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Key Points:

- We develop an unsupervised machine learning approach to resolving directivity modes in earthquake populations
- The problem is formulated as recovering the latent modes of rupture propagation that exist in the data naturally
- Across four strike-slip datasets in California with thousands of earthquakes, unilateral ruptures occur 63–73% of the time

Supporting Information:

- Supporting Information S1
- Data Set S1
- Data Set S2
- Data Set S3
- Data Set S4
- Data Set S5
- Data Set S6
- Data Set S7
- Data Set S8

Correspondence to:

Z. E. Ross,
zross@caltech.edu

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Directivity Modes of Earthquake Populations with Unsupervised Learning

Zachary E. Ross¹ , Daniel T. Trugman² , Kamyar Azizzadenesheli³ ,
and Anima Anandkumar³

¹Seismological Laboratory, California Institute of Technology, Pasadena, CA, USA, ²Geophysics Group, Los Alamos National Laboratory, Los Alamos, NM, USA, ³Computing and Mathematical Sciences, California Institute of Technology, Pasadena, CA, USA

Abstract We present a novel approach for resolving modes of rupture directivity in large populations of earthquakes. A seismic spectral decomposition technique is used to first produce relative measurements of radiated energy for earthquakes in a spatially compact cluster. The azimuthal distribution of energy for each earthquake is then assumed to result from one of several distinct modes of rupture propagation. Rather than fitting a kinematic rupture model to determine the most likely mode of rupture propagation, we instead treat the modes as latent variables and learn them with a Gaussian mixture model. The mixture model simultaneously determines the number of events that best identify with each mode. The technique is demonstrated on four datasets in California, each with compact clusters of several thousand earthquakes with comparable slip mechanisms. We show that the datasets naturally decompose into distinct rupture propagation modes that correspond to different rupture directions, and the fault plane is unambiguously identified for all cases. We find that these small earthquakes exhibit unilateral ruptures 63–73% of the time on average. The results provide important observational constraints on the physics of earthquakes and faults.

1. Introduction

The kinematics of the earthquake source process have a first-order effect on seismic ground motions. In particular, it is well-known that for moving sources, rupture directivity induces azimuthally dependent changes in the frequency content of seismic waves (Haskell, 1964). Often earthquake ruptures are simplified as falling into one of two end-member modes: bilateral (symmetric) ruptures and fully unilateral ruptures. These rupture modes produce distinct signatures in the seismic radiation. Some studies discuss a continuum of scenarios in between these modes for which there is some degree of asymmetry in the distribution of seismic moment (e.g., Boatwright, 1984).

Various studies of earthquake physics have suggested various links between rupture propagation and properties of fault zones. For example, large earthquakes that saturate the seismogenic zone are often viewed as being constrained geometrically to have elongated ruptures (McGuire et al., 2002). The bimaterial hypothesis (Andrews & Ben-Zion, 1997; Weertman, 1980) predicts that for faults with a velocity contrast across them, the rupture direction that is aligned with the direction of slip in the more compliant medium will be favored due to a dynamic reduction of normal stress at the crack tip. This model has led to a considerable body of numerical and observational studies (e.g., Ampuero & Ben-Zion, 2008; Brietzke et al., 2009; Zaliapin & Ben-Zion, 2011) and also generated discussion over its role in earthquake physics (Andrews & Harris, 2005; Ben-Zion, 2006a, 2006b; Harris & Day, 2005). Other models may anticipate earthquakes to rupture with unilateral directivity as a frictional phenomenon, such as when the nucleation size is much smaller than the size of a seismogenic patch (e.g., Lin & Lapusta, 2018; Michel et al., 2017). It is desirable to better understand the relevance of these models to earthquakes in nature and whether there are resolvable differences in the properties of faults that can lead to different physics. Providing observational constraints on some of these physical models would require assembling directivity measurements for large numbers of earthquakes in order to perform a statistical analysis.

The extent of rupture directivity is readily determined for $M \geq 6$ earthquakes with various seismological methods (e.g., McGuire et al., 2001; Van Houtte & Denolle, 2018; Ye et al., 2016). These events are

Figure 1. Earthquake source spectra are calculated for events in a compact cluster, as recorded by many stations with varying azimuths. Then, relative values of radiated energy are calculated for all stations and events. The set of energy values for all earthquakes is decomposed into K distinct rupture propagation modes. The decomposition simultaneously determines the fraction of events that belong to each mode.

recorded typically by many stations and are sufficiently large that a slip model can be obtained using seismic waveforms recorded at teleseismic distances. However, the number of these large events on any single fault is too few to determine the statistical properties of rupture directivity patterns. Small earthquakes on the other hand are plentiful on individual faults due to the well-known power law magnitude distribution. There have been various approaches to estimating rupture directivity in small earthquakes, such as the use of higher-order moment tensors (McGuire, 2004), fitting directivity functions (Abercrombie et al., 2017; Boatwright, 2007; Kane et al., 2013; Tan & Helmberger, 2010; Wang & Rubin, 2011), measuring spectral splitting (Ross & Ben-Zion, 2016; Calderoni et al., 2015), measuring variations in apparent duration among repeating earthquake sequences (Lengliné & Got, 2011), and azimuthal analyses of ground motion (Kurzon et al., 2014). For most of these approaches, empirical Green's functions are needed to correct the data for propagation and site effects (e.g., Hough & Dreger, 1995; Mueller, 1985; Prieto et al., 2004), and then some kind of geophysical inverse problem is solved to obtain measurements about directivity.

