AFOSR-TN-57-116

ASTIA Document No. AD 120 469

CALIFORNIA INSTITUTE OF TECHNOLOGY

Antenna Laboratory

Technical Report No. 9

A NEW JUNCTION TRANSISTOR HIGH-FREQUENCY CIRCUIT

by

R. D. Middlebrook

A Report on Research conducted under the

U. S. Air Force Office of Scientific

Research, Contract AF18(600)-1113

March 1957

ABSTRACT

A small-signal equivalent circuit for a junction transistor is presented which is applicable to alloy or grown types of p-n-p or n-p-n transistors, and which is valid from d-c up to twice the cutoff frequency. The equivalent circuit is in the form of four short-circuit admittances, each of which can be represented by a simple network of lumped elements constant with frequency. The derivation is based on physical principles and takes into account base widening and collector barrier capacitance. Equations for the equivalent circuit element values are given either in terms of physical parameters or in terms of six practical measurements. The four-admittance representation is given both for common-emitter and common-base connections, and a relation between the common-emitter and the common-base cutoff frequencies is derived and experimentally verified. Measurements of the real and imaginary parts of the four admittances as functions of frequency for several transistors show excellent agreement with the values predicted by the equivalent circuit.

I. Introduction

Analysis of the simplest form of one-dimensional carrier flow across the base region of a triode junction transistor leads directly to relations between the terminal small-signal voltages and currents. These relations are most conveniently expressed in terms of four short-circuit admittances which are functions of frequency and which are not easily representable by a small number of constant lumped network elements. Further, such analysis deals only with the so-called 'intrinsic' transistor, and for a more adequate representation of a practical transistor the collector barrier capacitance and majority carrier base resistance must be added to the equivalent circuit of the intrinsic transistor.

The complexity of the equivalent circuit derived from physical analysis, and the fact that the result is in terms of the physical and geometrical properties of the device, makes this equivalent circuit unsuitable for application of transistors to practical circuits. Here, one is concerned with the frequency dependence of the transistor circuit parameters and methods of predicting the values of these parameters from a small number of measurements on a given transistor: measurements which determine circuit properties and not physical and geometrical properties of the device.

The gulf between the viewpoint and requirements of the device engineer and those of the circuit engineer can only be bridged by a suitable equivalent circuit--one which is based on physical principles and yet which is suitable for practical application. Obviously, some approximation will be involved, and many such attempts have been made[1], all of which have particular advantages for certain applications. In general, the requirement placed upon an equivalent circuit is that it should provide a suitable compromise between extreme accuracy with consequent great complexity, and extreme simplicity with consequent great inaccuracy.

The equivalent circuit presented in this paper is an attempt to describe the circuit properties of a junction transistor as a small-signal linear amplifier. It is based on physical principles, yet does not require a knowledge of the physical or geometrical properties of the transistor for quantitative application to practical circuits. The equivalent circuit is in the form of four short-circuit admittances, is valid from d-c up to about twice the cutoff frequency, and is applicable to alloy or grown types of p-n-p or n-p-n transistors in which diffusion is the principal mechanism of current flow across the base region. Each of the four short-circuit admittances can be represented by a simple network of lumped elements, constant with frequency, whose values are given in terms of the physical properties of the transistor or may be obtained numerically from six practical measurements on the complete transistor. The procedure is applicable to configurations in which the emitter, base, or collector is common, but results for the first two only will be given here. The method is as follows.

The solution of the continuity equation for minority carriers in the base region of a transistor, taking into account the base widening effect and collector barrier capacitance, leads to a representation for the intrinsic transistor in terms of four short-circuit admittances which are functions of frequency[2-5]. By a suitable approximation procedure, a simple rational function expression has been obtained for the frequency variation of the intrinsic transistor short-circuit current gain.[4-6]

By an extension of this approximation procedure, the four admittances representing the intrinsic transistor are each expressed in terms of a parallel conductance and capacitance. These are then converted to a set of four short-circuit admittances which represent a transistor in common-emitter connection. The extrinsic base resistance is then added and algebraic manipulation leads to four short-circuit admittances which represent the complete practical transistor in common-emitter connection. By similar manipulations, four admittances are obtained which represent the transistor in common-base connection. The admittances in both connections are each expressed in rational function form.

By partial fraction expansion, each admittance is expressed in a form which can be represented by a simple network of lumped elements constant with frequency, where the element values are in terms of the physical and geometrical constants of the transistor, or can be obtained from six practical measurements on the transistor.

By comparison of the four short-circuit admittance representations of the common-emitter and the common-base connections, a relation between the common-emitter and common-base cutoff frequencies is obtained which is verified by measurements. This result is particularly useful since the common-base cutoff frequency is a useful figure of merit of the transistor, yet the common-emitter cutoff frequency is much more easily measured.

Results are presented for several transistors which show that the real and imaginary parts of each common-base admittance as determined by direct measurement are in excellent agreement with the values predicted by the equivalent circuit from d-c up to about twice the cutoff frequency.

II. Intrinsic Transistor Equivalent Circuit

Well-established procedures[2-5] lead to the following expressions for the small-signal diffusion admittances of the intrinsic transistor:

$$y_{11b}{}^{i} = G \coth ct \tag{1}$$

$$-y_{21b}{}^{i} = G \operatorname{csch} ct \tag{2}$$

$$y_{22b}{}^{i} = KG \coth ct \tag{3}$$

$$-y_{12b}{}^{i} = KG \operatorname{csch} ct \tag{4}$$

where $c = (1 + s\tau)^{1/2}$, in which τ is the lifetime of minority carriers in the base region, $s = j\omega$, and G, K, and t are constants dependent on the transistor material and geometry. The subscript b indicates that the four admittances refer to the common base connection*, and the superscript i indicates that the admittances refer only to the intrinsic transistor in which the collector barrier capacitance and the majority carrier base resistance are omitted. Equations (1) to (4) are valid for small a-c signals superimposed on normal biases (emitter forward-biased, collector reverse-biased to saturation), and are based on the following major approximations: carrier flow is one-dimensional; minority carrier flow in the base region is entirely by diffusion and not by drift; the emitter efficiency and the collector multiplication factor are each unity.

It has been shown[6] that (1) to (4) may be approximated by

$$y_{11b}{}^{i} = g_{11b}{}^{i}\,(1 + sT) \tag{5}$$

$$-y_{21b}{}^{i} = \alpha_0{}^{i}\, g_{11b}{}^{i}\,(1 - smT) \tag{6}$$

---

*Where possible, the notation used will be that recommended in IRE Standard 56 IRE 28. S1 ("IRE standards on letter symbols for semiconductor devices, 1956", Proc. IRE, Vol. 44, pp. 934-937; July 1956).

$$y_{22b}^{i} = g_{22b}^{i} (1 + sT) \tag{7}$$

$$-y_{12b}^{i} = \alpha_0^{i} g_{22b}^{i} (1 - smT) \tag{8}$$

where g_{11b}^{i} and g_{22b}^{i} are the input and output low-frequency short-circuit conductances of the intrinsic transistor, and α_0^{i} is the low-frequency short-circuit current gain of the intrinsic transistor. The parameters m and T are determined by arranging that the magnitude and phase angle of the frequency-dependent short-circuit current gain $\alpha^{i} = -y_{21b}^{i}/y_{11b}^{i}$ at the cutoff frequency $\omega_{\alpha b}^{i}$ given by (5) and (6) should be the same as the magnitude and phase given by (1) and (2). The resulting values of m and T are dependent on α_0^{i}, but for practical purposes the average values

$$m \approx 0.2 \tag{9}$$

$$T \approx \frac{1.04}{\omega_{\alpha b}^{i}} \tag{10}$$

are satisfactory. The short-circuit current gain of the intrinsic transistor is then closely approximated in magnitude and phase by the expression[6]

$$\alpha^{i} = \alpha_0^{i} \frac{1 - smT}{1 + sT} \tag{11}$$

Since (11) is a good approximation to the ratio of y_{21b}^{i} and y_{11b}^{i}, it does not necessarily follow that (5) and (6) will also be good approximations for y_{11b}^{i} and y_{21b}^{i} separately: in fact, they are not. More accurate representations could be set up by assuming expressions of the form

$$y_{11b}^{i} = g_{11b}^{i} (1 + sT') \tag{12}$$

$$-y_{21b}{}^{i} = \alpha_0{}^{i}\, g_{11b}{}^{i}/(1 + sm'T') \tag{13}$$

$$y_{22b}{}^{i} = g_{22b}{}^{i}\,(1 + sT') \tag{14}$$

$$-y_{12b}{}^{i} = \alpha_0{}^{i}\, g_{22b}{}^{i}/(1 + sm'T') \tag{15}$$

in which case the expression for α^i would be

$$\alpha^i = \frac{\alpha_0{}^i}{(1 + sm'T')(1 + sT')} \tag{16}$$

and the parameters m' and T' could be obtained in a way similar to that described for m and T. Such expressions for α^i have been proposed.[8] However, there are certain advantages in choosing the approximations (5) to (8) in which all four admittances are of the same form: the algebraic manipulations which form the main body of this paper are greatly simplified, and the resulting equivalent circuit contains fewer elements than if the more accurate approximations (12) to (15) were used. The choice between the two is determined by the compromise desired between accuracy and simplicity, and it will be shown that the less accurate approximations of (5) to (8) give equivalent circuit results which are quite good enough for practical purposes. It is believed that the greater equivalent circuit complexity which results from use of the more accurate approximations (12) to (15) is not justified by the slightly improved accuracy. The approximations (5) to (8) are adequate in practice because a transistor is normally used as a current amplifier and not as a voltage amplifier, and therefore a good approximation for the current gain α^i is more important than good approximations for the individual admittances.

