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THREE-DIMENSIONAL LATTICES WITH ISOTROPIC DIELECTRIC PROPERTIES

by

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ABSTRACT

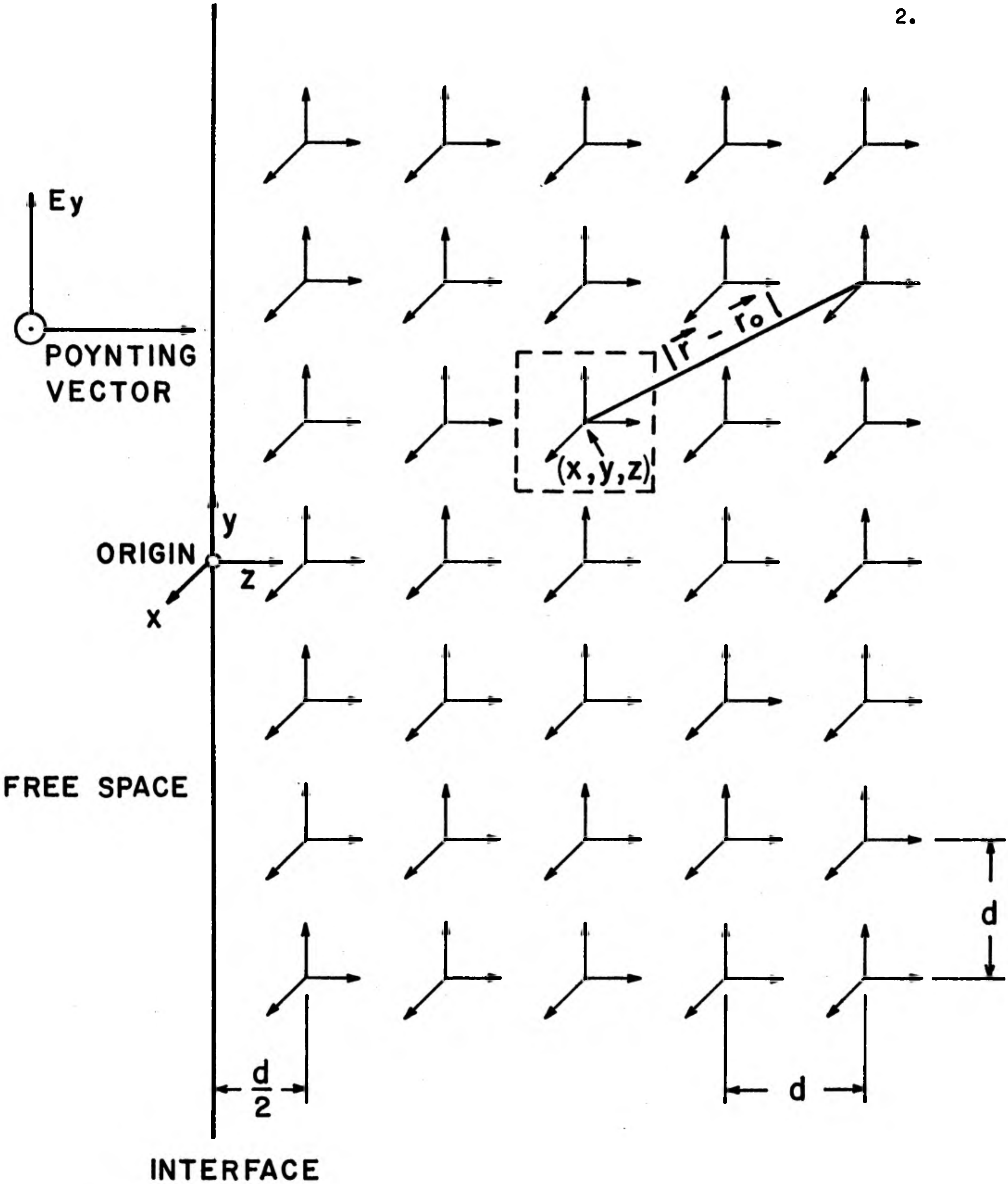
Expressions are derived for the constitutive dielectric parameters of a cubical lattice whose elements consist of a triad of mutually perpendicular polarizable elements. The analysis gives the fundamental relations for the simulation by suitably disposed dipoles, of three-dimensionally isotropic dielectrics with dielectric constants greater than, equal to, and less than unity. Three different approaches have been used. One of these is a complete and rigorous solution which gives the dielectric tensor for the general case of unrestricted spacing to wavelength ratio. This rigorous analysis shows that the Clausius-Mosotti relation often used in predicting the properties of such lattices is a satisfactory approximation only if the spacing is very small with respect to wavelength. Using the general principles developed in the paper, conditions are derived for the realizability of reflectionless media.

