Intermodulation distortion in a directly modulated semiconductor injection laser

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A most important quantity in high-frequency analog transmission is the intermodulation distortion product. Experimental studies of the third order intermodulation distortion products in the modulation response of high-speed semiconductor lasers give very low values (< -60 dB) at low frequencies, an increase at a rate of 40 dB/dec as the modulation frequency is increased, and a leveling off at one-half of the relaxation oscillation resonance frequency. These experimental results can be well explained by a theory based on a perturbative analysis of laser dynamics.

Recent investigations in very high speed direct modulation of semiconductor lasers have identified three key laser parameters of practical relevance that directly affect the modulation bandwidth, namely, the differential optical gain constant, the photon lifetime, and the internal photon density at the active region.[1] Successful tackling of each of these three parameters led to the first semiconductor lasers with a direct modulation bandwidth exceeding 10 GHz.[2,3] A direct modulation bandwidth of 12 GHz was obtained with a short cavity buried heterostructure laser fabricated on semi-insulating substrate (BH on SI) operating at -50 °C,[2] and an equally large bandwidth was achieved at room temperature with a "window" BH on SI laser.[3] One important application for multigigahertz bandwidth semiconductor laser is multichannel frequency division multiplexed transmission of analog or microwave signals. An obvious quantity of concern here is the nonlinear distortion characteristics of the laser. It is well known that a well behaved semiconductor laser (i.e., those with a linear light-current characteristic without kinks and instabilities above lasing threshold) exhibits very little nonlinear distortion when modulated at low frequencies (below a few tens of megahertz).[4] This is to be expected since at such low modulation speed, the laser is virtually in a quasi-steady state as it is ramping up and down along the light-current curve, and consequently the linearity of the modulation response is basically that of the cw light-current characteristic, which is excellent in well behaved laser diodes. Measurements and analysis have shown that second harmonic distortions of lower than -60 dB can be readily accomplished at the low-frequency range.[4] However, it was also shown that as the modulation frequency increases, the harmonic distortions increase very rapidly—at modulation frequencies above 1 GHz, the second and third harmonics can be as high as -15 dBc at a moderate optical modulation depth ($\sim 70\%$).[5-7] These results can be well explained by a perturbative analysis of the laser rate equations, which depicts the interaction of the photon and electron fluctuations as the origin of the large harmonic distortions observed at high frequencies.

In actual applications of multichannel signal transmission where baseband signals from different channels are carried on a number of well separated high-frequency carriers, second (or higher order) harmonic distortions generated by signals in a channel are actually of little concern since they generally do not fall within the frequency band of that particular channel (and, if the frequency bands are properly chosen, any other channels). The relevant quantity of concern here is the third order *intermodulation* (IM) product of the laser transmitter: two signals at frequencies ω_1 and ω_2 within a certain channel can generate intermodulation products at frequencies $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$, which in most likelihood will lie within that particular channel and is thus undesirable. There has been very little quantitative evaluation, experimental and theoretical, of this parameter which is extremely important in practical system applications. Questions which need answer are the dependence of IM product on modulation depth, signal frequencies, magnitude of relaxation oscillation, etc. These are the topics of consideration in this paper.

We have studied the IM characteristics of high-speed laser diodes capable of being modulated at multigigahertz frequencies. The experimental study consists of modulating the lasers with two sinusoidal signals, 20 MHz apart, and observing the various sum, difference, and harmonic frequencies thus generated. The major distortion signals considered here are shown in Fig. 1. The principal distortion signals of practical concern, as mentioned above, are the third order IM products, at frequencies $2\omega_1 - \omega_2$ and

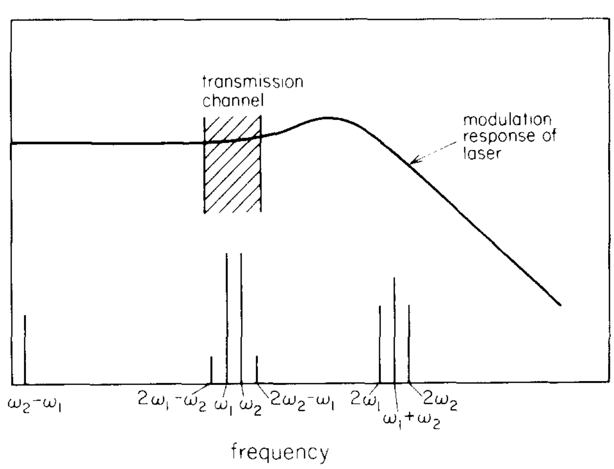

FIG. 1. Sidebands and harmonics generated by a two-tone modulation of a laser diode. This simulates narrowband signal transmission at a high carrier frequency.