Equations (5) to (8) suggest that each admittance of the intrinsic transistor may be represented by the parallel combination of a conductance and a capacitance, where each element is independent of frequency, and the representation is valid up to at least the cutoff frequency $\omega_{\alpha b}{}^{i}$. The capacitances $g_{11b}{}^{i}T$ and $g_{22b}{}^{i}T$ associated with the emitter and collector admittances are commonly known as diffusion capacitances. A more complete equivalent circuit of the transistor requires the addition of the emitter and collector barrier capacitances and the majority carrier base resistance r_b'; however, the emitter barrier capacitance is usually negligible compared to the emitter diffusion capacitance in standard triode structures. The practical transistor may therefore be represented by the equivalent circuit of Fig. 1, in which the admittance $y_{22b}{}^{i}$ is redefined to include both the collector diffusion capacitance and the collector barrier capacitance C_c. Equations (5) to (8) may then be written

$$y_{11b}{}^{i} = g_1(1+sT) \tag{17}$$

$$-y_{21b}{}^{i} = \alpha_1 g_1(1-smT) \tag{18}$$

$$y_{22b}{}^{i} = g_2(1+sT) + sC_\delta \tag{19}$$

$$-y_{12b}{}^{i} = \alpha_1 g_2(1-smT) \tag{20}$$

where the substitutions

$$g_1 \equiv g_{11b}{}^{i} \tag{21}$$

$$g_2 \equiv g_{22b}{}^{i} \tag{22}$$

$$\alpha_1 \equiv \alpha_0{}^{i} \tag{23}$$

have been made to simplify the notation.

In the equivalent circuit of some types of alloy junction transistors it has been suggested[8] that the collector barrier capacitance C_c should be connected to a tap on the base resistance, but it has been found that this modification leads to considerably greater complexity in the final equivalent circuit and is in any case a small effect.[4] In the equivalent circuit of grown junction transistors it has been mentioned[9] that the element $r_b{}'$ is not a pure resistance, but this effect will not be considered here.

Under the approximations already mentioned, Fig. 1 and (17) to (20) represent an equivalent circuit for a practical junction transistor. However, for many purposes this form is inconvenient because the two current generators $y_{21b}{}^{i}\, V_{1b}{}^{i}$ and $y_{12b}{}^{i}\, V_{2b}{}^{i}$ are functions of the internal junction voltages $V_{1b}{}^{i}$ and $V_{2b}{}^{i}$ which are not accessible to measurement. It is desirable, therefore, to modify the representation so that the current generators are functions of the external junction voltages V_{1b} and V_{2b}. The general form of the required result is shown in Fig. 2, in which the letter subscript of the voltage and admittance symbols is omitted since this is a general form applicable to any of the three circuit configurations (emitter, base, or collector common). For each configuration, it is required that each admittance should be representable by a simple network of lumped elements constant with frequency, and that an equation for each element should be obtained in terms of measurable terminal properties. In the following sections these results will be set up for the common-emitter and common-base configurations.

III. Rational Functions for the Equivalent-Circuit Admittances

For the common-emitter configuration, the equivalent form of Fig. 2 may be obtained from that of Fig. 1 in two steps. The first step leads to the form shown in Fig. 3, in which

$$y_{11e}^{i} = y_{11b}^{i} + y_{21b}^{i} + y_{22b}^{i} + y_{12b}^{i} \tag{24}$$

$$-y_{21e}^{i} = y_{21b}^{i} + y_{22b}^{i} \tag{25}$$

$$y_{22e}^{i} = y_{22b}^{i} \tag{26}$$

$$-y_{12e}^{i} = y_{12b}^{i} + y_{22b}^{i} \tag{27}$$

The second step leads to the form shown in Fig. 2 in which all symbols now take an additional subscript e to indicate common-emitter configuration, and in which

$$y_{11e} = \frac{y_{11e}^{i}}{1 + r y_{11e}^{i}} \tag{28}$$

$$y_{21e} = \frac{y_{21e}^{i}}{1 + r y_{11e}^{i}} \tag{29}$$

$$y_{22e} = \frac{y_{22e}^{i} + r(y_{11e}^{i} y_{22e}^{i} - y_{21e}^{i} y_{12e}^{i})}{1 + r y_{11e}^{i}} \tag{30}$$

$$y_{12e} = \frac{y_{12e}^{i}}{1 + r y_{11e}^{i}} \tag{31}$$

To find the y_e admittances as functions of the transistor physical and geometrical properties and of frequency, it is necessary only to substitute (17) to (20) into (28) to (31), with the help of

(24) to (27). The resulting equations, expressed as rational functions of the frequency variable s, are

$$y_{11e} = \frac{(g_1 + g_2)(1 - \alpha_1) + s\left[n(g_1 + g_2)T + C_c\right]}{k(1 + sb)} \tag{32}$$

$$-y_{21e} = \frac{(-\alpha_1 g_1 + g_2) + s\left[(\alpha_1 m g_1 + g_2)T + C_c\right]}{k(1 + sb)} \tag{33}$$

$$y_{22e} = \frac{g_2\left[1 + rg_1(1 - \alpha_1^2)\right] + s\left\{(1 + rg_1)C_c + g_2T\left[1 + 2rg_1(1 + \alpha_1^2 m)\right]\right\} + s^2\left[C_c + g_2T(1 - \alpha_1^2 m^2)\right]rg_1T}{k(1 + sb)} \tag{34}$$

$$-y_{12e} = \frac{g_2(1 - \alpha_1) + s(C_c + ng_2T)}{k(1 + sb)} \tag{35}$$

where

$$n = 1 + \alpha_1 m \tag{36}$$

$$k = 1 + rg_1(1 - \alpha_1) \tag{37}$$

$$b = \frac{r\left[n(g_1 + g_2)T + C_c\right]}{k} \tag{38}$$

For the common-base configuration, the equivalent circuit form of Fig. 2 may be obtained either from the previous result for the common-emitter configuration by the equations

$$y_{11b} = y_{11e} + y_{21e} + y_{22e} + y_{12e} \tag{39}$$

$$-y_{21b} = y_{21e} + y_{22e} \tag{40}$$

$$y_{22b} = y_{22e} \tag{41}$$

$$-y_{12b} = y_{12e} + y_{22e} \tag{42}$$

or from Fig. 1 by the equations

$$y_{11b} = \frac{y_{11b}^{\,i} + r\,\Delta^i}{1 + r\,\Gamma^i} \tag{43}$$

$$-y_{21b} = \frac{-y_{21b}^{\,i} + r\,\Delta^i}{1 + r\,\Gamma^i} \tag{44}$$

$$y_{22b} = \frac{y_{22b}^{\,i} + r\,\Delta^i}{1 + r\,\Gamma^i} \tag{45}$$

$$-y_{12b} = \frac{-y_{12b}^{\,i} + r\,\Delta^i}{1 + r\,\Gamma^i} \tag{46}$$

where

$$r \equiv r_b' \tag{47}$$

$$\Delta^i \equiv y_{11b}^{\,i}\,y_{22b}^{\,i} - y_{21b}^{\,i}\,y_{12b}^{\,i} \tag{48}$$

$$\Gamma^i \equiv y_{11b}^{\,i} + y_{21b}^{\,i} + y_{22b}^{\,i} + y_{12b}^{\,i} \tag{49}$$

Rational functions for the y_b admittances are then obtained by use of (17) to (20):

$$y_{11b} = \frac{g_1\left[1 + rg_2(1-\alpha_1^2)\right] + s\left\{g_1T + rg_1\left[C_c + 2g_2T(1+\alpha_1^2 m)\right]\right\} + s^2\left[C_c + g_2T(1-\alpha_1^2 m^2)\right]rg_1T}{k(1+sb)} \tag{50}$$

$$-y_{21b} = \frac{g_1\left[\alpha_1 + rg_2(1-\alpha_1^2)\right] + s\left\{-\alpha_1 m g_1T + rg_1\left[C_c + 2g_2T(1+\alpha_1^2 m)\right]\right\} + s^2\left[C_c + g_2T(1-\alpha_1^2 m^2)\right]rg_1T}{k(1+sb)} \tag{51}$$

$$y_{22b} = \frac{g_2\left[1 + rg_1(1 - \alpha_1^2)\right] + s\left\{(1 + rg_1)C_c + g_2T\left[1 + 2rg_1(1 + \alpha_1^2 m)\right]\right\} + s^2\left[C_c + g_2T(1 - \alpha_1^2 m^2)\right]rg_1T}{k(1 + sb)} \tag{52}$$

$$-y_{12b} = \frac{g_2\left[\alpha_1 + rg_1(1 - \alpha_1^2)\right] + s\left\{rg_1C_c + g_2T\left[-\alpha_1 m + 2rg_1(1 + \alpha_1^2 m)\right]\right\} + s^2\left[C_c + g_2T(1 - \alpha_1^2 m^2)\right]rg_1T}{k(1 + sb)} \tag{53}$$

where n, k, and b are as defined in (36) to (38).

IV. Equations for the Equivalent-Circuit Elements

It will be noticed that (32) to (35) for the y_e parameters and (50) to (53) for the y_b parameters are each of the general form

$$y = \frac{a_1 + a_2 s + a_3 s^2}{1 + bs} \tag{54}$$

although $a_3 = 0$ for some of the admittances. This form suggests that each admittance can be represented by a network of lumped elements as shown in Fig. 4, where the form of Fig. 4a is more convenient if

$$a_2 - a_1 b - a_3/b > 0 \tag{55}$$

and the form of Fig. 4b is more convenient if

$$a_2 - a_1 b - a_3/b < 0 \tag{56}$$

In terms of the coefficients of (54), the elements in Fig. 4a are given by

$$g^a = a_1 \tag{57}$$

$$C^a = a_2 - a_1 b - a_3/b \tag{58}$$

$$g^b = C^a/b \tag{59}$$

$$C^b = a_3/b \tag{60}$$

and those in Fig. 4b are given by

$$g^a = a_2/b - a_3/b^2 \tag{61}$$

$$g^b = a_1 - g^a \tag{62}$$

$$L^a = b/g^b \tag{63}$$

$$C^b = a_3/b \tag{64}$$

Expressions for the element values in Fig. 4a or 4b may be found for each of the y_e or y_b parameters by comparing (54) with (32) to (35) or (50) to (53). The results are summarized below.