In this paper, we present a data-driven approach to learning modes of directivity in earthquake populations that can scale to large datasets (Figure 1). We demonstrate that for a population of earthquakes, the azimuthal distribution of radiated energy naturally decomposes into distinct modes of rupture propagation without assuming a fault plane geometry or solving a geophysical inverse problem. The modes generally represent unilateral ruptures in different directions as well as bilateral ruptures. This decomposition simultaneously determines the fraction of events in each mode. We demonstrate the method on four datasets from California with thousands of events each and clearly identify the fault plane and rupture directivity modes for all four cases. We show that the earthquakes in these clusters have predominantly unilateral ruptures, with bilateral ruptures being relatively infrequent.

2. Methods

Our approach to resolving directivity modes is based on the idea that an individual earthquake within a population can be represented as a random sample from one of a handful of end-member rupture scenarios. For strike-slip faults, the number of observable cases might be three: a bilateral rupture with symmetric rupture propagation and two unilateral rupture modes that propagate in opposite directions along a fault. Alternatively, one might want to only consider unilateral directivity modes, with bilateral ruptures corresponding to a superposition of unilateral modes. The problem can then be formulated as one of recovering the distinct rupture modes that exist in the data. Our approach can be summarized with the following steps:

1. A cluster of earthquakes is identified for detailed analysis. This cluster should be compact enough that the separation between events is much less than the source-receiver distances.
2. The seismograms from these earthquakes are used with a seismic spectral decomposition algorithm to obtain apparent source spectra for as many events and stations as possible (Figure 1, Step 1).
3. Relative values of radiated energy at each station are determined by integrating the square of the velocity spectrum (Figure 1, Step 2).
4. We apply an algorithm for filling in missing data values by learning a low-rank approximation to the entire dataset.
5. Directivity modes are learned with a Gaussian mixture model (GMM) (Figure 1, Step 3).
6. Each mode can be visually interpreted as an azimuthal directivity function for a rupture mode that exists repeatedly in the data. The modes are obtained without solving a geophysical inverse problem.

We now discuss each of these steps in detail.

2.1. Data Preprocessing and Quality Control

The first step of the procedure involves selecting a cluster of earthquakes for analysis of rupture propagation. In this study, we use spatially compact clusters of dimension 5–8 km such that the azimuth to each station is roughly the same for all events. We restrict the minimum source-receiver distance to 20 km to help ensure this approximation is valid, although most stations are further away than this. For each event, we obtain all available automated and manual *P* wave picks, and for each associated waveform, we use these picks to define a 1.5-s signal window starting 0.1 s before the listed *P* wave arrival and a background noise window of equal length immediately preceding it. The noise windows are used to calculate signal-to-noise ratio (SNR). We use a multitaper algorithm (Krischer, 2016; Prieto et al., 2009) to compute seismic amplitude spectra on all available vertical-component channels with sampling rates ≥ 100 Hz. We convert each spectrum to units of displacement and interpolate where necessary to a uniform frequency spacing of 2/3 Hz with minimum and maximum values of 0 and 50 Hz (the Nyquist frequency for 100 Hz sampling).

For the purposes of quality control, we further consider only spectra with SNR greater than 3 within three frequency bands spanning the 3 to 25 Hz range. We focus our analysis on small-to-moderate (M1–4) earthquakes, relying on the SNR criteria to determine whether an individual spectrum should be used for smaller events. The time window for the spectral estimate is long enough to encompass the expected *P* wave duration of most events in our dataset, without extending into the *S* wave window at most stations. Clipped waveforms are commonly observed on short-period and broadband stations for moderate earthquakes, and to mitigate this, we further exclude spectra flagged by an automated clipped detection algorithm based on the fourth moment of the time domain waveform amplitudes observed in the signal window (Trugman & Shearer, 2017). The remaining displacement spectra after these waveform preprocessing steps and quality control steps are the inputs for the spectral decomposition method described below.