Introduction

The study of the dielectric properties of lattices composed of identical metallic or dielectric elements of various geometries has received considerable attention in recent years in connection with the practical application of such structures for polarizing devices, microwave lenses and radome materials. The objective of this paper is the study of a new type of lattice medium of cubical geometry in which the lattice components consist of a triad of mutually perpendicular, (see Fig. 1) polarizable elements of identical geometry.

If the geometry of the elements of this triad is restricted so that their induced fields may be described by a set of three mutually perpendicular dipoles on the lattice points, the resulting structure will be three-dimensionally isotropic and may be used to simulate lightweight dielectrics for the microwave region of the electromagnetic spectrum. The above restriction on element geometry to shapes that will scatter only dipole fields is based on the fact that if the lattice is to be isotropic, then the higher order multipoles, which have tensor moments, must be excluded.

In this analysis the induced dipole moments mentioned above are first chosen to be static moments, and then as dynamic moments, in order to consider retardation effects. The results for these two models are next compared in order to examine the range of validity of the static approximation. Furthermore, this evaluation of the static approximation is useful in any wave problem which is reduced to a molecular analogy by transforming a lattice of scatterers into a continuum of low frequency static dipoles.



Plane Polarized Wave Incident on Region $z > 0$ Containing Triad Elements in Cubic Lattice of Side d .

I. Static Model of Triad Medium

The constitutive dielectric parameters for uniform space arrays of generalized structural geometry composed of similarly oriented elements of completely generalized material and shape have been derived by the author[1]. The theoretical procedure employed in evaluating these parameters is analogous to the classical method used in the study of the dielectric properties of nonpolar media, and assumes that the disturbing action of each element on a uniform static field can be allowed for if each generalized particle is replaced by a set of three mutually perpendicular static dipoles. This assumption implies the restriction that element size and spacing be small compared to wavelength.

The lattice structure under consideration in this paper is a uniform lattice of elements, each of which is a triad of three mutually perpendicular elements, and will hereafter be referred to as a "triad medium". The fundamental dielectric parameters for this lattice structure may be obtained from the results of the aforementioned article. The dielectric constant tensor (k_e) when interaction between lattice elements is neglected, is given by

$$(k_e) = (1) + \frac{N(\delta)}{\epsilon_o} \tag{1}$$

where (δ), the polarizability tensor is defined by the relation

$$\vec{P} = N\vec{p} = N(\delta) \cdot \vec{E}_o \quad ,$$

N = number of elements per unit volume, $\vec{p} = p_x \vec{a}_x + p_y \vec{a}_y + p_z \vec{a}_z$ denotes

(1) Z. A. Kaprielian, J.Appl.Phys. 27, 24 (1956).

the resultant dipole moment of the triad at lattice points, ϵ_o = permittivity of free space, $\vec{E}_o$ = the incident electric vector, $\vec{P}$ = the polarization vector. The derivation of Equation 1 assumes that the field acting on each individual element in the presence of the others remains equal to the externally applied field. If the contribution of all elements in the exciting field of each element is taken into consideration, the dielectric constant is given by

$$(k_e) = (1) - \frac{N}{\epsilon_o}(T) \qquad (2)$$

where (T) is the diagonal tensor given by

$$(T) = \begin{bmatrix} \dfrac{1}{\dfrac{N}{3\epsilon_o} - \dfrac{1}{\delta}} & 0 & 0 \\ 0 & \dfrac{1}{\dfrac{N}{3\epsilon_o} - \dfrac{1}{\delta}} & 0 \\ 0 & 0 & \dfrac{1}{\dfrac{N}{3\epsilon_o} - \dfrac{1}{\delta}} \end{bmatrix} .$$

II. Dynamic Model of Triad Medium - Method 1

Equations (1) and (2) are valid only if the spacing between the elements of the lattice remains small compared to the wavelength. This restriction can be removed by taking into consideration the total field scattered by the dipole elements rather than simply the static dipole contribution. As has already been emphasized, such a computation would be valuable in determining the approximation involved in lattice problems with specialized element geometries such as disks, strips, etc. where the static approximation has frequently been used by many authors.