1. Common-emitter configuration. Equivalent-circuit as in Fig. 2, where the y_e admittances have the forms shown in Fig. 5 and the element symbols are as defined in the same figure. The equations for the element values are as follows.

y_{11e}, short-circuit input admittance:

$$g_{11e}{}^a = g_{11b}(1 - \alpha_1) \tag{65}$$

$$C_{11e}{}^a = \frac{ng_{11b}T}{k} \tag{66}$$

$$g_{11e}{}^b = \frac{1}{kr} \tag{67}$$

y_{21e}, short-circuit forward-transfer admittance:

$$g_{21e}{}^a = -\frac{\alpha_1 m}{nr} \tag{68}$$

$$g_{21e}{}^b = \alpha_1 g_{11b}\left(1 + \frac{m}{nrg_{11b}}\right) \tag{69}$$

$$L_{21e}{}^{a} = nrT \Big/ \alpha_1 \left(1 + \frac{m}{nrg_{11b}}\right) \tag{70}$$

y_{22e}, short-circuit output admittance:

$$g_{22e}{}^{a} = g_2\left[1 + \alpha_1 r g_{11b}(1 - \alpha_1)\right] \tag{71}$$

$$C_{22e}{}^{a} = \left(\frac{1}{k} + rg_{11b} - \frac{1}{n}\right)C \tag{72}$$

$$g_{22e}{}^{b} = \left(\frac{1}{k} + rg_{11b} - \frac{1}{n}\right)C \Big/ nrg_{11b}T \tag{73}$$

$$C_{22e}{}^{b} = C/n \tag{74}$$

y_{12e}, short-circuit reverse-transfer admittance:

$$g_{12e}{}^{a} = \frac{g_2(1 - \alpha_1)}{k} \tag{75}$$

$$C_{12e}{}^{a} = C/k \tag{76}$$

$$g_{12e}{}^{b} = \frac{C}{knrg_{11b}T} \tag{77}$$

2. Common-base configuration. Equivalent circuit as in Fig. 2 where the y_b admittances have the forms shown in Fig. 6 and the element symbols are as defined in the same figure. The equations for the element values are as follows:

y_{11b}, short-circuit input admittance:

$$g_{11b}{}^{a} = \frac{1 + rC/T}{nr} \tag{78}$$

$$g_{11b}{}^{b} = g_{11b} - \frac{1 + rC/T}{nr} \tag{79}$$

$$L_{11b}{}^{a} = \frac{nrg_{11b}T}{g_{11b} - (1 + rC/T)/nr} \tag{80}$$

$$C_{11b}{}^{b} = C/n \tag{81}$$

y_{21b}, short-circuit forward-transfer admittance:

$$g_{21b}{}^{a} = -\frac{\alpha_1 m - rC/T}{nr} \tag{82}$$

$$g_{21b}{}^{b} = \alpha_1 g_{11b} + \frac{\alpha_1 m - rC/T}{nr} \tag{83}$$

$$L_{21b}{}^{a} = \frac{nrg_{11b}T}{\alpha_1 g_{11b} + (\alpha_1 m - rC/T)/nr} \tag{84}$$

$$C_{21b}{}^{b} = C/n \tag{85}$$

y_{22b}, short-circuit output admittance:

$$g_{22b}{}^{a} = g_2\left[1 + \alpha_1 r g_{11b}(1 - \alpha_1)\right] \tag{86}$$

$$C_{22b}{}^{a} = \left(\frac{1}{k} + rg_{11b} - \frac{1}{n}\right)C \tag{87}$$

$$g_{22b}{}^{b} = \left(\frac{1}{k} + rg_{11b} - \frac{1}{n}\right)C\Big/nrg_{11b}T \tag{88}$$

$$C_{22b}{}^{b} = C/n \tag{89}$$

y_{21b}, short-circuit reverse-transfer admittance:

$$g_{12b}{}^{a} = g_2\left[\alpha_1 + rg_{11b}(1 - \alpha_1)\right] \tag{90}$$

$$C_{12b} = \left(rg_{11b} - \frac{1}{n}\right)C \tag{91}$$

$$g_{12b}{}^{b} = \left(rg_{11b} - \frac{1}{n}\right)C\Big/nrg_{11b}T \tag{92}$$

$$C_{12b}{}^{b} = C/n \tag{93}$$

In (65) to (93), the quantities n, g_{11b}, and C are defined by

$$n \equiv 1 + \alpha_1 m \tag{94}$$

$$g_{11b} \equiv g_1/k \tag{95}$$

$$C \approx C_c + g_2 T\left[1 + \alpha_1^{\,2}(1 + 2m)\right] \tag{96}$$

The significance of n, g_{11b} and C is of interest. It has been shown[4,5] that n is a slowly-varying function of $\alpha_0^{\,i} = \alpha_1$ and an average value

$$n = 1 + \alpha_1 m \approx 1.2 \tag{97}$$

is satisfactory for practical purposes. The quantity g_{11b} is the low-frequency short-circuit input conductance in the common-base configuration, as may be seen from (78) and (79). Examination of Fig. 1 leads to the conclusion[4,5] that the quantity C is the capacitance which would be measured at low frequency at the external base-collector terminals of a transistor when the emitter and collector are biased to their normal d-c operating points but the emitter is open-circuited for a-c. In other words, the quantity C is the sum of the collector barrier capacitance and a combination of the output and reverse-transfer diffusion capacitances.

Two major approximations have been made in obtaining (65) to (93). The first is that the collector low-frequency short-circuit conductance of the intrinsic transistor is much less than the emitter low-frequency short-circuit conductance of the intrinsic transistor, or

$$g_2 << g_1 \tag{98}$$

The second is that the collector diffusion capacitance of the intrinsic transistor is much smaller than the collector barrier capacitance, or

$$g_2 T << C_c \tag{99}$$

For a more complete discussion of the approximations involved in deriving the equations for the element values, the reader is referred elsewhere.[4,5]

Equations (65) to (93), together with the definition of the quantities therein, are sufficient to show how the circuit performance of a junction transistor depends on the physical and geometric properties of the device. Forms more suitable for application to practical circuits will be given in Section VI.

V. Relation between Common-Emitter and Common-Base Cutoff Frequencies

Quantities which are of especial practical interest are the low-frequency short-circuit current gain and the cutoff frequency of the short-circuit current gain in both the common-emitter and common-base configurations. Some useful relationships between these quantities will be derived in this section.

The forward short-circuit current gain in the common-base configuration is defined by

$$\alpha_{fb} = -\frac{y_{21b}}{y_{11b}} \tag{100}$$

Expressions for y_{11b} and y_{21b} have been given in (50) and (51), and hence

$$\alpha_{fb} = \frac{(\alpha_1 - \omega^2 rCT) + j\omega(-\alpha_1 mT + rC)}{(1 - \omega^2 rCT) + j\omega(T + rC)} \tag{101}$$

where use has been made of (96), (98) and (99), and $s = j\omega$.

If the value of α_{fb} at low frequencies is defined as α_{fb0}, it follows from (23) and (101) that

$$\alpha_{fb0} = \alpha_1 = \alpha_0^{\,i} \tag{102}$$

Thus the low-frequency short-circuit current gain in the common-base configuration is the same for the practical transistor as for the intrinsic transistor. Henceforward, therefore, α_1 will be replaced by α_{fb0}.

The angular cutoff frequency of the short-circuit current gain in the common base configuration, $\omega_{\alpha b}$, is by definition the frequency at which $|\alpha_{fb}|/\alpha_{fb0} = 1/\sqrt{2}$. By manipulation of (101), it may be shown[5] that the parameter T is related to $\omega_{\alpha b}$ by the approximate relation

$$T = \frac{1}{\omega_{\alpha b}(1 - 2m^2)^{1/2}} - \frac{2rC(1 + m)}{\alpha_{fb0}(1 - 2m^2)} \tag{103}$$

With use of the average value $m = 0.2$, (103) becomes

$$T = \frac{1.04}{\omega_{\alpha b}} - \frac{2.6rC}{\alpha_{fb0}} \tag{104}$$

which is an expression from which T may be calculated if $\omega_{\alpha b}$, α_{fb0}, r, and C are known. From (10) and (104),

$$\frac{1}{\omega_{\alpha b}} = \frac{1}{\omega_{\alpha b}^{1}} + \frac{2.5rC}{\alpha_{fb0}} \tag{105}$$

which shows that the presence of r and C causes the cutoff frequency of the practical transistor to be lower than that of the intrinsic transistor.

The forward short-circuit current gain in the common-emitter configuration is defined by

$$\alpha_{fe} = \frac{y_{21e}}{y_{11e}} \tag{106}$$

Expressions for y_{11e} and y_{21e} have been given in (32) and (33), and hence

$$\alpha_{fe} = \alpha_{fe0}\,\frac{1 - j\omega mT}{1 + j\omega mT/(1 - \alpha_{fb0})} \tag{107}$$

where

$$\alpha_{fe0} = \frac{\alpha_1}{1 - \alpha_1} = \frac{\alpha_{fb0}}{1 - \alpha_{fb0}} \tag{108}$$

and where terms in g_2 and in C_c have been neglected (which are good approximations). The quantity α_{fe0} is the low-frequency value of α_{fe}.

The angular cutoff frequency of the short-circuit current gain in the common-emitter configuration, $\omega_{\alpha e}$, is by definition the frequency at which $|\alpha_{fe}|/\alpha_{fe0} = 1/\sqrt{2}$. From (107) it is easily shown that

$$T = \frac{1 - \alpha_{fb0}}{n\omega_{\alpha e}} \tag{109}$$

to a good approximation. With use of the average value $n = 1.2$, (109) becomes

$$T = \frac{1 - \alpha_{fb0}}{1.2\omega_{\alpha e}} \tag{110}$$

and is a more convenient expression for calculating the value of T than that of (104), partly because r and C do not enter into the relation, and partly because $\omega_{\alpha e}$ is lower than $\omega_{\alpha b}$ and is hence more easily measured. By combining (104) and (110) the following relation between $\omega_{\alpha b}$ and $\omega_{\alpha e}$ is obtained:

$$\frac{1}{\omega_{\alpha b}} = \frac{1 - \alpha_{fb0}}{1.25\omega_{\alpha e}} + \frac{2.5rC}{\alpha_{fb0}} \tag{111}$$

The above result shows that, even if the effect of r and C is negligible, calculation of $\omega_{\alpha b}$ from $\omega_{\alpha e}$ by the usually-quoted formula[10]

$$\frac{1}{\omega_{\alpha b}} = \frac{1 - \alpha_{fb0}}{\omega_{\alpha e}} \tag{112}$$

introduces an error of 25 per cent in the result if (111) is valid. To test (111), measurements of $\omega_{\alpha e}$, α_{fb0}, r, and C were made on several transistors, and the values of $\omega_{\alpha b}$ calculated by (111) were compared with directly measured values. Table I shows the results, from which it is apparent that (111) is a more accurate relation than (112).

VI. Practical Formulas and Experimental Verification

For practical application, it is desirable to have equations for the equivalent circuit elements in terms of a minimum number of quantities which can be measured on a complete practical transistor. Examination of (65) to (96) will show that there are six independent quantities, and thus six appropriate measurements on a given transistor are sufficient to obtain numerical values in the equivalent circuits of Figs. 5 or 6, and thus to characterize the transistor at all frequencies up to the cutoff frequency.

