moment for the $11S_{1/2} \rightarrow 9P_{3/2}$ transition to be $(3)^{1/2}(6.0)$ Debye and the corresponding oscillator strength to be 0.048. Theoretically the $11S_{1/2} \rightarrow 9P_{1/2}$ transition is a factor of 2 weaker.[14] Consequently, the total oscillator strength for $11S \rightarrow 9P$ should be 0.072. In comparison, Anderson and Zilitis's[15] semiempirical calculation gives $f = 0.156$ which is about a factor of 2 higher. Since theoretical calculation of oscillator strengths involves much uncertainty, the agreement is satisfactory.

Note that the dipole moment for the $5S \rightarrow 11P$ pump transition is only $\sim 10^{-1}$ D.[15] Therefore, the dynamic Stark effect produced by the uv field is negligible.

By using a TEA CO_2 laser it should be possible to obtain a reasonably uniform infrared field three orders of magnitudes more intense than in the present case. For the $P(16)$ line, the condition $\nu_R \ll \Delta\nu$ will then be greatly violated. However, by increasing $\Delta\nu$ to ~ 10 cm^{-1}, the above condition can be satisfied again and a tuning range of $\sim \pm 1$ cm^{-1} will be obtained. Tunable radiation in other parts of the FIR spectrum can be generated by applying the same scheme to other transitions in metal vapors and by using other infrared lasers.

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Image phase compensation and real-time holography by four-wave mixing in optical fibers[a)]

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It is proposed that real-time holography can be performed inside multimode fibers (or optical waveguides) using four-wave optical mixing. Of particular interest is the generation of complex-conjugate replicas of input fields for image transmission and compensation of propagation distortion. A theoretical analysis and a numerical estimate are presented.

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It has been proposed recently[1] that nonlinear optical mixing in crystals can be used to compensate for modal dispersion and to make possible long-distance image transmission in fibers. The mixing is used to generate a phase conjugate ("time-reversed") replica of the transmitted field.

The application of four-wave (third-order nonlinearity) mixing to the generation of phase-conjugated waves[2] and for amplification and oscillation[3] has been considered theoretically[2,3] and demonstrated experimentally.[4–6]

In this note we examine the possibility of four-wave mixing in multimode optical fibers. We find that complex image fields can be phase conjugated and amplified *without loss* of spatial information, i.e., without mode mixing. A numerical calculation shows that this can be done on a cw basis with moderate (~ 1 W) pump powers in a few meters of fiber. This suggests that four-wave mixing in fibers is a serious candidate for real-time holographic applications including image transmission and compensation for distorting media.

The basic geometry involved in the experiment is illustrated in Fig. 1. An input field E_i is incident on a fiber whose core medium possesses an appreciable third-order nonlinear coefficient $\chi^{(3)}$. The input field E_i may correspond to an image field transmitted by another fiber or to some other field whose phase conjugate is sought. The fiber is multimode with a number of propagating modes essentially equal to the number

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of resolution elements contained in the incident field E_i. The fiber is pumped simultaneously by two strong oppositely traveling fields, E_{p1} and E_{p2}, whose frequency is the same as that of E_i.

We are looking for a reflected wave E_r generated by the four-wave mixing, and will show that it is a complex conjugate of the input field E_i and that for sufficiently intense pumping intensity, $|E_r|>|E_i|$, it is amplified. The four-wave mixing can thus be used to obtain an amplified phase-conjugate version of the input field E_i to the fiber.