In this dynamic model, in which total scattered fields are considered, a semi-infinite region of the lattice is considered as shown in Fig. 1. A plane wave polarized in the y-direction is incident from the left. The total field in the lattice medium is given by

$$E_y = E_{yinc} + E_{ys} \tag{3}$$

where E_y is the y-component of the exciting electric field at (X,Y,Z), and consists of the incident field E_{yinc} plus the contributions of the scattered fields E_{ys} from the remaining elements (see Fig. 1). The amplitude of the field scattered by a triad of mutually perpendicular dipoles is given by:

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \vec{\nabla} \times \vec{\nabla} \times \left(\vec{p}\frac{e^{-jkr}}{r}\right)$$

$$= \frac{1}{4\pi\epsilon_o} \vec{\nabla} \times \vec{\nabla} \times \left[(\delta)\cdot\vec{E}\,\frac{e^{-jkr}}{r}\right]$$

where $\vec{E}$ is the incident field on the triad and r is the distance from the center of the triad to the point of observation. Since all the elements in the plane $z = Z$ are influenced by the exciting field, in the same way, the exciting electric and magnetic fields are a function of the z-coordinate only. The only restriction that will be imposed is that the geometry of the elements is such as to insure a predominantly dipole field; the higher multipole excitation should be small to insure isotropy. There are no restrictions on element spacing.

The bulk constants, as used here, are such that the usual results derivable from Maxwell's equation in an ordinary dielectric medium hold in any macroscopic region of the loaded material. The idea of macroscopic field expresses the concept of an average field in a region large compared to the mean spacing between the loading particles.

Using the expression for the field scattered by a single triad of dipoles, the total scattered field at the point (X,Y,Z) equals

$$\vec{E}_s = \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=0}^{\infty} \frac{1}{4\pi\epsilon_0} \left[\nabla(\nabla \cdot \vec{A'}) + k^2 \vec{A'} \right]$$

where

$$A' = \frac{e^{-jk\left|\vec{r}-\vec{r}_o\right|}}{\left|\vec{r}-\vec{r}_o\right|} \quad (\delta) \cdot \vec{E}(m_3 d)$$

and

$$\left|\vec{r}-\vec{r}_o\right| = \left[(m_1 d - X)^2 + (m_2 d - Y)^2 + (m_3 d - Z)^2\right]^{1/2}$$

$\sum'$ signifies that the field arising from the element (X,Y,Z) is to be omitted from the summation. Since the expression for $\vec{E}_s$ is slowly varying with respect to m_1, m_2, and m_3, the summation can be replaced by an integration by using the trapezoidal rule. Changing variables to $\alpha = m_1 d$, $\beta = m_2 d$, $\gamma = m_3 d$ the triple summation becomes

$$E_{ys} = \frac{1}{4\pi\epsilon_0 d^3} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \left(\frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} + k^2 A_y \right) d\alpha\, d\beta\, d\gamma \right.$$

$$\left. - \int_{x-d/2}^{x+d/2} \int_{y-d/2}^{y+d/2} \int_{z-d/2}^{z+d/2} \left(\frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} + k^2 A_y \right) \right] d\alpha\, d\beta\, d\gamma \quad (4)$$

where

$$A = \frac{e^{-jk\sqrt{(\alpha - X)^2 + (\beta - Y)^2 + (\gamma - Z)^2}}}{\sqrt{(\alpha - X)^2 + (\beta - Y)^2 + (\gamma - Z)^2}} \, (\delta) \cdot E(\gamma)$$