Six suitable measurements are listed below, all of which are made at some specified d-c operating point:

g_{11b}, common-base short-circuit input conductance at low frequency

g_{22b}, common-base short-circuit output conductance at low frequency

α_{fe0}, common-emitter short-circuit current gain at low frequency

$r = r_b'$, high frequency majority carrier base resistance.

C, common-base low-frequency output capacitance with emitter open-circuited for a-c.

$\omega_{\alpha e}$ or $\omega_{\alpha b}$, common-emitter or common-base short-circuit current gain cutoff frequency.

Various methods of measuring the above six quantities are possible.[11] As examples, the General Radio Vacuum Tube Bridge is suitable[4] for determining g_{11b}, g_{22b}, and α_{fe0} (test frequency 1 kc/s). The capacitance C may be measured[4] on a capacitance bridge or on a Wayne Kerr Radio Frequency Bridge. One possible method[4] of determining r_b' is to measure the open-circuit emitter-base a-c voltage when a known a-c current is injected at the collector-base terminals at a frequency ω sufficiently high that $\omega \gg 1/r_b'C$. The cutoff frequencies $\omega_{\alpha e}$ and $\omega_{\alpha b}$ may (in principle) be measured directly by determining the frequency at which the magnitude of the appropriate short-circuit current gain is 3 db below its low-frequency value.

The series of equations (65) to (96) may be rearranged to give the equivalent circuit element values in terms of the six measured quantities, where the average values $n = 1.2$, $m = 0.2$ may be used. For convenient reference, the procedure to be followed in practical applications is summarized below.

From the six measurements described above, compute the quantities

$$\alpha_{fb0} = \frac{\alpha_{fe0}}{1 + \alpha_{fe0}} \tag{113}$$

$$g_2 = \frac{g_{22b}}{1 + \alpha_{fb0} r g_{11b}(1 - \alpha_{fb0})} \tag{114}$$

$$k = 1/[1 - rg_{11b}(1 - \alpha_{fb0})] \tag{115}$$

$$T = \frac{1 - \alpha_{fb0}}{1.2\omega_{\alpha e}} \tag{116}$$

or

$$T = \frac{1.04}{\omega_{\alpha b}} - \frac{2.6rC}{\alpha_{fb0}} \tag{117}$$

The relation (115) is obtained from (37) and (95).

<u>Common-Emitter Configuration.</u> Equivalent circuit as in Fig. 2, where the y_e admittances have the form shown in Fig. 5 and the element symbols are as defined in the same figure. Compute the element values in the given order:

y_{11e}, short-circuit input admittance:

$$g_{11e}{}^{a} = g_{11b}(1 - \alpha_{fb0}) \tag{118}$$

$$C_{11e}{}^{a} = \frac{1.2g_{11b}T}{k} \tag{119}$$

$$g_{11e}{}^{b} = \frac{1}{kr} \tag{120}$$

y_{21e}, short-circuit forward-transfer admittance:

$$g_{21e}{}^{a} = -\frac{\alpha_{fb0}}{5.8r} \tag{121}$$

$$g_{21e}{}^{b} = \alpha_{fb0}g_{11b}\left(1 + \frac{1}{5.8rg_{11b}}\right) \tag{122}$$

$$L_{21e}{}^{a} = \frac{1.2rg_{11b}T}{g_{21e}{}^{b}} \tag{123}$$

y_{22e}, short-circuit output admittance:

$$g_{22e}{}^{a} = g_{22b} \tag{124}$$

$$C_{22e}{}^{a} = \left[\alpha_{fb0}rg_{11b} + (1/6)\right]C \tag{125}$$

$$g_{22e}{}^{b} = \frac{C_{22e}{}^{a}}{1.2rg_{11b}T} \tag{126}$$

$$C_{22e}{}^{b} = C/1.2 \tag{127}$$

y_{12e}, short-circuit reverse-transfer admittance:

$$g_{12e}{}^{a} = \frac{g_2(1 - \alpha_{fb0})}{k} \tag{128}$$

$$g_{12e}{}^{a} = C/k \tag{129}$$

$$g_{12e}{}^{b} = \frac{C}{1.2krg_{11b}T} \tag{130}$$

Common-base Configuration. Equivalent circuit as in Fig. 2 where the y_b admittances have the forms shown in Fig. 6 and the element symbols are as defined in the same figure. Compute the element values in the given order:

y_{11b}, short-circuit input admittance:

$$g_{11b}{}^{a} = \frac{1 + rC/T}{1.2r} \tag{131}$$

$$g_{11b}{}^{b} = g_{11b} - g_{11b}{}^{a} \tag{132}$$

$$L_{11b}{}^{a} = \frac{1.2rg_{11b}T}{g_{11b}{}^{b}} \tag{133}$$

$$C_{11b}{}^{b} = C/1.2 \tag{134}$$

y_{21b}, short-circuit forward-transfer admittance:

$$g_{21b}{}^{a} = \frac{0.2\alpha_{fb0} - rC/T}{1.2r} \tag{135}$$

$$g_{21b}{}^{b} = \alpha_{fb0}g_{11b} + g_{21b}{}^{a} \tag{136}$$

$$L_{21b}{}^{a} = \frac{1.2rg_{11b}T}{g_{21b}{}^{b}} \tag{137}$$

$$C_{21b}{}^{b} = C/1.2 \tag{138}$$

y_{22b}, short-circuit output admittance:

$$g_{22b}{}^{a} = g_{22b} \tag{139}$$

$$C_{22b}{}^{a} = \left[\alpha_{fb0}rg_{11b} + (1/6)\right] C \tag{140}$$

$$g_{22b}{}^{b} = \frac{C_{22b}{}^{a}}{1.2rg_{11b}T} \tag{141}$$

$$C_{22b}{}^{b} = C/1.2 \tag{142}$$

y_{12b}, short-circuit reverse-transfer admittance:

$$g_{12b}{}^{a} = g_2\left[\alpha_{fb0} + rg_{11b}(1 - \alpha_{fb0})\right] \tag{143}$$

$$C_{12b}{}^{a} = \left[rg_{11b} - (1/1.2)\right] C \tag{144}$$

$$g_{12b}{}^{b} = \frac{C_{12b}{}^{a}}{1.2rg_{11b}T} \tag{145}$$

$$C_{12b}{}^{b} = C/1.2 \tag{146}$$

Equations (118) to (146) permit the element values of the equivalent circuit in the common-emitter or common-base configurations to be determined numerically from six independent measurements on a transistor. A similar procedure may be set up for the common-collector configuration, but is not given here.

The numerical results obtained in this way are sufficient to determine the circuit parameters of a transistor at any frequency up to the common-base cutoff frequency, since the initial equations (5) to (8) are adequate approximations up to this frequency. In order to verify the results, the following procedure was carried out.

From the six independent measurements on each transistor, the equivalent-circuit element values in the common-base configuration were calculated as described above. Each of the four y_b admittances was then expressed in the form

$$y_b = g(\omega) + j\omega C(\omega) \tag{147}$$

in which the real part $g(\omega)$ and the imaginary part $C(\omega)$ were determined from the appropriate network in Fig. 6 and the numerical values of the elements therein. The real and imaginary part of each admittance calculated in this way was plotted as a function of frequency and compared with direct measurements made at various frequencies with a Wayne Kerr Radio Frequency Bridge. These comparisons were made for several transistors including p-n-p alloy and n-p-n grown types. The results for four of these units are shown in Tables II and III and Figs. 7 to 10, in which the solid lines are the predicted values and the points are the directly measured values. Similar plots for other transistors may be found elsewhere.[4]

It is apparent that the agreement between the predicted and directly measured values of the real and imaginary parts of each admittance is extremely good, and in fact, extends beyond the cutoff frequency in every case. It may be concluded, therefore, that the equivalent circuit developed in this paper is sufficiently accurate for practical purposes. Additional confirmation of the form of the equivalent circuit has been obtained by G. T. Lake[12] and his group at the Defence Research Telecommunications Establishment, Ottawa, Canada, and by Dr. A. B. Credle[13] at the I.B.M. Laboratories in Poughkeepsie, New York. Mr. Lake has transformed the equivalent circuit into a form first proposed by Giacoletto[14] and applied the results to the design of common-emitter high-frequency amplifiers. Dr. Credle's paper discusses the temperature dependence of the equivalent circuit elements from minus 190° to plus 28° C .

It is believed that the equivalent circuit developed in this paper will be of value in the design of wide-band and video transistor amplifiers in which the frequency range of interest approaches the cutoff frequency of the individual stages, and to this end it is proposed in a later paper to discuss the application of this equivalent circuit to the problem of wide-range frequency compensation of transistors.

VII. Conclusions

A small-signal equivalent circuit has been developed for alloy or grown types of standard triode junction transistors. The equivalent circuit is in the form of four short-circuit admittances, each of which can be represented by a simple network of lumped elements constant with frequency whose values are given in terms of the physical properties of the transistor or may be obtained numerically from six practical measurements on the complete transistor. The four-admittance representation has been given both for the common-emitter and the common-base configurations, but the procedure is also applicable to the common-collector configuration. The range of validity of the equivalent circuit is from d-c up to about twice the common-base cutoff frequency. Results have been presented for several transistors which show that the real and imaginary parts of each common-base admittance as determined by direct measurement are in excellent agreement with the values predicted by the equivalent circuit. An experimentally-verified relation between the common-emitter and common-base short-circuit cutoff frequencies has also been derived.

Acknowledgments

This work was performed in part at Stanford University under Office of Naval Research Contract N6onr 251(07) (NR 373 360). The author wishes to thank Dr. J. M. Pettit, Dr. G. S. Bahrs, Dr. R. M. Scarlett, and K. G. Sorenson, all of Stanford University, for valuable assistance and discussion. The work was completed at the California Institute of Technology under Air Force Office of Scientific Research Contract AF 18(600)-1113. The numerical values in Table I were obtained by R. L. Walker at the Stanford Electronics Laboratories, and are part of a larger series of measurements used by him to verify a rather more accurate expression for

the cutoff frequency of the common-base practical transistor than that given in (111). Figures 7 through 10 are reproduced by kind permission of John Wiley and Sons, Inc.

References

1. R. L. Pritchard, "Electric network representation of transistors--a survey", IRE Trans. on Circuit Theory, Vol. CT-3, pp. 5-21; March, 1956.

