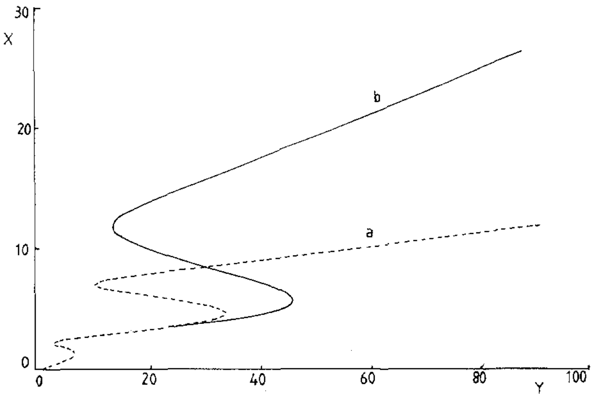

Fig. 5. Double bistability, $T_1/T_2 = 10$, $C = 20$, $\Omega = 24$, $\Phi = 8$, (a) $\delta = -0.1$, (b) $\delta = 0.1$.

stability. The optical multistability may be useful for multi-logic computers and signal processing.

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REFERENCES

[1] E. Giacobimo, M. Devaud, F. Biraben, and G. Grynberg, "Doppler-free two-photon dispersion and optical bistability in rubidium vapour," *Phys. Rev. Lett.*, vol. 45, pp. 434-437, 1982.
[2] F. T. Arecchi and A. Politi, "Optical bistability in a resonant two-photon absorber," *Lett. Al. Nuovo. Cimento*, vol. 23, pp. 65-69, 1978.
[3] G. P. Agrawal and C. Flytzanis, "Two-photon double-beam optical bistability," *Phys. Rev. Lett.*, vol. 44, pp. 1058-1061, 1980.
[4] J. A. Hermann, "Stability condition for two-photon optical bistability in a ring cavity," *Opt. Commun.*, vol. 37, pp. 431-436, 1981.
[5] D. F. Walls, C. V. Kunasz, P. D. Drummond, and P. Zoller, "Bifurcations and multistability in two-photon processes," *Phys. Rev.*, vol. 24A, pp. 627-630, 1981.
[6] B. V. Thompson and J. A. Hermann, "Modified optical bistability in two-photon absorption," *Phys. Lett.*, vol. 83A, pp. 376-378, 1981.

On the High Power Limit of the Laser Linewidth

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Abstract—**The quantum mechanical limit of the laser linewidth is shown to imply a residual constant linewidth rather than obey an inverse power dependence as is usually assumed.**

THE question of the limiting spectral linewidth of the field of a single mode laser was addressed in the early stages of the laser's development. Schawlow and Townes, the first to concern themselves with this problem, obtained [1]

$$(\Delta\nu)_{\text{laser}} = \frac{2\pi h\nu(\Delta\nu_{1/2})^2}{P}. \tag{1}$$

Equation (1) was subsequently modified to [2]

$$(\Delta\nu)_{\text{laser}} = \frac{2\pi h\nu(\Delta\nu_{1/2})^2}{P} \frac{N_2}{[N_2 - N_1(g_2/g_1)]_{\text{th}}} \tag{2}$$

where the factor

$$\mu = \frac{N_2}{[N_2 - N_1(g_2/g_1)]_{\text{th}}} \tag{3}$$

accounts for the fact that when $t_1 \neq 0$, the finite population N_1 of the lower laser level requires a corresponding increase in N_2 in order for the gain to remain equal to the loss. This increases the spontaneous emission noise power, which is proportional to N_2, and hence $(\Delta\nu)_{\text{laser}}$.

The common interpretation of (2) is that it predicts an inverse dependence of $(\Delta\nu)_{\text{laser}}$ on the power output P.

It is the purpose of this letter to point out that, according to (2), there should remain a residual laser linewidth, even as $P \to \infty$. This is due to the fact that, unless t_1 is zero, as P increases, N_1 must increase since the increased net-induced transition rate into level 1 must equal in steady state N_1/t_1 the rate of emptying of level 1. This causes the population N_2 to increase in order to keep $N_2 - N_1(g_2/g_1)$, and thus the gain, a constant. At sufficiently high values of P, N_2 becomes and stays proportional to P and the ratio N_2/P in (2) approaches a constant value, thus leading to a residual power independent linewidth.