$2\omega_2 - \omega_1$. The various distortion signals are systematically studied as one varies the frequency (ω) of the modulating signals (the two signals are at ω and $\omega + 2\pi \times 20$ MHz), the optical modulation depth (OMD), and laser bias level. The OMD is defined as B/A, where B is half of the peak-to-peak amplitude of the modulated optical waveform and A is the optical output from the laser at the dc bias level. The major observed features are summarized as follows.

(1) At low modulation frequencies (a few hundred MHz) all the lasers tested exhibit very low IM products of below -60 dBc (relative to the signal amplitude) even at an OMD of approaching 100%. (2) The second harmonics of the modulation signals increase roughly as the square of the OMD, while the IM products increase as the cube of the OMD. (3) The relative amplitude of the IM produce (relative to the signal amplitude) increases at a rate of 40 dB/decade as ω increases, reaching a plateau at one-half of the relaxation oscillation frequency, and picks up the 40 dB/decade increment as ω exceeds the relaxation oscillation frequency. A typical value of the IM product at the plateau is -45 to -50 dBc at an OMD of 50%. (4) In some lasers the IM product may show a peak at one-half of the relaxation oscillation frequency. The magnitude of this peak is found to roughly correspond to the magnitude of the relaxation oscillation resonance peak in the small-signal modulation response of the laser.

Figure 2 shows the IM and harmonic distortions of a high-frequency laser diode under the two-tone modulation

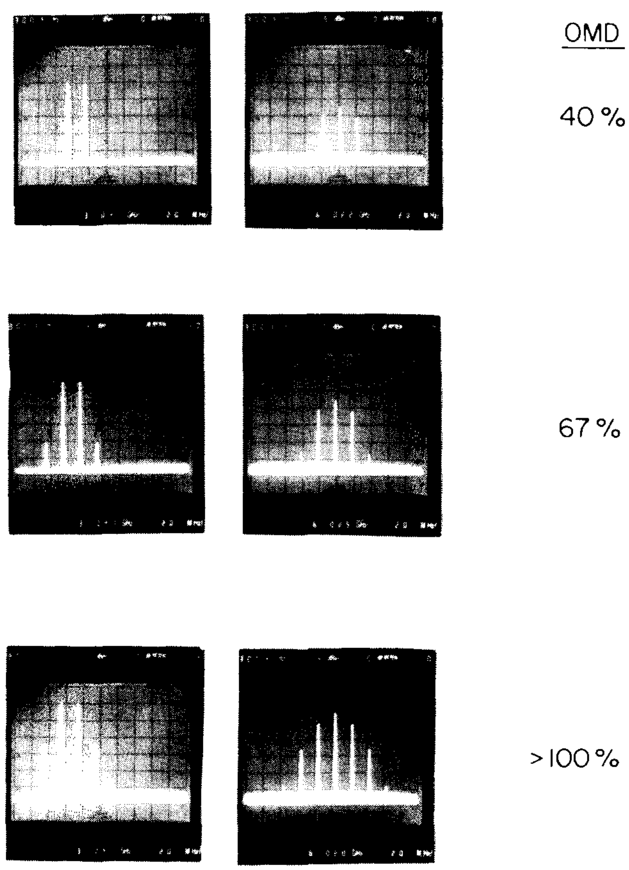

FIG. 2. Harmonics and IM products generated in a high-speed laser diode under a two-tone modulation. The two tones are 20 MHz apart centered at $\simeq 3$ GHz.

