to estimate the value of a unit of the security that, in turn, serves as the individual demand for a unit. According to this model, individuals do not acquire information about the dividend from the buying and selling activities of others. The equilibrium is a price range so the figure contains the upper and lower bounding prices of the range. The MLE model assumes that individuals are able to extract full posterior probabilities of the dividend given the sample of all signals pooled. Market equilibrium prices are based on such assumptions.

The figure contains a reliable impression of the statistics reported in the result. The variance of contract prices declines over time and prices move toward the private information

equilibrium. The same pattern exists for the case where the grid exists and the case where the grid does not exist. Of course, exceptions can be found.

Journal Pre-proof

Figure 7. Market Prices, MLE Prediction, Median Signal and Actual Dividend. Experiments 2.2 (grid) and 2.5 (no grid) differ according to the existence of a grid. Payoffs and information are identical. Shown are the time series of contract prices. Aggregated information for each period include maximum likelihood, median signals and actual dividend.

Result 9: The private information market equilibrium is accurate with or without a grid.

Support. There are four natural candidates for price equilibrium in the case of uncertain and asymmetric information. They are the median private signal (private information equilibrium), the median of individual maximum likelihood estimates, the full maximum likelihood estimate based on the aggregated private signals, and the true value of the dividend. In Table 13 we report the second moment of the difference between the price at which the trades equilibrate towards and the four possible candidates and compare them. This shows that for both experiments with the grid and without the grid, the price convergence is closest to the median signal, i.e., the private information equilibrium.

Table 13: Second Moment of the Difference between Prices and Theoretical Predictions

	Median Signal	Median MLE	Full MLE	True Value
Grid	21.6	23.29	25.51	27.67
No Grid	29.27	29.82	32.1	31.86

For each period, let the converging price be p_t and the median signal be $p_{t,ms}$. Then the value in the table is $\sqrt{\sum_t (p_t - p_{t,ms})^2}$.

We could use an alternative measure to test which of the equilibrium is most accurate. With a grid, 64.4% of total trades happened in the neighborhood of the median signal as opposed to 57.6% of total trades occurring in the neighborhood of the rational expectations equilibrium, the full MLE.¹³

6.3 The Average Trading Price

The information content of average contract prices is an important issue, particularly if a grid might obscure what might be revealed by unrestricted prices or order flow. The next result is that the average contract price contains key information even when a grid is in effect. Section 5

¹³One possible explanation as to why private information equilibrium outperforms rational expectations equilibrium has to do with the distribution we used for the parameters. By setting up a lower bound and an upper bound and using a truncated normal distribution, the maximum likelihood estimate is skewed towards extreme endpoints especially if the value of the dividend is not near the center.

demonstrates (under certainly) that the average contract price is close to the VCE; but now the VCE is unknown because values founded on beliefs cannot be observed.

Due to the uncertainty of individual signals, the price discovery process should explore a larger ranges of possibilities. Therefore, instead of using the average price of all trades, we focus on the final five trades where convergence is more likely. We treat the median signal as a private information equilibrium and use its difference from the average trade price as an inverse measure of the information content of prices. We note that precision is lost because the median of the private signals is a range, i.e multiple medians. Furthermore, due to the non-unique values of the median the distinction between the CE and VCE can be obscure so in the next result we use only the median of the private signals.

Result 10. The average trading price moves toward the median of private signals (a proxy for the CE). In all cases convergence can be observed with tightest fits existing at the end of periods,

Support. Table 14 reports that the discrepancy between prices and the median of private signals is larger for earlier periods compared to later periods. This is consistent with price convergence toward aggregated information revealed by the order flow of bids and asks. Coincidentally, it suggests consistency with features of cascades and herds in which individuals place too much weight on private information and not enough weight on information produced in the market. Under all conditions, convergence is evident. Finally, the first five trades of each period exhibit a much larger difference with the median signal intimating that convergence takes place within a period as well. Only in the last trades of the late periods, we observe the discrepancy between average price and the median signal is lower for the no-grid case.

In all cases, the average discrepancy is greater in the no grid case and the variance is larger. This somewhat surprising result reveals, at a minimum, that a grid does not impede information aggregation very much, if at all. We hesitate to conclude, however, that a grid actually improves information aggregation because we do not have a sharp test of statistical significance between comparable cells. The average price of late trades does in fact carry price information, which overcomes any impediments of a grid for the grid sizes we have imposed.

Table 14. The Difference between the Average Trade Prices and the Median Private Signal

	Grid			No Grid		
	All trades	First five trades	Last five trades	All trades	First five trades	Last five trades
All Periods	23.27 (0.93)	27.18 (1.67)	18.09 (1.15)	28.6 (1.13)	31.49 (2.29)	24.88 (1.78)
Early periods (2-6)	23.61 (1.54)	31.06 (2.84)	16.15 (1.87)	31.49 (1.58)	35.68 (3.88)	24.63 (2.91)
Late periods (7-12)	22.95 (1.10)	23.96 (1.89)	19.81 (1.43)	24.88 (1.55)	29.48 (2.66)	17.8 (1.95)

Standard errors in parentheses.