2. J. M. Early, "Effects of space charge layer widening in junction transistors", Proc. IRE, Vol. 40, pp. 1401-1406; Nov. 1952.

3. J. Zawels, "Physical theory of new circuit representation for junction transistors", J. App. Phys., Vol. 25, pp. 976-981; 1954.

4. R. D. Middlebrook, "A junction-transistor high-frequency equivalent circuit", Technical Report No. 83, Electronics Research Laboratory, Stanford University, May 1955.

5. R. D. Middlebrook, An Introduction to Junction Transistor Theory, John Wiley and Sons, Inc., New York, Jan. 1957.

6. R. D. Middlebrook and R. M. Scarlett, "An approximation to alpha of a junction transistor", IRE Trans. on Electron Devices, Vol. ED-3, pp. 25-29, Jan. 1956.

7. A. B. Macnee, "Approximating the alpha of a junction transistor", Proc. IRE, Vol. 45, p. 91; Jan. 1957.

8. J. M. Early, "Design theory of junction transistors", Bell Syst. Tech. J., Vol. 32, pp. 1271-1312; Nov. 1953.

9. R. L. Pritchard and W. N. Coffey, "Small-signal parameters of grown-junction transistors at high frequencies", IRE Convention Record, Part III, pp. 80-98; 1954.

10. A. W. Lo et al, Transistor Electronics, Prentice-Hall, New York, 1955.

11. "IRE standards on solid-state devices: methods of testing transistors, 1956", (IRE Standard 56 IRE 28. S2) Proc. IRE, Vol. 44, pp. 1542-1561; Nov. 1956.

12. G. T. Lake, "Design of high frequency amplifiers", 1957 Transistor and Solid State Circuits Conference, University of Pennsylvania, February, 1957.

13. A. B. Credle, "Effects of low temperatures on transistor characteristics", 1957 Transistor and Solid State Circuits Conference, University of Pennsylvania, February 1957.

14. L. J. Giacoletto, "Study of p-n-p alloy junction transistors from d-c through medium frequencies", RCA Rev., Vol. 15, pp. 506-562, Dec.1954.

TABLE I

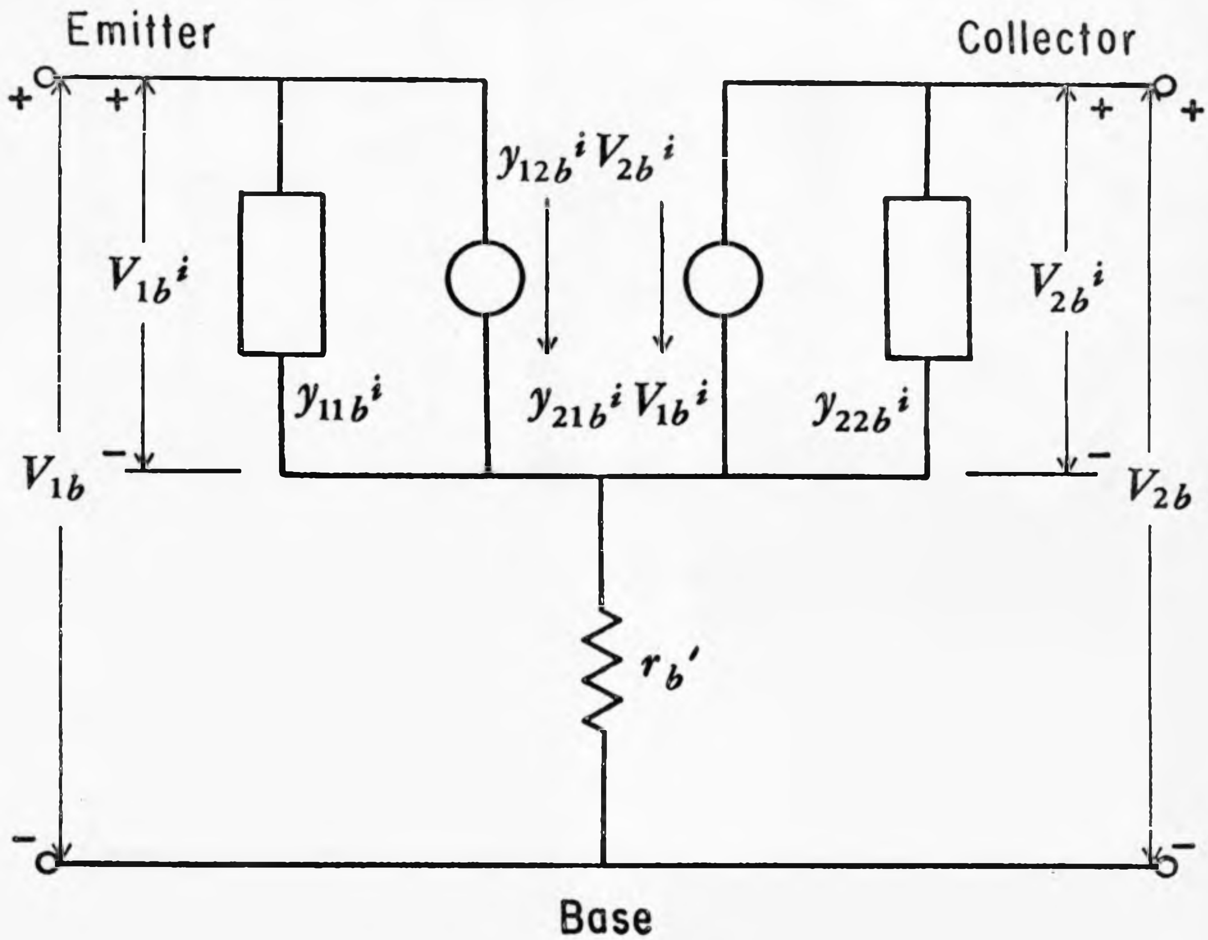
	Measured					Calculated from (111)
Unit	α_{fe0}	r_b' Ω	C $\mu\mu f$	$\omega_{\alpha e}/2\pi$ kc	$\omega_{\alpha b}/2\pi$ Mc	$\omega_{\alpha b}/2\pi$ Mc
2N123 #20	87	128	8.6	74	6.8	7.1
#50	111	145	8.8	85	8.8	9.6
SB100 #20	17	211	2.5	1850	37	31
#50	12.4	224	2.1	1650	22	23

TABLE II

Unit	$1/g_{eeb}$ Ω	r_b' Ω	C $\mu\mu f$	α_{fb0}	$\omega_{\alpha b}/2\pi$ Mc	$1/g_{ccb}$ $k\Omega$
2N36 #1	59.2	210	38.1	0.978	0.77	149
RR34 #6065	78.7	450	23.5	0.956	0.55	200
1858 #3	68.0	320	10.1	0.980	1.90	476
200 #2	65.5	330	13.9	0.952	1.56	556

TABLE III

Unit	$1/g_{eeb}$ [a] Ω	$1/g_{eeb}$ [b] Ω	L_{eeb} [a] μh	$-1/g_{ceb}$ [a] Ω	$1/g_{ceb}$ [b] Ω	L_{ceb} [a] μh	$1/g_{ccb}$ [a] kΩ	$1/g_{ccb}$ [b] kΩ	C_{ccb} [a] μμf	$1/g_{ecb}$ [a] kΩ	$1/g_{ecb}$ [b] kΩ	C_{ecb} [a] μμf	C_b [b] μμf
2N36 #1	240	78.5	64.9	1569	58.3	48.2	149	5.99	138	152	8.03	103	31.9
RR34 #6065	517	92.8	175	3400	80.4	151	200	14.3	132	205	16.4	114	19.7
1858 #3	367	83.5	37.2	2380	67.4	30.1	476	9.25	48.2	484	11.4	39.1	8.47
200 #2	375	79.3	45.3	2680	67.1	38.3	556	8.27	69.0	573	9.76	58.4	11.7



Emitter
Collector
+
+
+
+
V_{1b}^{i}
$y_{12b}^{i} V_{2b}^{i}$
V_{2b}^{i}
y_{11b}^{i}
$y_{21b}^{i} V_{1b}^{i}$
y_{22b}^{i}
V_{1b}
V_{2b}
r_b'
Base