2.2. Spectral Decomposition

Spectral decomposition is a technique designed to separate the observed displacement spectra $d(f)$ of earthquakes i recorded and a set of seismic stations j into source, path, and site terms. As described in detail by Trugman and Shearer (2017), for densely recorded datasets, each earthquake will be recorded by many stations, each approximate source-station path will be sampled many times, and each station will record many earthquakes. If these conditions hold, then by working in the log spectral domain, relative source terms s_i , path terms p_{ij} , and site terms t_j can be estimated at each frequency point as part of an overdetermined inverse problem defined by the linear equation:

$$d_{ij}(f) = s_i(f) + p_{ij}(f) + t_j(f) + \epsilon_{ij}(f). \quad (1)$$

Here, $\epsilon_{ij}(f)$ are the residual terms. Following Shearer et al. (2006) and Trugman and Shearer (2017), we assume azimuthally isotropic path terms p that depend only on the source-receiver travel-time. We limit our analysis to stations with epicentral distances less than 100 km, and to ensure that the basic assumptions of spectral decomposition hold, we require each station used in the analysis to have recorded at least 20 earthquakes. The spectral decomposition technique does not have specific requirements on the distribution of focal mechanisms or dimensions of the cluster of events examined; the path and site terms can be determined even if all of the events are of similar location and focal mechanism. We estimate the source, path, and site terms defined by equation (?) using an iterative, robust least-squares inversion that uses Huber-norm weighting to suppress the influence of outliers on the final solution (Shearer et al., 2006).

Our primary observational constraints come not from the source spectra $s(f)$ themselves, but instead the apparent source spectra $a(f)$ observed at each station and how they vary as a function of azimuth:

$$a_{ij}(f) = d_{ij}(f) - p_{ij}(f) - t_j(f). \quad (2)$$

The apparent source spectrum $a(f)$ is thus a path- and site-corrected form of the observed displacement spectrum, whereas the source spectrum $s(f)$ itself is the azimuthal average of apparent source spectra across all stations (to good approximation). Rupture directivity can cause significant azimuthal variations in apparent spectra, with higher amplitudes along azimuths aligned with rupture direction. One general limitation of the spectral decomposition method is that the estimated terms only resolve relative differences between each source, each path, and each site, as one can add an arbitrary function to all source terms and subtract the same function from all site or path terms without effecting the misfit in the spectral inversion. While this presents a challenge for estimating earthquake source parameters like corner frequency and stress drop (Shearer et al., 2019), it does not affect the results presented in this study, which are based upon on the relative variations in spectral energy as a function of azimuth, and not the absolute energy values.

An assumption in using the spectral decomposition for our purposes is that the events in each cluster have similar focal mechanisms. Under these conditions, the radiation pattern will be comparable for all events at a given station, and as a result, the azimuthal variations in the radiation pattern will map into the $t_j(f)$ terms. For the datasets examined in this study, such conditions are generally appropriate; however, for a cluster of events with heterogeneous focal mechanisms, radiation pattern effects will remain in the $a_{ij}(f)$ terms and may partially obscure the directivity signal.

2.3. Building the Data Matrix

By this point, we have obtained apparent source spectra for many events at many stations. From here, we proceed to calculate apparent radiated energy values for the spectra. For an apparent source displacement spectrum, $a(f)$, the apparent radiated energy (up a multiplicative constant) can be calculated as

$$E_R = 4\pi^2 \int_{f_{min}}^{f_{max}} f^2 |a(f)|^2 df. \quad (3)$$

The optimal values of f_{min} and f_{max} generally depend on the instrument response, sampling rate, and the magnitude range of the events for which the directivity analysis will be performed. They further depend on the SNR of the spectra. Our spectra generally have low SNR below 2 Hz and above 30 Hz, so we set f_{min} and f_{max} equal to these values.

Next, we normalize the E_R values separately for each event by dividing by the median value calculated over all of the stations. This results in a set of values that indicate whether E_R at a given station is greater than or less than the median. In doing so, we require a minimum of 10 stations for which E_R values are available, or else, the event is skipped altogether. We then remove all stations from the dataset which have fewer than 50 spectra over all the events. Then, we identify outlier values by calculating the median absolute deviation (in log units) and looking to see whether there are any values for a single event that are more than 5 deviations away from the median. If so, these values are removed from the dataset. If the outlier removal process results in fewer than 10 values for a single event, the event is skipped.

Applying the steps described in this section results in a set of relative E_R values for many events and stations (e.g., Figure 1, Step 2). However, of the complete set of stations available, few if any events will have E_R values at all of them. This is especially true for the smallest events in the dataset, which are much more frequent than the larger ones. To fit a latent variable model to the data, there cannot be any missing values, and therefore, they must be filled.

In applied mathematics, there is an extensive literature on data imputation algorithms for filling missing data. The simplest form of these algorithms fills missing values with the mean or median for each variable. Other algorithms are more sophisticated, trying to learn a lower dimensional representation of the data from the observations that can be used to reconstruct the missing values. Algorithms of this type have seen some use in geophysics, for example, to fill gaps in GPS data prior to performing principal component analysis (e.g., Kositsky & Avouac, 2010). In this study, we chose an imputation algorithm that learns a low-rank approximation to the existing data, which can then be used to fill in missing values. This technique is based

Figure 2. Examples of relative $\log E_R$ distributions for five earthquakes. The data are shown before (red circles) and after (black circles) applying the imputation algorithm. Blue text indicates fraction of missing data for each example.

on thresholding a singular value decomposition of the data (Hastie et al., 2014; Mazumder et al., 2010). Some examples of applying this technique are shown in Figure 2, where real values are shown alongside imputed values. Filled values have a tendency to be generally close to zero (the mean) unless there is significant evidence in the data otherwise.