7. SUMMARY OF CONCLUSIONS

Grids and tick sizes have not been studied experimentally. The advantages of an experimental approach are made apparent by a lack of consensus in the policy related literature. The experimental approach produces a connection with the theory of markets, which has a successful history unmasking basic principles of market behavior. Traditional analysis pivots on the competitive equilibrium, its game theory variants and the dynamics it suggests about disequilibrium adjustments. When a tick-related price grid is imposed on a market, the fundamental tool, the competitive equilibrium, need not exist. No consensus exists about appropriate theory. Thus, the question backs into a broad question of what theories might apply. Our exploratory experiments point to a possible understanding of these real-world phenomena.

Our study begins with experiments involving large tick sizes, a stationary market and no uncertainty about private values. The main discovery is that the appropriate model appears to be stable outcomes from matching theory. With that background model in mind, we study suggestions from related literature and examine issues raised in the policy related literature.

Principles taken from the stable matching model make precise predictions for cases when a competitive equilibrium exists and cases when its existence is destroyed by the imposition of a grid. Result 1, Result 2 and Result 3 of Section 5 demonstrate the power of a stable outcome model as a predictor of the experimental outcomes. Result 4 demonstrates that the average of the bouncing of prices created by the non-existence of the competitive equilibrium actually approximates the competitive equilibrium price. Result 5 and Result 6 address issues of speculation, market depth and liquidity. In these experiments, the imposition of a grid increases

liquidity and volume but it does so at the expense of market efficiency and brings losses for those seduced into speculation.

Order flow in these stationary markets bears similarity to that in markets without a tick size grid. Namely, the direction of order flow reflects excess demand/supply. It remains to be determined if such patterns continue to hold as parameters change or substantial asymmetric information is introduced.

A second series of experiments examine a market trading an asset with an uncertain (dividend) payoff. The results reported in Section 6 inherit important properties of the certainty case. Private information channels behavior consistent with private value aggregated market demand and market supply (Result 7) and Result 8 demonstrates that grid impeded contracts still converge to the bounding prices of a virtual competitive equilibrium (VCE.) The VCE that emerges is a private information equilibrium as established by Result 9 and Result 10. That is, the successful model assumes that reservation prices are based on private information as opposed to wildly unstable beliefs or information based on the aggregation of all information that would reflect the maximum likelihood of aggregated information. Interestingly, while the prices do not approximate the maximum likelihood of the security payoff, prices do carry key information. Result 10 demonstrates that the average trading price is a close approximation of the VCE determined by the median of the information distribution. Thus, while the competitive equilibrium does not exist, the average market price tells us where the competitive equilibrium would be if it were not prevented by the grid and that it operates to facilitate information aggregation. Thus, while the imposition of a grid clearly impacts the structure of order flow and has the capacity to disrupt major features of the price discovery power of markets, markets seem to work around it but do so following a more general set of principles than those traditionally modeled.

References

- Anshuman VR, Kalay A. Can splits create market liquidity? theory and evidence. *Journal of Financial Markets*. 2002;5(1):83-125.
- Bacidore J, Battalio RH, Jennings RH. Order submission strategies, liquidity supply, and trading in pennies on the New York stock exchange. *Journal of Financial Markets*. 2003;6(3):337-362.
- Bessembinder H. Trade execution costs and market quality after decimalization. *Journal of*

- Financial and Quantitative Analysis*. 2003;38(04):747-777.
- Chakravarty S, Wood RA, Van Ness RA. Decimals and liquidity: A study of the NYSE. *J Financ Res*. 2004;27(1):75-94.
- Chao Y, Yao C, Ye M. Tick size constraints, two-sided markets, and competition between stock exchanges. *Working Paper*. 2015.
- Chung KH, Charoenwong C, Ding DK. Penny pricing and the components of spread and depth changes. *Journal of Banking & Finance*. 2004;28(12):2981-3007.
- Dayri K, Rosenbaum M. Large tick assets: Implicit spread and optimal tick size. *Market Microstructure and Liquidity*. 2015;1(01):1550003.
- Demsetz H. The cost of transacting. *The quarterly journal of economics*. 1968:33-53.
- Gao X, Ritter JR, Zhu Z. Where have all the IPOs gone? *Journal of Financial and Quantitative Analysis*. 2013;48(06):1663-1692.
- Garman MB. Market microstructure. *J Financ Econ*. 1976;3(3):257-275.
- Gibson S, Singh R, Yerramilli V. The effect of decimalization on the components of the bid-ask spread. *Journal of Financial Intermediation*. 2003;12(2):121-148.
- Harris L. Trading in pennies: A survey of the issues. *Unpublished working paper, University of Southern California*. 1999:1-13.
- Harris L. Decimalization: A review of the arguments and evidence. *Unpublished working paper, University of Southern California*. 1997.
- Harris LE. Minimum price variations, discrete bid-ask spreads, and quotation sizes. *Rev Financ Stud*. 1994;7(1):149-178.
- Hatfield JW, Kominers SD, Nichifor A, Ostrovsky M, Westkamp A. Stability and competitive equilibrium in trading networks. *Journal of Political Economy*. 2013;121(5):966-1005.
- Hatfield JW, Plott CR, Tanaka T. ,“Understanding Price Controls and Non-Price competition with Matching Theory”, *American Economic Review*, 2012, 102(3):371-375
- Hatfield JW, Plott CR, Tanaka T. Price controls, non-price quality competition, and the nonexistence of competitive equilibrium. *Games and Economic Behavior*. 99(2016):134-163.
- Isaac. M.R. and Plott, C.R. “Price Controls and the behavior of auction markets: An experimental Examination, *American Economic Review*, 1981,71 (3), 448-450.
- Kelso Jr AS, Crawford VP. Job matching, coalition formation, and gross substitutes.

