BINARY NEUTRON STARS IN QUASI-EQUILIBRIUM CIRCULAR ORBIT: A FULLY RELATIVISTIC TREATMENT

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We present a numerical scheme that solves the initial value problem in full general relativity for a binary neutron star in quasi-equilibrium. While Newtonian gravity allows for a strict equilibrium, a relativistic binary system emits gravitational radiation, causing the system to lose energy and slowly spiral inwards. However, since inspiral occurs on a time scale much longer than the orbital period, we can adopt a quasi-equilibrium approximation. In this approximation, we integrate a subset of the Einstein equations coupled to the equations of relativistic hydrodynamics to solve the initial value problem for binaries of arbitrary separation, down to the innermost stable orbit.

Neutron star binaries are of interest for several reasons. They exist, even within our own galaxy, and are among the most promising sources for gravitational wave detectors like LIGO, VIRGO and GEO. More fundamentally the two-body problem is one of the outstanding unsolved problems in classical general relativity.

So far most researchers have treated binary neutron stars in Newtonian theory [1]. In post-Newtonian treatments [2] the stars are usually treated as point-sources, so that hydrodynamical effects are absent. More recently, Nakamura [3] and Wilson and Mathews [4] have initiated studies of binary neutron stars in general relativity.

In our work we assume that the two stars have equal mass, are co-rotating and obey a polytropic equation of state, $P = K\rho_0^\Gamma$. In Newtonian gravity, a strict equilibrium solution for two stars in circular orbit can be found. Since this solution is stationary, the hydrodynamical equations reduce to the Bernoulli equation,

$$\frac{\Gamma}{\Gamma-1}\frac{P}{\rho_0} + \Phi - \frac{1}{2}\Omega^2(x^2+y^2) = C, \tag{1}$$

where C is a constant, Φ the gravitational potential, Ω the angular velocity, and where the rotation is about the z-axis. The gravitational potential satisfies Poisson's equation, $\nabla^2\Phi = 4\pi\rho_0$. These equations comprise a coupled system, containing a linear elliptic PDE in 3D, which must be solved numerically.

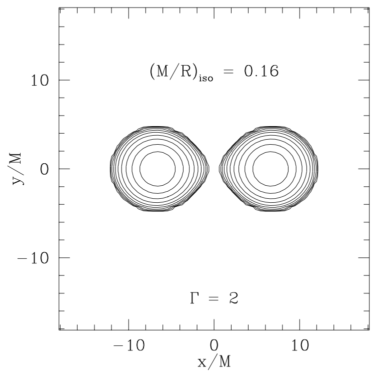

Figure 1: Rest density contours of a close, highly relativistic neutron star binary in the equatorial plane. The contours span densities logarithmically between the central density and 1 percent of that value.

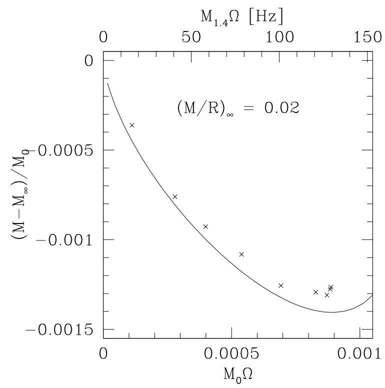

Figure 2: Binding energy of a $\Gamma = 2$ polytrope. The solid line is a result of a post-Newtonian calculation for compressible ellipsoids [5]; crosses are the result of this work.

Because of the emission of gravitational waves, a binary in general relativity cannot be in strict equilibrium. However, up to the innermost stable circular orbit (ISCO) the timescale for orbital decay by radiation will be much longer than the orbital period so that the binary can be considered to be in "quasi-equilibrium". This allows us to neglect both gravitational waves and wave-induced deviations from a circular orbit to good approximation.

To minimize the gravitational wave content we choose the 3-metric to be conformally flat [6]. The field equations then reduce to a set of coupled, quasi-linear elliptic equations for the lapse, shift, and the conformal factor. Neglecting small deviations from circular orbit, the fluid flow is again stationary, and the hydrodynamical equations reduce to the relativistic Bernoulli equation. Solving these equations yields a valid solution to the initial value (constraint) equations and an approximate solution to the full Einstein equations at any given moment, prior to plunge.

As in the Newtonian case, a system of coupled elliptic equations must be solved, and since the two problems have a very similar structure, they can both be solved with very similar numerical methods. We have developed parallel FAS multigrid solvers for both applications. Because of the symmetries of the problem, is is sufficient to work in one octant. The codes are written in cartesian coordinates and use the DAGH infrastructure that has been developed as part of the Binary Black Hole Grand Challenge project. For code development we typically run on 8 processors on the IBM SP2 parallel cluster at Cornell. We use up to 5 levels of refinement, which gives a $(64)^3$ grid on the finest level. The matter is covered by about 20 gridpoints in each direction.

We are interested in quasi-equilibrium models in their own right, but we also plan to use the models as initial data for fully relativistic evolution codes. We show a density profile of a close neutron star binary in Figure 1. In isolation, each star would have a compaction of $(M/R)_\infty = 0.16$, showing that this configuration is highly relativistic. The maximum mass configuration for this Γ satisfies $(M/R)_\infty = 0.22$.

We construct quasi-equilibrium sequences for binaries of fixed rest mass. Up to the ISCO, these sequences approximate evolutionary sequences. We plot in Figure 2 the binding energy $(M - M_\infty)/M$ versus the separation, parameterized by the angular velocity, for a mildly relativistic sequence ($(M/R)_\infty = 0.02$). The turning point of this curve indicates the onset of orbital instability at the ISCO and the angular velocity there. Note that we are restricted to co-rotating sequences. Sequences of conserved circulation are probably more realistic, since maintaining co-rotation would require excessive viscosity [7].

In the near future we plan to implement adaptive mesh refinement (AMR) and increase the accuracy of our calculation. We will then explore the physics of fully relativistic binary neutron stars for different polytropic indices, separations, and values of $(M/R)_\infty$.

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