2.4. Gaussian Mixture Models

GMMs are a type of latent variable model that assumes a generative process can be represented as a superposition of two or more Gaussian distributions. Each Gaussian represents a distinct mode of occurrence, and there is a probability w_i that a realization $x \in \mathbb{R}^d$ will be drawn from mode i . Specifically, the model can be written as

$$p(x) = \sum_{k=1}^K w_k \mathcal{N}(x|\mu_k, \Sigma_k), \quad (4)$$

$$\sum_{k=1}^K w_k = 1, \quad (5)$$

where K is the number of components in the GMM and μ_k and Σ_k are the mean vector and covariance matrix of the multivariate Gaussian distribution, $\mathcal{N}(x|\mu_k, \Sigma_k)$. Since μ_k and Σ_k are not directly observed, they are latent variables; however, they can be recovered by fitting a GMM to some data. The most common approach to estimating the parameters of a GMM is with the Expectation-Maximization algorithm (Dempster et al., 1977), which is the method we use in this paper. We also tested the method of moments algorithm (Anandkumar et al., 2014) that matches the empirical moments to the theoretical ones through a tensor decomposition algorithm. We found that the Expectation-Maximization algorithm was more effective at separating the modes for the datasets considered in this study, which probably relates to assumptions about the statistical properties of the data.

Figure 3. Results of applying the full procedure (imputation and GMM) to three synthetic datasets. Circles indicate average energy values at individual stations, colored by the mode (i.e., the values of μ_k). Black lines denote ground truth. Here, $K = 2$. The method recovers the modes in all three cases, although when 75% of the data is missing, there is a noticeable drop in performance.

Our measured $\log E_R$ values for each earthquake correspond to the x in the GMM, and we therefore wish to obtain the μ_k and Σ_k for the K modes in the data. For this paper, we will work with strike-slip faults and consider both $K = 2$ and $K = 3$ cases. The $K = 2$ scenario would ideally recover two unilateral directivity modes, while the $K = 3$ scenario would correspond to a bilateral rupture mode and two opposite unilateral modes. We will further assume that the covariances are spherical, that is, $\Sigma_k = \sigma_k^2 I$. With this formulation, each μ_k is a vector with dimensionality equal to the number of stations used in the analysis, and the measured $\log E_R$ values of an individual earthquake (from all of the stations) are modeled as a random sample from the GMM.

The spherical covariance assumption results in a model where, for each mode K , the mean energy is determined separately for each station, but there is a single common variance for all stations. In other words, we assume that while the average radiated energy may differ at each station, the event-to-event variability does not. This allows for a concise model that captures the relative energy amplification on each station within each mode. From models of earthquake ruptures, it is known that strong directivity focuses the energy flux into a relatively narrow solid angle (e.g., Kaneko & Shearer, 2015; Venkataraman & Kanamori, 2004), and our approach is well-suited to handle these cases provided that there are stations present to observe them. Therefore, while our method does not have an explicit requirement for uniform sampling over the focal sphere, having a well-sampled azimuthal distribution allows for these energy focusing effects to be better resolved, therefore enabling more efficient modal decomposition with the GMM. While in reality there is strong azimuthal correlation, we opted for using a spherical covariance model as learning a full covariance matrix would substantially increase the number of free parameters. We found that even a model with spherical covariances has sufficient expressiveness to capture the desired phenomena. Parametric models for the covariance between stations as a function of azimuth could be used (e.g., radial basis functions), but this we leave to future work.

Figure 4. Map of the Hayward fault and surrounding region. Earthquakes are shown as red dots. Seismic stations used in this study are indicated by blue triangles.

2.5. Uncertainty Analysis

To estimate the uncertainty in the model parameters, we use a bootstrapping approach. We resample the events with replacement 1,000 times and re-fit the model each time. Since the order of the elements of w is randomly determined when the model is fit, we obtain all permutations of the columns of μ and calculate the ℓ_1 norm of the difference between the best fitting μ^{best} and each of the permutations. The permutation with the smallest ℓ_1 value is taken as the w for that particular resampling. Then we calculate a 95% confidence interval for each of the w_k .

2.6. Synthetic Tests

Before applying the full method to real data, it is instructive to first examine the performance on synthetic examples. We generate synthetic data using a cosine azimuthal function with a strike direction of 0° . In these datasets, the rupture direction can be in either of the two strike orientations. Each dataset consists of 300 synthetic events. We create 20 stations uniformly spaced over the azimuth range, and to further demonstrate the performance in the presence of azimuthal gaps, we create a 90° gap over the azimuth range 0° to 90° (Figure 3). Furthermore, rather than make each mode equally likely, we set the probability for one mode to be 75% and the other therefore 25%. Gaussian noise with $\mu = 0$ and $\sigma = 0.1$ is added to each value. We then create three datasets with increasing amounts of data removed. In the first dataset, we then randomly remove 25% of the values; in the second, this increases to 50%, and for the third, 75% of the values are missing. We then apply the imputation algorithm to fill in missing values and fit the GMM to the imputed values.