Fig. 1

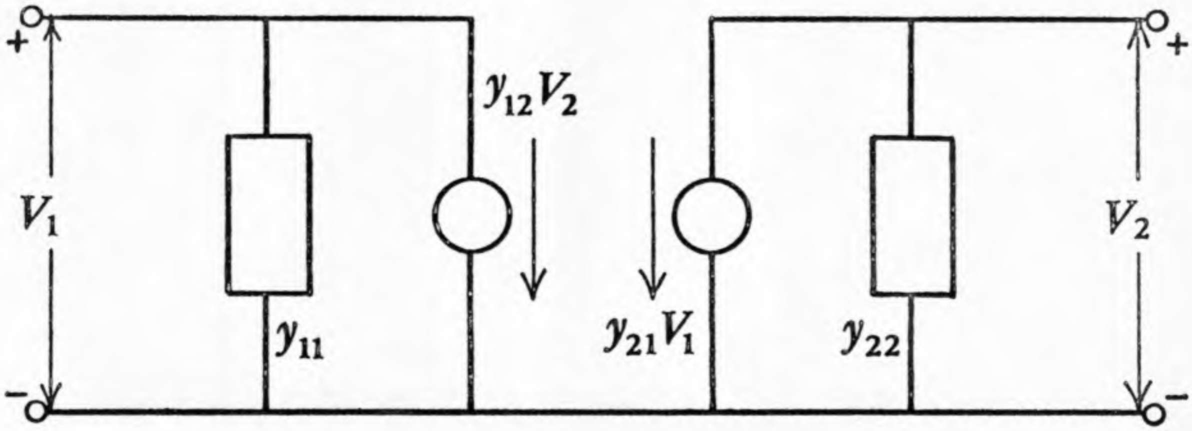


Fig. 2

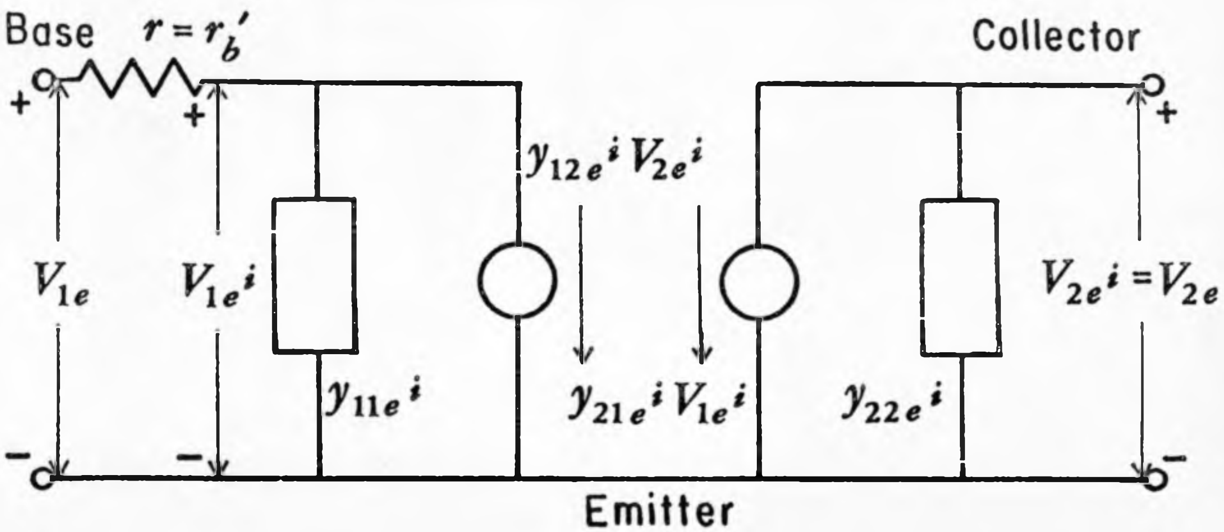

Base
$r = r_b'$
Collector
+
+
+
V_{1e}
V_{1e}^i
$y_{12e}^i V_{2e}^i$
y_{11e}^i
$y_{21e}^i V_{1e}^i$
y_{22e}^i
$V_{2e}^i = V_{2e}$
−
−
−
Emitter

Fig. 3

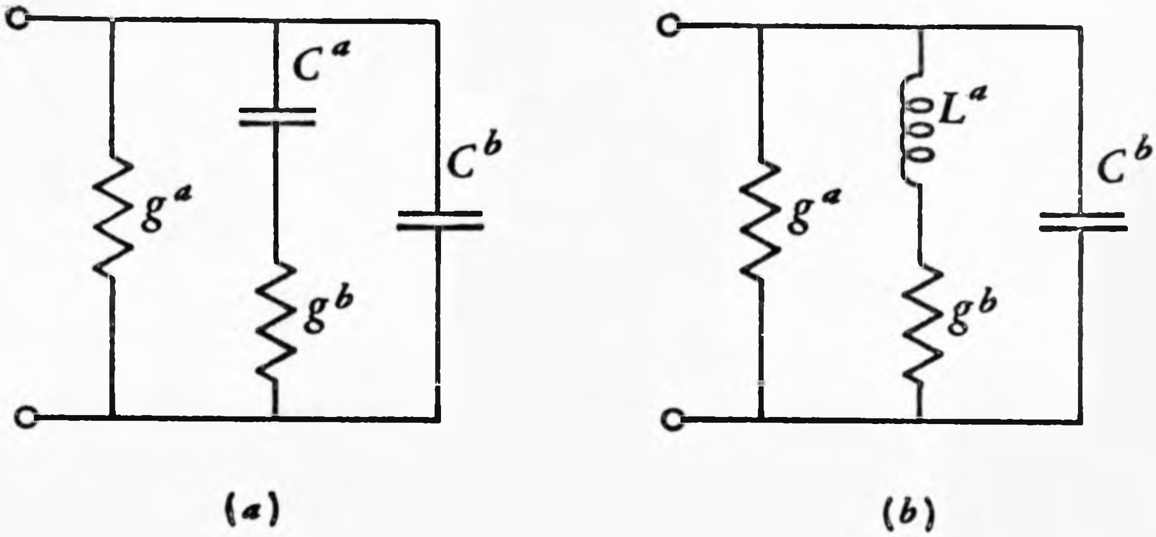
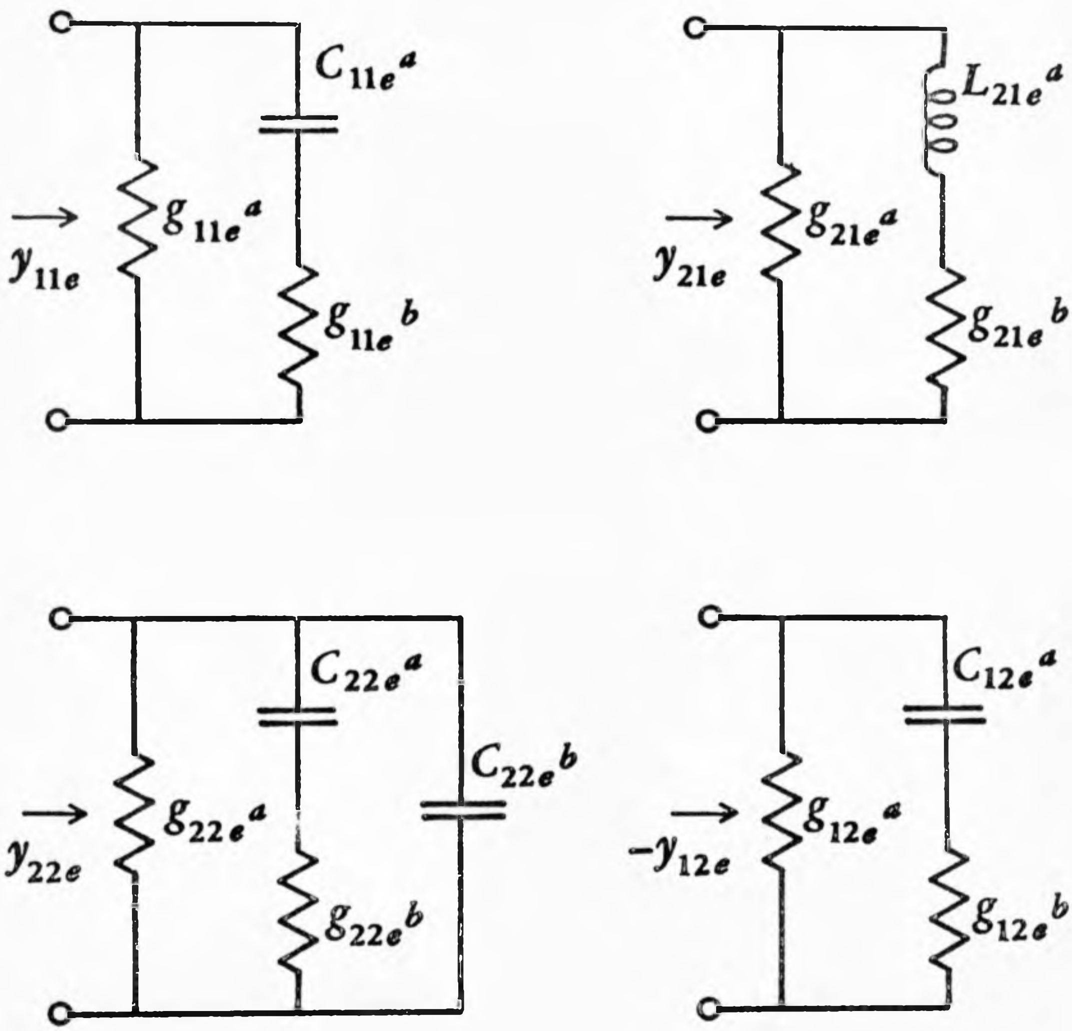
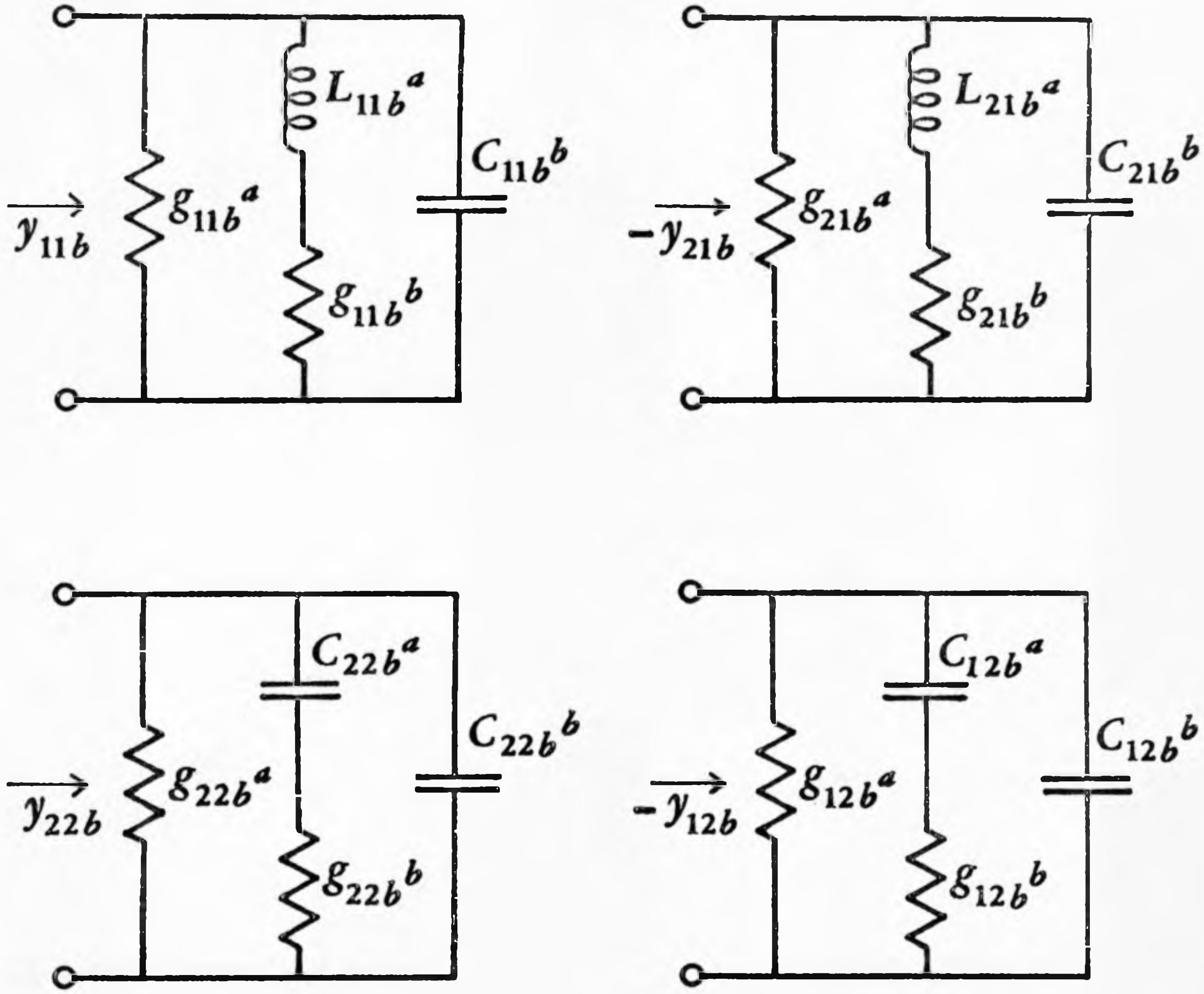