The oppositely traveling waves are taken as

$$E_{p1}=E_{p1}(x,y)\exp[i(\omega t-\beta_p z)],$$

$$E_{p2}=E_{p2}(x,y)\exp[i(\omega t+\beta_p z)].$$

These pump fields will be coupled ideally into the lowest-order propagating mode of the fiber so that their confinement over the full interaction length is assured. The input signal wave E_i and the reflected output wave E_r are expanded in terms of the propagating eigenmodes $\mathcal{E}_l(x,y)$ of the fiber as

$$E_i=\tfrac{1}{2}\sum_m B_m(z)\,\mathcal{E}_m(x,y)\exp[i(\omega t-\beta_m z)]+\text{c.c.},$$
$$E_r=\tfrac{1}{2}\sum_l A_l(z)\,\mathcal{E}_l(x,y)\exp[i(\omega t+\beta_l z)]+\text{c.c.} \tag{1}$$

The complex amplitudes $B_m(0)$ are determined by the input conditions. We are seeking a solution for $B_m(z)$ and $A_l(z)$. The wave equation is

$$\nabla^2\mathbf{E}-\mu\epsilon(\mathbf{r})\frac{\partial^2\mathbf{E}}{\partial t^2}=\mu\frac{\partial^2}{\partial t^2}P_{\rm NL}(\mathbf{r},t). \tag{2}$$

The functions $\mathcal{E}_l(x,y)$ satisfy (2) with $P_{\rm NL}=0$,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\beta_l^2\right)\mathcal{E}_l(x,y)+\omega^2\mu\epsilon(\mathbf{r})\mathcal{E}_l(x,y)=0. \tag{3}$$

We also have

$$\int_{-\infty}^{\infty}\mathcal{E}_l(x,y)\mathcal{E}_m(x,y)\,dx\,dy=\delta_{lm}. \tag{4}$$

Putting Eq. (1) in Eq. (2)

$$\sum_l\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\beta_l^2+\omega^2\mu\epsilon(r)\right)\frac{A_l}{2}+i\beta_l\frac{dA_l}{dz}+\frac{1}{2}\frac{d^2A_l}{dz^2}\right]\mathcal{E}_l$$
$$\times\exp[i(\omega t+\beta_l z)]+\text{c.c.}$$
$$=\mu\frac{\partial^2}{\partial t^2}P_{\rm NL}(\mathbf{r},t). \tag{5}$$

Using Eq. (3) to eliminate the first factor inside the square brackets, and assuming that $|d^2A_l/dz^2|\ll|\beta_l\,dA_l/dz|$ leads to

$$\sum_l i\beta_l\frac{dA_l}{dz}\mathcal{E}_l\exp[i(\omega t+\beta_l z)]+\text{c.c.}=\mu\frac{\partial^2}{\partial t^2}P_{\rm NL}(\mathbf{r},t). \tag{6}$$

In a similar fashion we obtain for the input field E_i

$$\sum_m -i\beta_m\frac{dB_m}{dz}\mathcal{E}_m\exp[i(\omega t-\beta_m z)]+\text{c.c.}=\mu\frac{\partial^2}{\partial t^2}P_{\rm NL}(\mathbf{r},t) \tag{7}$$

(note change in sign due to opposite direction of propagation). The nonlinear polarization is third order in the field amplitudes

$$\mathbf{P}_{\rm NL}=\chi^{(3)}:\mathbf{E}\cdot\mathbf{E}\cdot\mathbf{E} \tag{8}$$

and in our scalar analysis is taken as

$$P_{\rm NL}^{(\omega_4=\omega_1+\omega_2-\omega_3)}=\chi^{(3)}E_1E_2E_3^*. \tag{9}$$

We substitute Eq. (9) in Eq. (6), take E_1 as E_{p1}, E_2, as E_{p2}, and E_3 as E_i, yielding

$$\sum_l\beta_l\frac{dA_l}{dz}\mathcal{E}_l(x,y)\exp(i\beta_l z)$$
$$=\frac{i\omega^2\mu\chi^{(3)}}{2}E_{p1}E_{p2}\sum_m B_m^*(z)\mathcal{E}_m(x,y)\exp(i\beta_m z). \tag{10}$$