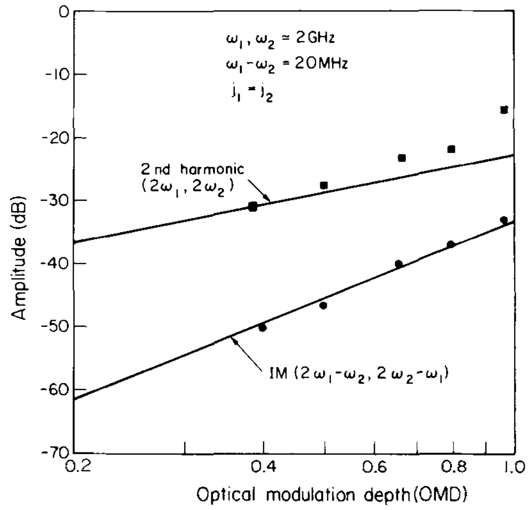

FIG. 3. Plots of second harmonic and third order IM amplitudes (relative to the signal amplitude) as a function of optical modulation depth (OMD), with the signal fregqency at 2 GHz. Experimental data points obtained with a high-frequency laser diode are also shown.

as described above, at $\omega = 2\pi \times 3$ GHz, at various OMD's. The relaxation oscillation frequency of this laser is at 5.5 GHz. The observed data of IM and second harmonic as functions of OMD and ω are plotted in the graphs in Fig. 3 and 4 respectively; the various curves in those graphs are from theoretical calculations as described below. The analytical results, based on the simplest rate equation model, can explain the above observed features very well.

The spirit of the analysis follows closely that employed in the previous harmonic distortion perturbative analysis. One starts out with the simple rate equations and assumes that the harmonics are much smaller than the fundamental. The photon and electron fluctuations at the fundamental modulation frequency are thus obtained with a standard small-signal analysis, neglecting terms of higher harmonics. The fundamental terms are then used as drives for the higher harmonic terms. In IM analysis where more than one fundamental drive frequencies are present, we shall concentrate on the distortion terms as shown in Fig. 1. We shall assume the

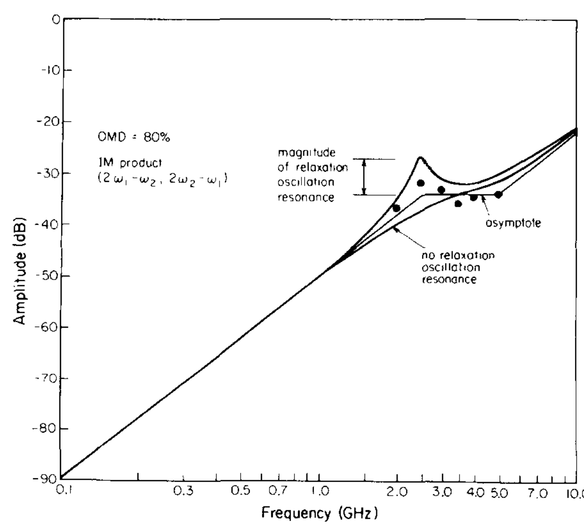

FIG. 4. Plots of third order IM amplitudes (relative to the signal amplitude) as a function of signal frequency, at an OMD of 80%.

TABLE I. Driving terms of the various harmonic and IM signals.

ω	$D(\omega)$	$G(\omega)$
$\omega_{1,2}$	$j_{1,2}$	0
$2\omega_{1,2}$	$\frac{1}{2}n^{\omega_{1,2}}p^{\omega_{1,2}}$	
$\omega_1 - \omega_2$	$\frac{1}{2}[n^{\omega_1}(p^{\omega_2})^* + p^{\omega_1}(n^{\omega_2})^*]$	
$\omega_1 + \omega_2$	$\frac{1}{2}(n^{\omega_1}p^{\omega_2} + p^{\omega_1}n^{\omega_2})$	same as $D(\omega)$
$2\omega_1 - \omega_2$	$\frac{1}{2}[n^{2\omega_1}(p^{\omega_2})^* + p^{2\omega_1}(n^{\omega_2})^* + n^{\omega_1 - \omega_2}p^{\omega_1} + p^{\omega_1 - \omega_2}n^{\omega_1}]$	
$2\omega_2 - \omega_1$	interchange ω_1 and ω_2 above	

following: the amplitudes of the fundamental terms (ω_1, ω_2) $\gg$ those of second order terms $(2\omega_1, 2\omega_2, \omega_1 \pm \omega_2)$ $\gg$ those of third order terms $(2\omega_{1,2} - \omega_{2,1})$. The perturbative analysis then follows in a straightforward manner. Denote the steady state photon and electron densities by P_o and N_o, and the fluctuations of the photon and electron densities by lower case n and p at a frequency given by the superscript. For each of the eight frequency components shown in Fig. 1, the small-signal photon and electron density fluctuations are given by the following coupled linear equations:

$$i\omega n^{\omega} = -(N_o p^{\omega} + P_o n^{\omega} + n^{\omega} + D^{\omega}) \tag{1a}$$

$$i\omega p^{\omega} = \gamma(N_o p^{\omega} + P_o n^{\omega} - p^{\omega} + \beta n^{\omega} + G^{\omega}). \tag{1b}$$