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To obtain the power (P) dependence of μ, we solve the conventional laser rate equations

$$\frac{dN_2}{dt} = R - \frac{N_2}{t_2} - \left[N_2 - N_1 \frac{g_2}{g_1}\right] W_i$$
$$\frac{dN_1}{dt} = -\frac{N_1}{t_1} + \left[N_2 - N_1 \frac{g_2}{g_1}\right] W_i + \frac{N_2}{t_2}$$
$$\frac{dp}{dt} = \left[N_2 - N_1 \frac{g_2}{g_1}\right] W_i - \frac{p}{t_c} \quad (4)$$

where N_2 and N_1 are the level populations, $g_{1,2}$ are their degeneracies, t_1 and $t_2 = t_{\text{spont}}$ are the lifetimes, t_c is the passive resonator photon lifetime, R is the pumping rate into level 2, W_i is the induced transition rate, and p is the number of photons in the oscillating mode.

At equilibrium, $d/dt = 0$, we can solve (4) to obtain

$$N_2 - N_1 \frac{g_2}{g_1} = \frac{R[t_2 - t_1(g_2/g_1)]}{1 + W_i t_2}$$
$$N_2 = \frac{R t_2 [1 + W_i t_1 (g_2/g_1)]}{(1 + W_i t_2)} \quad (5)$$

so that

$$\mu = \frac{N_2}{[N_2 - N_1(g_2/g_1)]_{\text{th}}} = \frac{t_2}{t_2 - t_1(g_2/g_1)} \left(1 + W_i \frac{g_2}{g_1} t_1\right) \quad (6)$$

where the subscript "th" indicates the value at threshold. The power output, including "wall losses" of the laser, is

$$P = \left[N_2 - N_1 \frac{g_2}{g_1}\right]_{\text{th}} W_i h\nu \quad (7)$$

which, when used together with (6) in (2), gives

$$(\Delta\nu)_{\text{laser}} = \frac{2\pi h\nu(\Delta\nu_{1/2})^2}{P} \left[\frac{t_2}{t_2 - t_1(g_2/g_1)}\right] + \frac{c\Delta\nu_{1/2}\lambda_0^2}{8\pi n^3 (\Delta\nu)_{\text{gain}} V} \left[\frac{t_1}{t_2(g_1/g_2) - t_1}\right] \quad (8)$$

where $\Delta\nu_{1/2} \equiv 1/2\pi t_c$ and $(\Delta\nu_{\text{gain}})$ is the linewidth of atomic transition responsible for the laser gain. V is the mode volume. In obtaining (8), we use

$$\left[N_2 - N_1 \frac{g_2}{g_1}\right]_{\text{th}} = \frac{8\pi\nu^2 n^3 (\Delta\nu)_{\text{gain}} V t_2}{c^3 t_c} \qquad (t_2 = t_{\text{spont}}). \quad (9)$$

The first term on the right-hand side of (8) is the conventional Schawlow–Townes expression containing the inverse P dependence. The second term is *power independent* and corresponds to a residual linewidth as $P \to \infty$.

To get an idea of the magnitudes involved, we consider the case of a 0.6328 μm HeNe laser with mirror reflectivities of $R = 0.99$, a resonator length of $l = 30$ cm, and take $t_1/t_2 = 0.1$. We obtain

$$\Delta\nu_{1/2}(\text{Hz}) = \frac{(1 - R)c}{2\pi n l} = 1.6 \times 10^6$$

and

$$\Delta\nu_{\text{laser}}(\text{Hz}) \simeq \frac{10^{-3}}{P(m\omega)} + 3.8 \times 10^{-4}.$$

The residual linewidth thus dominates at power levels exceeding a few milliwatts.

In a semiconductor laser the situation is more complicated. The dynamics of pumping are fundamentally different from those in a simple atomic laser. Charge neutrality will dictate that each injected electron is accompanied by the injection of a hole which would tend to clamp N_2 above threshold. For this reason we expect the above-described linewidth mechanism, if at all present, to have a negligible effect on the linewidth of a semiconductor laser.

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References

[1] A. L. Schawlow and C. H. Townes, "Infrared and optical masers," *Phys. Rev.*, vol. 112, pp. 1940–1949, 1958.
[2] A. Yariv, *Quantum Electronics*, 1st ed. New York: Wiley, 1967.