The results for each dataset are shown in Figure 3. It can be seen that the procedure recovers both modes with almost perfect accuracy even with 50% of the data missing. For the dataset with 75% missing values, the modes are somewhat distorted in shape from the truth, but the general azimuthal pattern is still clearly discernible. Thus, the results show that full azimuthal coverage is not necessary to use the procedure, although this can help to improve interpretability of the modes. We have tested an even wider range of scenarios than

Figure 5. Two-mode decomposition for the Hayward fault. The modes correspond directly to the w_k determined by fitting the GMM (Table 1). Black lines denote a cosine azimuthal function aligned on the strike of the fault.

just these and have in general found the method to function quite well even for cases in which it might have been expected to fail due to data limitations.

3. Related Work

In this study, our goal is to estimate the fraction of events for each rupture directivity mode. We are interested in working with hundreds of earthquakes at a time to ensure that the statistics are robust. To date, there are several studies that have examined directivity on such a scale.

Wang and Rubin (2011) calculated spectral ratios using empirical Green's functions and fit three different models (bilateral and two unilateral) to each event separately to determine the most likely rupture mode. They perform this analysis in the creeping section of the San Andreas fault and test the method on more than 900 earthquakes. They concluded that roughly 40% had bilateral ruptures, and of the unilateral ruptures, most had southeast directivity.

Kane et al. (2013) used a spectral decomposition technique to obtain apparent source spectra for thousands of events at the Parkfield section of the San Andreas fault. They fit a unilateral directivity model to the apparent spectra for each event individually and concluded that there was slightly more events with southeast ruptures than northwest ruptures. They did not consider bilateral ruptures.

Table 1
Modal Results for the Hayward Fault

Mode	$w (K = 2)$	$w (K = 3)$
NW	0.54 [0.32, 0.57]	0.47 [0.39, 0.56]
SE	0.46 [0.43, 0.68]	0.29 [0.21, 0.38]
Bilateral		0.27 [0.18, 0.30]

Figure 6. Three-mode decomposition for the Hayward fault. The modes correspond directly to the w_k determined by fitting the GMM (Table 1).

Building on the results of Calderoni et al. (2015) and Pacor et al. (2016) for the L'Aquila, Italy, earthquake sequence, Calderoni et al. (2017) used azimuthal variations in S wave spectra to demonstrate that along-strike directivity is a common feature of normal faulting earthquakes in the central Apennines. Earthquakes from the Umbria Marche, L'Aquila, and 2016–2017 central Italy sequences exhibit temporally persistent and spatially coherent directivity patterns, which suggests that in situ fault and geologic properties play an important role in the preferred rupture direction.

A different approach was developed by Lengliné and Got (2011), who exploited repeating earthquake sequences in Parkfield to simultaneously invert for relative perturbations to apparent duration for all earthquake pairs and stations. They used P wave signals for their analysis, and their kinematic directivity model only allowed for unilateral ruptures. Lengliné and Got (2011) concluded that the majority of earthquakes analyzed exhibited southeast rupture signals.

A time domain method based on ground-motion prediction equations was developed by Kurzon et al. (2014). They proposed an empirical directivity index based on observed azimuthal variations in ground motion amplitude. They applied the method to two clusters of earthquakes in the central San Jacinto fault zone.

4. Experiments

Now, we demonstrate the method on four different datasets from California. These datasets use P wave seismograms only and were chosen for several reasons. First, each dataset is a spatially compact cluster with thousands of events. Second, two of them have been used in previous studies to analyze directivity and can serve as a point of reference.

4.1. Hayward Fault

The first dataset analyzed in this paper is the Hayward fault. The seismicity here occurs over a long stretch of the fault, and so we chose a compact cluster to work with to ensure that the azimuths were similar to all events. There were 4118 events from 2002 to 2019 over the whole area, and we used these to determine the

Figure 7. Map of the Cahuilla swarm and surrounding region. Earthquakes are shown as red dots. Seismic stations used in this study are indicated by blue triangles.

source spectra (Figure 4). All of the data are publicly available from the Northern California Earthquake Data Center. The Hayward fault has a velocity contrast in the range 3–8% (Allam et al., 2014).

After all of the preprocessing, the final Hayward dataset comprises 206 earthquakes at 59 stations (Data Sets S1 and S2 in the supporting information). First, we show the results for $K = 2$ in Figure 5 and Table 1. In this plot, each mode is assigned a different color. The elements of μ_k represent the average value of $\log E_R$ at a given station for mode k . Since there are two modes, each station has two different colored circles present. Values larger than zero indicate that $\log E_R$ at that station are amplified relative to the median (for that mode). μ_1 and μ_2 have peaks along the strike of the fault but in opposite directions. There is more than a factor of 3 difference in the E_R between these opposing directions. Thus, the method has been able to identify the fault plane, and these modes represent unilateral directivity. The data separate clearly into two modes with directivity behavior in NW and SE directions, which aligns with the strike of the fault. The w values for the modes vary significantly for the best-fitting values, but after examining the confidence intervals, these differences are found to not be statistically significant.