Fig. 4



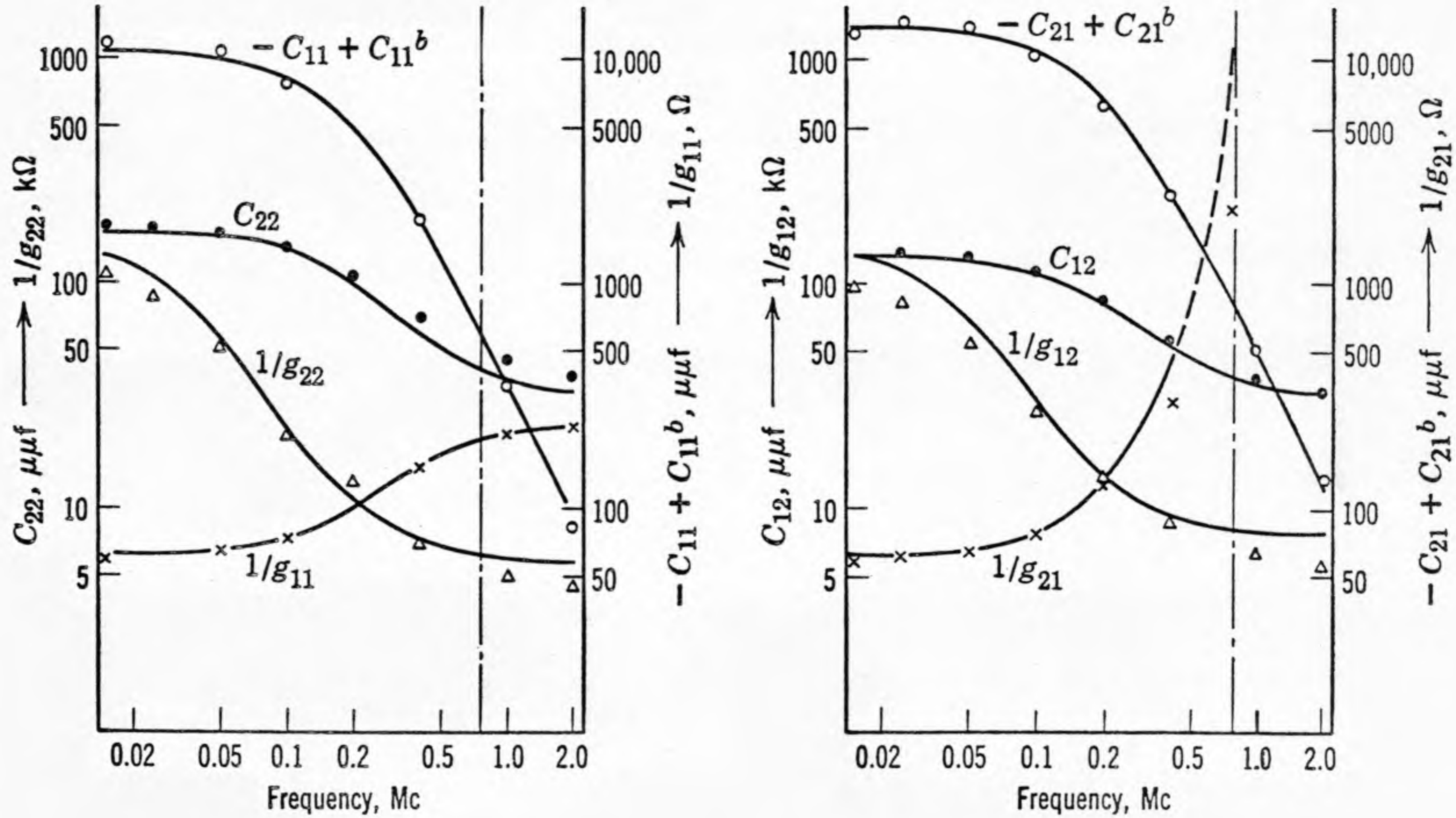
$C_{11e}{}^a$
$g_{11e}{}^a$
y_{11e}
$g_{11e}{}^b$
$L_{21e}{}^a$
$g_{21e}{}^a$
y_{21e}
$g_{21e}{}^b$
$C_{22e}{}^a$
$C_{22e}{}^b$
$g_{22e}{}^a$
y_{22e}
$g_{22e}{}^b$
$C_{12e}{}^a$
$g_{12e}{}^a$
$-y_{12e}$
$g_{12e}{}^b$

Fig. 5



$L_{11b}{}^{a}$
$C_{11b}{}^{b}$
$g_{11b}{}^{a}$
y_{11b}
$g_{11b}{}^{b}$
$L_{21b}{}^{a}$
$C_{21b}{}^{b}$
$g_{21b}{}^{a}$
$-y_{21b}$
$g_{21b}{}^{b}$
$C_{22b}{}^{a}$
$C_{22b}{}^{b}$
$g_{22b}{}^{a}$
y_{22b}
$g_{22b}{}^{b}$
$C_{12b}{}^{a}$
$C_{12b}{}^{b}$
$g_{12b}{}^{a}$
$-y_{12b}$
$g_{12b}{}^{b}$

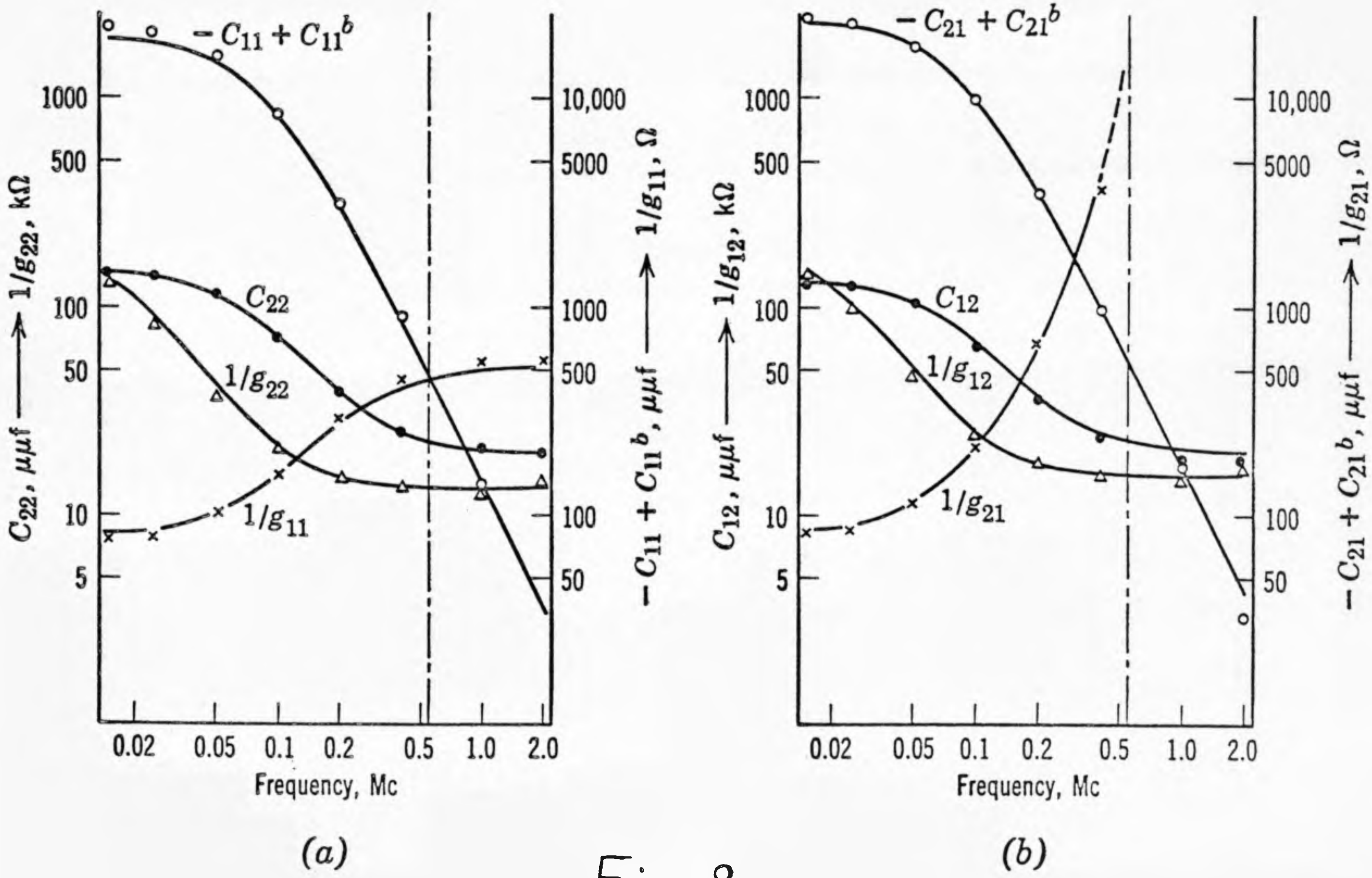
Fig. 6



$-C_{11} + C_{11}{}^b$
C_{22}
$1/g_{22}$
$1/g_{11}$
C_{22}, μμf ⟶ $1/g_{22}$, kΩ
$-C_{11} + C_{11}{}^b$, μμf ⟶ $1/g_{11}$, Ω
1000
500
100
50
10
5
10,000
5000
1000
500
100
50
0.02
0.05
0.1
0.2
0.5
1.0
2.0
Frequency, Mc
$-C_{21} + C_{21}{}^b$
C_{12}
$1/g_{12}$
$1/g_{21}$
C_{12}, μμf ⟶ $1/g_{12}$, kΩ
$-C_{21} + C_{21}{}^b$, μμf ⟶ $1/g_{21}$, Ω
Frequency, Mc