The driving terms, D^{ω} and G^{ω}, are given in Table I for each of the eight frequency components. The quantities j_1 and j_2 are the modulating currents at frequencies ω_1 and ω_2. The quantities γ and β in Eq. (1) are the ratio of the carrier lifetime to photon lifetime and the spontaneous emission factor, respectively. The ω's in Eq. (1) are normalized by $1/\tau_s$ where τ_s is the carrier lifetime and the n's, p's, and j's are normalized in the usual fashion.[8] One can solve for the n's and p's at each of the eight frequency components. To simplify algebra we shall consider a practical case of transmission of a single channel in a narrowband centered around a high-frequency carrier, as diagramatically depicted in Fig. 1. Specifically, the following is assumed: (1) $\omega_1 = \omega_c - \frac{1}{2}\Delta\omega$, $\omega_2 = \omega_c + \frac{1}{2}\Delta\omega$, $\Delta\omega \ll 1$, ω_c is the center frequency of the channel- (2) $\beta \ll 1$; $1 - N_o \sim \beta$. The first assumption implies that the carrier frequency ω_0 is much higher than $1/2\pi\tau_s$ (which is typically around 50 MHz). The second assumption is based on the fact that $\beta \lesssim 10^{-3}$. The amplitudes of the eight frequency components of Fig. 1 are as follows:

$$p^{\omega_{1,2}} = j_{1,2}/f(\omega_c), \tag{2}$$

where $f(\omega)$ takes the form of $1 + i\omega/\omega_0 Q + (i\omega/\omega_0)^2$, where $\omega_0 = \sqrt{\gamma P_0}$ is the relaxation oscillation frequency, and Q depends on, among other things, β and the bias level;

$$p^{2\omega_{1,2}} = (i\omega_c)^2 j_{1,2}^2/\gamma P_0^2 f(2\omega_c), \tag{3}$$

$$p^{\Delta\omega} = ij_1 j_2^* \Delta\omega/\gamma P_0^2, \tag{4}$$

$$p^{\omega_1 + \omega_2} = ij_1 j_2 (2\omega_c)^2/\gamma P_0^2 f(2\omega_c), \tag{5}$$

$$p^{2\omega_1 - \omega_2} = -\frac{1}{2}\frac{i j_1^2 j_2^* \omega_c^2(\omega_c^2 + \gamma P_0)}{\gamma^2 P_0^3 f(2\omega_c)}. \tag{6}$$

Taking $j_1 = j_2 = j$, the relative second harmonic $(p^{2\omega_{1,2}}/p^{\omega_{1,2}})$ and intermodulation $(p^{2\omega_1 - \omega_2}/p^{\omega_{1,2}})$, as given in Eqs. (2), (3), and (6), are plotted in Fig. 3 as a function of the OMD $(=2j/p_0)$, at a signal frequency of 2 GHz (i.e., $\omega_c/\tau_s = 2\pi \times 2$ GHz). The data points shown are obtained with a high-speed laser diode with the relaxation oscillation frequency at 5.5 GHz. The amplitude of the IM signal [Eq. (6)] is plotted in Fig. 4 as a function of carrier frequency ω_c, at a fixed OMD of 80%, assuming $\omega_0/\tau_s = 2\pi \times 5.5$ GHz. The IM characteristics at other values of OMD can be obtained by shifting the curve of Fig. 4 vertically by an amount as given in Fig. 3. The values of other parameters are $\tau_s = 4$ ns, $\tau_p = 1$ ps. The actual small-signal modulation response of the lasers tested showed almost no relaxation oscillation resonance, and the value of Q was taken to be 1 accordingly. The general trend of the experimental data agrees with theoretical predictions quite well.

The above results are significant in that (1) the linearity of the cw light-current characteristic (as well as distortion measurements at low frequencies) is not a reliable indication of the IM performance at high frequencies, (2) although the IM product initially increases at an alarming rate of 40 dB/dec at the modulation frequency is increased, it does settle to a steady value of ~ -45 dBc which is satisfactory for many applications, including, for instance, television signal transmission.

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