When applying a $K = 3$ model to the data (Figure 6), the modes are distinct and form smoothly varying patterns as a function of azimuth. The three modes also have sizable uncertainties as estimated by the bootstrapping, but we can say that unilateral modes occur between 70% and 82% of the time; thus, they are the most frequent mode of rupture.

It should be noted that the modal data plotted are the centroid vectors, which by definition define the center of a given mode. As the modes in a GMM are represented by not only the centroids but also the covariance matrices about these modes, the centroids themselves will represent some kind of middle ground. Thus, the most extreme unilateral ruptures will have azimuthal functions that look quite different than the functions shown in Figure 5. For reference, a cosine function aligned with the strike is shown (with amplitude

Figure 8. Two-mode decomposition for the Cahuilla swarm. The modes correspond directly to the w_k determined by fitting the GMM (Table 2).

scaled to match the observations), which is an oversimplification (Venkataraman & Kanamori, 2004), but qualitatively similar to our observed azimuthal dependence.

4.2. Cahuilla Swarm

This dataset contains seismograms for 11,631 events that occurred as part of the Cahuilla earthquake swarm in Southern California during 2016–2019 (Hauksson et al., 2019; Figure 7). The largest of these events is $M_{4.4}$, while most events are considerably smaller. These events were studied in detail by Hauksson et al. (2019), who observed that the events have right-lateral strike-slip focal mechanisms with generally little variation in strike. The waveform and metadata are all publicly available from the Southern California Earthquake Data Center.

The results of applying the method to the Cahuilla swarm data are shown in Figure 8 and Table 2. In total, there are 845 earthquakes and 85 stations used for the final GMM decomposition (Data Sets S3 and S4). For these two modes in the Cahuilla swarm (Table 2), $w_1 = 0.62$ and $w_2 = 0.38$. The confidence intervals suggest that these differences are statistically significant.

For the $K = 3$ decomposition (Figure 9), two of the modes look very similar to the $K = 2$ results. The third mode exhibits more complex behavior, with four peaks and four troughs. These peaks are essentially at conjugate orientations and suggest that there may be cross-faulting mixed in. Events have bilateral ruptures between 24% and 34% of the time (Table 2). Thus, it is the least likely of the three modes to occur.

Table 2
Modal Results for the Cahuilla Swarm

Mode	w ($K = 2$)	w ($K = 3$)
NW	0.62 [0.56, 0.69]	0.39 [0.35, 0.45]
SE	0.38 [0.31, 0.44]	0.31 [0.27, 0.35]
Bilateral		0.30 [0.24, 0.34]

Figure 9. Three-mode decomposition for the Cahuilla swarm. The modes correspond directly to the w_k determined by fitting the GMM (Table 2).

4.3. San Andreas (Creeping Section)

The third dataset analyzed in this paper is for the creeping section of the San Andreas fault. This area has a very active seismicity cluster that has produced 4,118 events from 2002 to 2019 (Figure 10). All of the data are publicly available from the Northern California Earthquake Data Center. This cluster was studied by Wang and Rubin (2011) and serves as an additional baseline for our work.

The $K = 2$ results for the creeping section of the San Andreas fault are shown in Figure 11 and Table 3. A total of 624 earthquakes and 68 stations were used to fit the model (Data Sets S5 and S6). As with the previous dataset, the fault plane is clearly identified for this segment, with prominent peaks in the NW azimuths for one mode and the SE azimuths for the other mode. The modes are relatively imbalanced, on average, but the large uncertainty as measured by the bootstrap analysis indicates that these differences are not statistically significant.

For the $K = 3$ decomposition, there are two unilateral modes (NW and SE) and one bilateral mode (Figure 12). The bilateral rupture mode is estimated to occur 32–43% of the time. Wang and Rubin (2011) found that about 40% of the earthquakes on this segment had bilateral ruptures, which is similar to our observations. They also found that of the unilateral ruptures, SE ruptures were more common; however, we find no evidence of this that is statistically significant.

4.4. San Andreas (Parkfield)

The final dataset is for the Parkfield section of the San Andreas fault in central California (Figure 13). We selected all 14,562 events in the NCEDC catalog that have occurred since 2002. This region was used for several directivity studies in the past (Kane et al., 2013; Lengliné & Got, 2011) and serves as a point of comparison for our results. The earthquakes at Parkfield have very homogeneous focal mechanisms (Thurber et al., 2006) which help to simplify the demonstrations of our approach.

In Figure 15, the results for the $K = 3$ decomposition are shown. We find that the bilateral mode occurs 28–46% of the time.

Figure 10. Map of the creeping section of the San Andreas fault. Earthquakes are shown as red dots. Seismic stations used in this study are indicated by blue triangles.