In a similar fashion from Eq. (7)

$$\sum_m\beta_m\frac{dB_m}{dz}\mathcal{E}_m(x,y)\exp(-i\beta_m z)$$
$$=-\frac{i\omega^2\mu\chi^{(3)}}{2}E_{p1}E_{p2}\sum_l A_l^*(z)\mathcal{E}_l(x,y)\exp(-i\beta_l z) \tag{11}$$

or by conjugation (and taking E_{p1}, E_{p2}, and $\chi^{(3)}$ as real)

$$\sum_m\beta_m\frac{dB_m^*}{dz}\mathcal{E}_m(x,y)\exp(i\beta_m z)$$
$$=\frac{i\omega^2\mu\chi^{(3)}}{2}E_{p1}E_{p2}\sum_l A_l(z)\,\mathcal{E}_l(x,y)\exp(i\beta_l z). \tag{12}$$

Multiply Eq. (10) by $\mathcal{E}_s$ and integrate over the cross section using Eq. (4);

$$\frac{dA_s}{dz}=\frac{i\omega^2\mu\chi^{(3)}}{2\beta_s}\sum_m B_m^*(z)\exp[i(\beta_m-\beta_s)z]$$
$$\times\int\int E_{p1}E_{p2}\mathcal{E}_m\mathcal{E}_s\,dx\,dy, \tag{13a}$$

and from Eq. (12)

$$\frac{dB_s^*}{dz}=\frac{i\omega^2\mu\chi^{(3)}}{2\beta_s}\sum_m A_m(z)\exp[i(\beta_m-\beta_s)z]$$
$$\times\int\int E_{p1}E_{p2}\mathcal{E}_m\mathcal{E}_s\,dx\,dy. \tag{13b}$$

Equations (13) are the basic coupling equations. We note that a given mode A_s of the reflected field may be coupled to all modes B_m of the incident field. This is undesirable, since it causes a loss of spatial information. Two physical facts conspire to remedy this mode

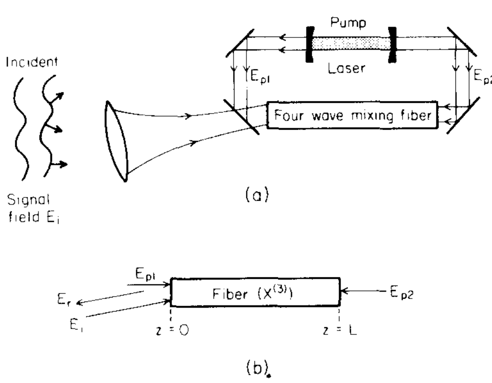

FIG. 1. (a) The basic geometry for four-wave phase conjugation in a fiber. Note that by injecting the signal E_i at an angle it should be possible to spatially filter the reflected conjugate wave E_r from the pump wave E_{p2}. (b) The coordinates and wave designation used in the analysis.

scrambling: (a) We note that for a length of fiber $L \gg 2\pi(\beta_m - \beta_s)^{-1}$ modes A_s and B_m are grossly mismatched so that no cumulative power exchange between them can take place unless $\beta_m = \beta_s$. (b) In addition, if the pump fields $E_{p1}(x,y)$ and $E_{p2}(x,y)$ are more or less uniform over the cross section, then the overlap integrals appearing in Eqs. (13) are zero unless $m = s$. The combined effect of (a) and (b) is that a given mode A_s of the reflected field couples almost exclusively to the incident field mode of the same index, i.e., to B_s. We thus have

$$\frac{dA_s}{dz} = \frac{i\omega^2\mu\chi^{(3)}}{2\beta_s} E_{p1}E_{p2}B_s^*, \qquad \frac{dB_s^*}{dz} = \frac{i\omega^2\mu\chi^{(3)}}{2\beta_s} E_{p1}E_{p2}A_s. \tag{14}$$