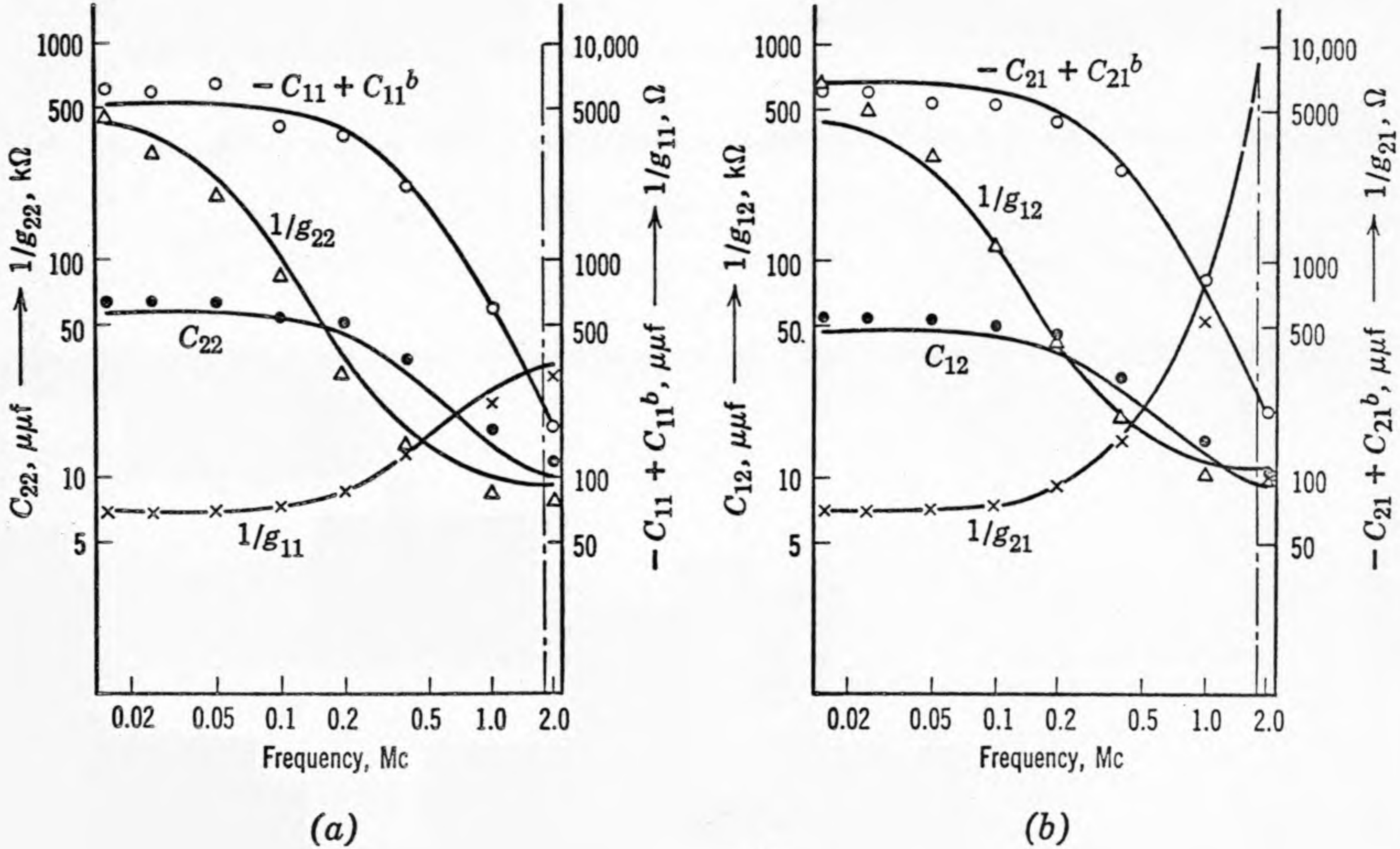
(a) (b)

Fig. 7



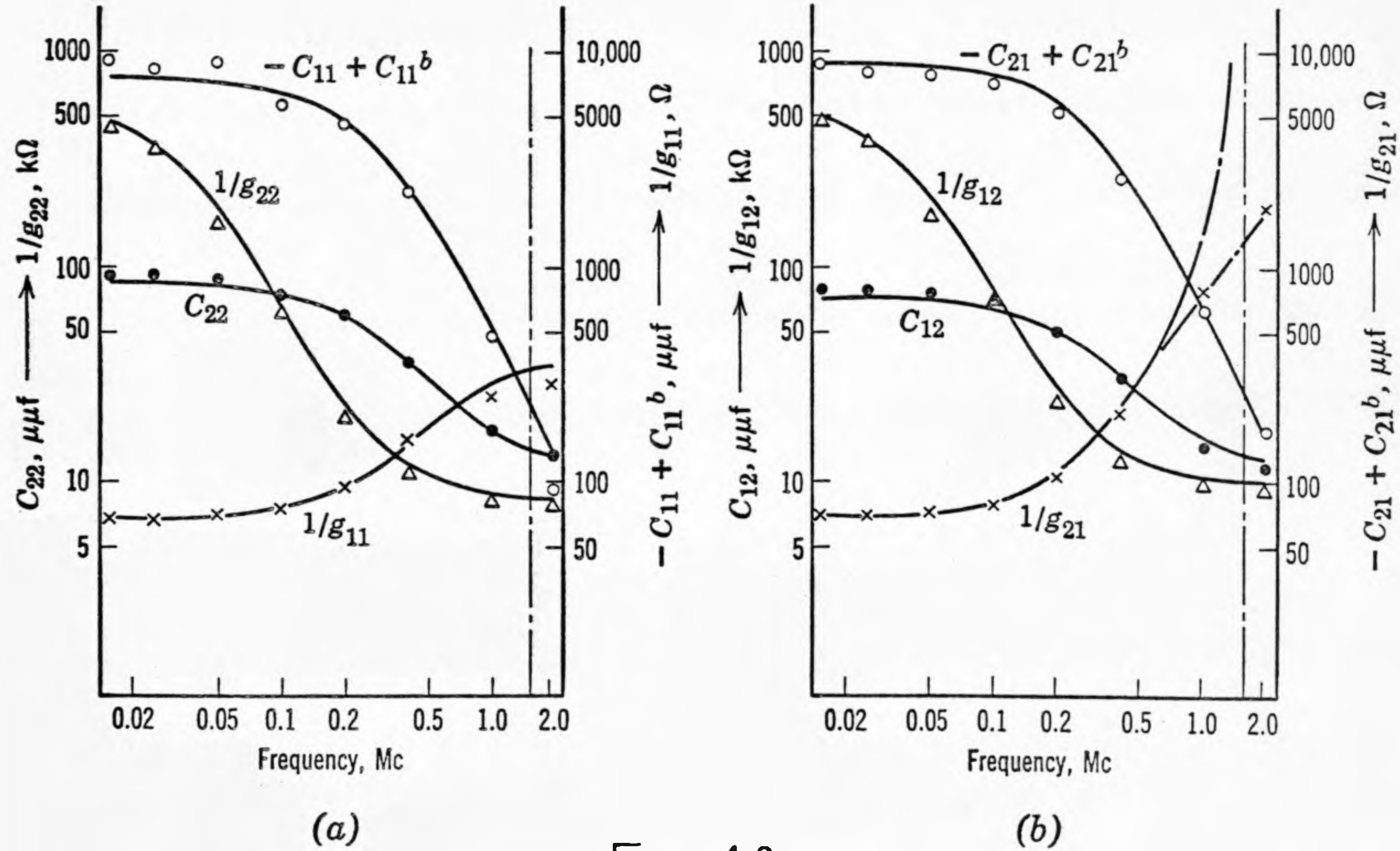
$-C_{11} + C_{11}^b$
1000
500
100
50
10
5
10,000
5000
1000
500
100
50
C_{22}
$1/g_{22}$
$1/g_{11}$
C_{22}, $\mu\mu f$ ⟶ $1/g_{22}$, kΩ
$-C_{11} + C_{11}^b$, $\mu\mu f$ ⟶ $1/g_{11}$, Ω
0.02
0.05
0.1
0.2
0.5
1.0
2.0
Frequency, Mc
(a)
$-C_{21} + C_{21}^b$
1000
500
100
50
10
5
10,000
5000
1000
500
100
50
C_{12}
$1/g_{12}$
$1/g_{21}$
C_{12}, $\mu\mu f$ ⟶ $1/g_{12}$, kΩ
$-C_{21} + C_{21}^b$, $\mu\mu f$ ⟶ $1/g_{21}$, Ω
0.02
0.05
0.1
0.2
0.5
1.0
2.0
Frequency, Mc
(b)

Fig. 8



$-C_{11} + C_{11}{}^b$
$1/g_{22}$
C_{22}
$1/g_{11}$
C_{22}, $\mu\mu$f ⟶ $1/g_{22}$, kΩ
$-C_{11} + C_{11}{}^b$, $\mu\mu$f ⟶ $1/g_{11}$, Ω
1000
500
100
50
10
5
10,000
5000
1000
500
100
50
0.02
0.05
0.1
0.2
0.5
1.0
2.0
Frequency, Mc
(a)
$-C_{21} + C_{21}{}^b$
$1/g_{12}$
C_{12}
$1/g_{21}$
C_{12}, $\mu\mu$f ⟶ $1/g_{12}$, kΩ
$-C_{21} + C_{21}{}^b$, $\mu\mu$f ⟶ $1/g_{21}$, Ω
Frequency, Mc
(b)

Fig. 9



1000
500
100
50
10
5
10,000
5000
1000
500
100
50
$-C_{11} + C_{11}{}^b$
$1/g_{22}$
C_{22}
$1/g_{11}$
C_{22}, $\mu\mu f$ ⟶ $1/g_{22}$, kΩ
$-C_{11} + C_{11}{}^b$, $\mu\mu f$ ⟶ $1/g_{11}$, Ω
0.02 0.05 0.1 0.2 0.5 1.0 2.0
Frequency, Mc
(a)
$-C_{21} + C_{21}{}^b$
$1/g_{12}$
C_{12}
$1/g_{21}$
C_{12}, $\mu\mu f$ ⟶ $1/g_{12}$, kΩ
$-C_{21} + C_{21}{}^b$, $\mu\mu f$ ⟶ $1/g_{21}$, Ω
Frequency, Mc
(b)

Fig. 10