Our results for the Parkfield data ($K = 2$) are shown in Figure 14 and Table 4. The model was fit to 463 events at 56 stations (Data Sets S7 and S8). The strike and fault plane are clearly identified, with prominent peaks in the modes along the NW and SE azimuths. While the SE mode is on average more frequent, after factoring in the uncertainty from the bootstrap, this difference disappears. We note that Lengliné and Got (2011) studied repeating sequences in this area and concluded that most of the events had SE rupture directivity signals.

5. Discussion

5.1. The Importance of Studying Directivity Modes

Directivity modes characterize the first-order kinematics of rupture propagation during earthquakes. It is important to understand these modes and their statistical tendencies because they can provide valuable observational constraints on the physics of earthquakes. One of the important findings of this study is that for the four examined fault segments, unilateral ruptures are more frequent than bilateral ruptures. Specifically, the best-fitting rates for bilateral rupture vary from 27% to 37% over the four regions tested, and the uncertainty estimates show that this principal conclusion is robust. We find evidence of a statistically preferred rupture direction only for the Cahuilla swarm. For the other regions, the uncertainties on the w_i values are larger than the differences between them. This does not mean a preferred direction does not exist; rather the question could potentially be addressed better in the future by incorporating more events or improving the data, which may tighten the confidence intervals. In all cases, however, there is no evidence of a single unilateral rupture direction being overwhelmingly likely.

Another noteworthy finding is that for these fault segments, there is a clear preference for ruptures to propagate horizontally more often than vertically. As discussed below, along-dip ruptures will generally

Figure 11. Two-mode decomposition for the San Andreas fault (creeping section). The modes correspond directly to the w_k determined by fitting the GMM (Table 3).

be included in the bilateral mode because there is little sensitivity to them. This means that our conclusions about the predominance of unilateral ruptures not only applies with respect to bilateral cases but also along-dip cases. A similar observation was made by McGuire et al. (2002), who observed that many large earthquakes have unilateral ruptures. For earthquakes large enough to rupture the full seismogenic zone, along-strike ruptures are expected purely from a geometrical perspective, although this says nothing about whether they should be unilateral or bilateral. However, for small earthquakes, our observations may be unexpected. There may be attributes of the faults that are breaking the symmetry and leading to horizontal ruptures being so prevalent.

An additional reason that these findings are important is because observational studies of earthquake source properties commonly assume that rupture areas are circular (e.g., Abercrombie, 2015; Calderoni et al., 2012; Ross & Ben-Zion, 2016). However, the geometry of the rupture area has an important influence on quantities like stress drop and radiated energy (Kaneko & Shearer, 2015). Our results suggest that for the examined datasets, unilateral ruptures are more frequent than bilateral ruptures, and this may indicate that some of the large scatter commonly observed in source properties may result from assuming the wrong rupture geometry.

One physical explanation for the systematic tendencies for unilateral directivity in earthquakes is from rate- and state-dependent friction. When the nucleation size h is much smaller than the potential seismogenic

Table 3
Modal Results for the Creeping Section of San Andreas

Mode	w ($K = 2$)	w ($K = 3$)
NW	0.67 [0.47, 0.74]	0.38 [0.33, 0.43]
SE	0.33 [0.26, 0.53]	0.25 [0.20, 0.29]
Bilateral		0.37 [0.32, 0.43]

Figure 12. Three-mode decomposition for the San Andreas fault (creeping section). The modes correspond directly to the w_k determined by fitting the GMM (Table 3).

region, this results in ruptures that tend to propagate unilaterally (e.g., Lin & Lapusta, 2018; Michel et al., 2017). However, the expected rates of unilateral ruptures for this type of physical model have not been analyzed rigorously. Future work in this area could be helpful to compare with observations.

Directivity modes are also important from a hazard perspective because the rupture propagation pattern has a significant effect on the strong ground motion. While stress drop is widely believed to influence ground motion amplitudes in certain contexts (e.g., Baltay et al., 2017; Oth et al., 2017; Trugman & Shearer, 2018), directivity has received less attention to date but is readily acknowledged to be a source parameter that does have a major impact (Bozorgnia et al., 2014; Douglas & Edwards, 2016). Therefore, being able to provide some expectations on the possible range of modal probabilities can better inform hazard estimates.

5.2. Data-Driven Approaches to Studying Directivity

There are a variety of advantages to studying directivity with a data-driven approach, as opposed to using techniques that fit kinematic rupture models. The first is that a single model can be fit to the data for all earthquakes simultaneously. This allows for weaker signals to be extracted from the data because the information that is common to all events can be averaged, which acts to suppress noise. Traditional approaches generally fit a model separately to each event, and then if performed for enough events, then the results can be averaged for some statistical estimate. However, this type of approach is limited by the capabilities of fitting a model to individual events.

Another reason is that there are generally fewer assumptions necessary with a data-driven approach, compared with traditional model fitting. For example, we do not need to assume that our physical model is correct or that we understand the statistical properties of the noise. However, as in most analyses of earthquake directivity, we assume that azimuthal variations in seismic spectra are caused primarily by source effects, rather than 3D variations in path effects. While this is likely a valid approximation to first order, in reality the observed spectra will contain hints of both effects.