Since the input field E_i is specified, the mode amplitudes B_s $(z=0)$ are given. Another boundary condition is that the reflected field E_r is zero at the output end $(z=L)$ of the fiber, so that $A_s(L)=0$. With these boundary conditions we can solve Eq. (14), obtaining

$$A_s(z) = iB_s^*(0)(\sin\kappa_s z - \tan\kappa_s L\cos\kappa_s z), \qquad B_s(z) = B_s(0)(\tan\kappa_s L\sin\kappa_s z + \cos\kappa_s z), \tag{15}$$

where $\kappa_s = \omega^2\mu\chi^{(3)}E_{p1}E_{p2}/2\beta_s$. Equations (15) display the same amplification and oscillation features of the plane wave case.[3] We are particularly interested in what happens to the image information. We find that at $z=0$

$$A_s(0) = -iB_s^*(0)\tan\kappa_s L. \tag{16}$$

We can reconstruct the total reflected image field using Eq. (1);

$$E_r(x,y,z=0) = -i\tan\kappa L \sum_l \tfrac{1}{2}B_l^*(0)\mathcal{E}_l(x,y)\exp(i\omega t) + \text{c.c.}, \tag{17}$$

where, neglecting the weak dependence of κ_s on s, we took $\kappa_s \equiv \kappa$. The original input field is, according to Eq. (1),

$$E_i(x,y,z=0) = \sum_l \tfrac{1}{2}B_l(0)\mathcal{E}_l(x,y)\exp(i\omega t) + \text{c.c.}, \tag{18}$$

so that as far as the total complex field amplitudes are concerned we have

$$E_r(x,y,z=0) = -i\tan(\kappa L)E_i^*(x,y,z=0). \tag{19}$$

The reflected field at the fiber input is thus an *amplified* (for $\kappa L > \frac{1}{4}\pi$) *complex-conjugate* replica of the input field.

The tremendous advantage of performing the phase conjugation inside a fiber is due to the fact that pump waves can be launched as fiber modes so that large pump intensities over the whole interaction path can result from moderate pump powers. If we consider, as an example, a multimode fiber with a core diameter of 20 μm which is filled with CS_2 and take the two pump wave powers as 1 W we obtain for $\lambda = 1$ μm (using $\chi^{(3)} \simeq 1.2\times10^{-12}$ esu) $\kappa \simeq 5\times10^{-4}$ cm^{-1}, so that $\kappa L \simeq 1$ is achieved with a fiber length of $L = 20$ m. It should be noted that if the pump propagates as a low-order mode, the overlap integrals of Eqs. (13) may be appreciable for near-lying modes, i.e., incident and reflected modes of different indices can interact. This interaction, however, is negligible since the phase mismatch $(\beta_s - \beta_m)$ can be shown to exceed 10^4 rad in the above example.

The theory and example presented above clearly demonstrate the advantage of performing four-wave mixing in long fibers. Under such conditions it is important to consider the effect of optical absorption. If we take the intensity absorption coefficient of the four waves involved in the interaction as α we obtain instead of Eq. (19)

$$E_r(x,y,0) = \frac{-2i\kappa\tan(\kappa_e L)\exp(-\frac{1}{2}\alpha L)}{\alpha\tan(\kappa_e L) + 2\kappa_e} E_i^*(x,y,0), \qquad \kappa_e \equiv [\kappa^2\exp(-\alpha L) - (\tfrac{1}{2}\alpha)^2]^{1/2}. \tag{20}$$

It follows from Eq. (20) that for substantial reflections we need fulfill $\alpha L \ll 1$, $\kappa > \frac{1}{2}\alpha$. If, as an example $L\alpha \gg 1$ we obtain

$$\frac{E_r(x,y,0)}{E_i^*(x,y,0)} = -i\exp(-\tfrac{1}{2}\alpha L)\,\kappa/\alpha.$$

The pump intensity in the above example is below that for the onset of stimulated Brillouin scattering in CS_2 and also is such that amplified spontaneous Stokes radiation is insignificant.

In conclusion, four-wave mixing in fibers is proposed and analyzed as a means for image phase compensation in fiber links,[1] for the generation of time-reversed phase conjugate images and other real-time holographic applications (Ref. 7).

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