Econometrica: Journal of the Econometric Society. 1982:1483-1504.

Kyle AS. Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*. 1985:1315-1335.

Noussair, Charles N., Charles R. Plott, and Raymond G. Riezman. "An Experimental Investigation of the Patterns of International Trade." *American Economic Review* 85.3 (1995): 462-491.

O'Hara, Maureen, Gideon Saar, and Zhuo Zhong. "Relative tick size and the trading environment." *Review of Asset Pricing Studies*, 2018.

Shapley LS, Shubik M. The assignment game I: The core. *International Journal of game theory*. 1971;1(1):111-130.

Smith V.L. and Williams A. "Efficiency of Non-binding price control in double auction trading, Ch.7, pp46-54, Plott, C.R and Smith V.L. Handbook of Experimental Economic Results, North Holland, 2008.

Weild D, Kim E, Newport L. The trouble with small tick sizes. *Capital Markets Series*. 2012.

Yao, Chen, and Mao Ye. "Why trading speed matters: A tale of queue rationing under price controls." *The Review of Financial Studies* 31.6 (2018): 2157-2183.

Zhao X, Chung KH. Decimal pricing and Information-Based trading: Tick size and informational efficiency of asset price. *Journal of Business Finance & Accounting*. 2006;33(5-6):753-766.

Appendix A: Proofs of propositions

Proposition 1. The Competitive Equilibrium is a stable outcome

Proof. Let the size of the grid be 0. In the proof of Proposition 2 let the upper support of the CE play the role of p^U and let the lower support of the CE play the role of p^L . With the change of notation, the proof of Proposition 1 follows as a corollary to Proposition 2.

Proposition 2. Stable outcomes cannot involve contracts above p^U or below p^L .

Proof. Any stable outcome containing the contract $\langle(i, j), p\rangle$ with $i \in D(p^U)$ and payment $p > p^U$ is blocked by the an outcome with a contract $\langle(i, j'), p^U\rangle$ where $j' \in S(p^U)$ because i prefers p^U to p and because there exists an unmatched j' due to the fact that $S(p^U)$ is larger than $D(p^U)$. A similar argument demonstrates that any outcome with $p < p^L$ is blocked.

Proposition 3. The stable outcomes always include buyer values at or above the upper bounding grid or seller costs that are at or below the lower bounding grid or both. Thus, the limitation of values at or outside the lower and upper price bounds influences the proportion of trades at the two bounding prices.

Proof. Suppose $A = \{\langle(i, j), p\rangle\}$ is a stable outcome. If $\langle(i^s, j^s), p^s\rangle \in A$ then either (1) $[i^s \in D: V(i) \geq p^U]$ or (2') $[j^s \in S: C(j) \leq p^L]$ or both. Notice that the existence of the grid dictates that $p^s \in \{p^U, p^L\}$. If neither 1 nor 2 are satisfied then either $V(i^s) - p^s < 0$ or $p^s - C(j^s) < 0$ so rationality is violated.

Proposition 4: Trading volume is in the range $(\text{Max}\{D(p^U), S(p^L)\}, \text{Min}\{D(p^L), S(p^U)\})$. Additional volume could reflect speculation.

Proof. Assume that the trading volume is smaller than $\text{Max}\{D(p^U), S(p^L)\}$. If $D(p^U) \geq S(p^L)$, then there exists an $i \notin D(p^U)$ that is unmatched. From Proposition 2, we know all matches are at either p^U or p^L . Furthermore, we know that there exists a $j \in S(p^U)$ that is unmatched since $S(p^U) > D(p^U)$. Then i and j become a profitable match at p^U . The argument is symmetric when $D(p^U) \leq S(p^L)$. Now assume that the trading volume is larger than $\text{Min}\{D(p^L), S(p^U)\}$. If $D(p^L) \geq S(p^U)$, then there exists a $j \notin S(p^U)$ that is matched. However, from Proposition 2, we know all matches at either p^U or p^L and this violates the individual rationality of j . The argument is symmetric when $D(p^L) \leq S(p^U)$.