Figure 13. Map of the Parkfield section of the San Andreas fault. Earthquakes are shown as red dots. Seismic stations used in this study are indicated by blue triangles.

5.3. Assumptions and Interpretability of Results

In this study, there are several key assumptions that underlie the analysis. The first, and arguably the most important, is that we assumed the number of distinct modes of rupture propagation a priori. By making an assumption that a certain number of modes exist in the data, we are able to search for and identify their centroids μ_k and variances. If these assumptions are violated, for example, in the case that there are more than three distinct modes, then the resulting μ_k vectors will be complicated and uninterpretable. In all four datasets that we tested here, the μ_k are generally simple, smoothly varying functions of azimuth, which suggests that the assumptions are justified. However, when applying the data to other datasets, such as clusters with more than one dominant fault strike, more care will be needed to determine the optimal number of modes in the data. There are various strategies for determining this objectively, including the use of information criteria or silhouette analysis. We applied a silhouette analysis to the data and in all regions, the metric favored $K = 2$. However, as we have shown, $K = 3$ models provide additional insights into the directivity physics and we believe that examining both scenarios together provides a more informed result.

Another assumption in this study is that the spectral decomposition technique is correctly separating the genuine source component from propagation and site effects. As this technique is used to produce the apparent source spectra that are the basis for our analysis, the effectiveness of the procedure will have an important influence on the final results. We believe that most of the observed variability of the modal shapes between different datasets is resulting from the spectral decomposition algorithm. Further improvements to this approach, including adding more picks to smaller earthquakes, may reduce this variability.

The GMM assumes that a finite number of discrete modes exist in the data and that an event can be drawn from one of them. While it is reasonable to assume that there may be only two to three end-member modes present in the data, in reality, there may be more of a continuum of rupture scenarios between bilateral and unilateral (e.g., Boatwright, 2007). Therefore, while the GMM is still useful for understanding the structure of these types of datasets, the learned modes should be viewed as not quite end-member cases; instead, it is

Figure 14. Two-mode decomposition for the San Andreas fault (Parkfield section). The modes correspond directly to the w_k determined by fitting the GMM (Table 4).

more instructive to view the learned modes as structure that persists across the dataset, and for any given event, we can ask which of the three learned modes most closely matches that particular event.

If the recovered modes reflect approximately end-member rupture scenarios, one way of assessing the degree of unilateral directivity for a given event is by calculating the probability that an event belongs to the best-matching mode. This could be useful for identifying the events with the strongest directivity signals in an objective manner. However, it should be mentioned that this strategy will only determine whether an event best matches the centroid of the mode; if an event has even stronger directivity signals than the centroid, the probability will be diminished in proportion to the deviation from the centroid. This simply means that events with the highest probability values will not be the events with the strongest directivity.

Another important aspect of the results is that the modal shapes which we have identified as bilateral are a bit more complicated in practice. In particular, if a rupture is purely along-dip, then without stations right on top of the rupture, it is very difficult to observe any directivity patterns. These ruptures will appear as generally symmetric directivity patterns and will likely be aggregated into the bilateral mode. Thus, the bilateral mode (particularly the w_k value associated with it) should be interpreted as an upper bound for the likelihood of bilateral ruptures. It should be understood that this results from a lack of sensitivity to along-dip ruptures. Some studies have included other information, such as depth phases (e.g., He & Ni, 2017), to help constrain these cases, but in general, it is a challenge common to all directivity analyses.

Table 4
Modal Results for the Parkfield Cluster

Mode	w ($K = 2$)	w ($K = 3$)
NW	0.46 [0.34, 0.62]	0.33 [0.23, 0.41]
SE	0.54 [0.38, 0.66]	0.33 [0.22, 0.40]
Bilateral		0.34 [0.28, 0.46]

Figure 15. Three-mode decomposition for the San Andreas fault (Parkfield section). The modes correspond directly to the w_k determined by fitting the GMM (Table 4).

The results in the paper do not depend on the particular model used (e.g., GMM). We tested several other latent variable models including BIRCH and spectral clustering, and when setting the number of clusters to $K = 2, 3$, recover modes that look very similar.

6. Conclusions

We developed a new approach to resolving modes of directivity in large earthquake populations. We formulate the problem as one of recovering K latent variable modes from the azimuthal energy distributions of many earthquakes, where each mode is a distinct state of rupture propagation. A GMM is used to determine these modes, which allows for simultaneous estimation of the fraction of events that best align with each mode of rupture propagation. In the process, we have not needed to fit a kinematic directivity model to the data; the decomposition is possible because the data exhibit this type of structure naturally. We applied the method to four large clusters of earthquakes and performed an assessment of the uncertainties for each mode of rupture, for each dataset. Our results indicate that for the examined strike-slip clusters, unilateral ruptures are more likely to occur than bilateral ruptures, even after incorporating the uncertainties.

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