The first integrand involves terms like

$$\frac{\partial^2}{\partial\alpha\,\partial\beta}\left[\frac{e^{-jk|\vec{r}-\vec{r}_o|}}{|\vec{r}-\vec{r}_o|}\,\delta\,E_x(\gamma)\right]$$

and these may be integrated by parts, the integrated part vanishing at both limits; the fourth term of the integrand integrated with respect to α and β gives

$$k^2\,\delta\,E_y(Z)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-jk|\vec{r}-\vec{r}_o|}}{|\vec{r}-\vec{r}_o|}\,\partial\alpha\,\partial\beta = -2j\,\pi\,k\,\delta\,e^{-jk|\gamma - z|}E_y(Z)\,. \qquad (5)$$

This integral is evaluated by performing a transformation to polar coordinate system centered at (X,Y). The second integration around the point (X,Y,Z) corresponds to the omission from the summation of the field of the particle at that point. If for this term the near field is considered, that is, the highest power of $\frac{1}{|\vec{r}-\vec{r}_o|}$ in the integrand, the second integral reduces to

$$\int_{X-d/2}^{X+d/2}\int_{Y-d/2}^{Y+d/2}\int_{Z-d/2}^{Z+d/2} E_y(Z)\,\frac{\partial^2}{\partial\alpha^2}\left(\frac{1}{|\vec{r}-\vec{r}_o|}\right)\Big|\,d\alpha\,d\beta\,d\gamma$$

$$= -\frac{4\pi}{3}E_y(Z) \qquad . \qquad (6)$$

Substitution of the results of equations (5) and (6) in equation (3) gives

$$E_y(Z) = E_{y\,inc} + \frac{\delta}{3\epsilon_o d^3}E_y(Z) - \frac{j\delta k}{2\epsilon_o d^3}\int_0^{\infty} e^{-jk|\gamma - Z|}E_y(\gamma)\,d\gamma\,. \quad (7)$$

The solution of equation (7) for the exciting field is obtained by operating on both sides of the equation with $\left(\frac{\partial^2}{\partial Z^2} + k^2\right)$. This gives

$$\left(\frac{\partial^2}{\partial Z^2} + k^2\right)\left(1 - \frac{\delta}{3\epsilon_o d^3}\right) E_y(Z) + \frac{j\delta k}{2\epsilon_o d^3}\left(\frac{\partial^2}{\partial Z^2} + k^2\right) \int_0^\infty E_y(\gamma)\, e^{-jk|\gamma - Z|}\, d\gamma = 0 \quad . \tag{8}$$

The expression under the integral sign yields

$$\int_0^\infty \left(\frac{d^2}{\partial Z^2} + k^2\right) e^{-jk|\gamma - Z|} E_y(\gamma)\, d\gamma = -2jk\, E_y(Z) \quad . \tag{9}$$

Substituting this result in equation (8) yields the differential equation

$$\left(\frac{d^2}{dZ^2} + k^2 + k^2 \frac{\delta/\epsilon_o d^3}{1 - \frac{\delta}{3\epsilon_o d^3}}\right) E_y(Z) = 0 \tag{10}$$

whose solution is

$$E_y(Z) = B\, e^{-j\mathcal{H}z} \; ; \quad \mathcal{H}^2 = k^2 + k^2 \frac{\delta/\epsilon_o d^3}{1 - \frac{\delta}{3\epsilon_o d^3}} \quad . \tag{11}$$

Since only the electric dipoles are considered, the relative permeability of the medium is unity and therefore

$$k_{ezz} = \frac{\mathcal{H}^2}{k^2} = 1 + \frac{\delta/\epsilon_o d^3}{1 - \frac{\delta}{3\epsilon_o d^3}} \quad . \tag{12}$$

It can easily be shown that

$$k_{exx} = k_{eyy} = k_{ezz}$$

and replacing $\frac{1}{d^3}$ by N , number of elements per unit volume, the dielectric tensor is given by

$$(k_e) = (1) - \frac{N}{\epsilon_o}(T) \tag{13}$$

(T) is defined by equation (2) .

This expression for the dielectric tensor (k) is identical with the expression obtained by electrostatic considerations (See Equation 2) and represents the Clausius-Mosotti relations for an isotropic dielectric. This result clearly demonstrates that consideration of the retardation effects of the fields induced in the elements of the array, for practical purposes, can only give results identical to those obtained from simple electrostatic considerations. This degeneration of the "exact" solution is the result of the approximation used in evaluating the summations occurring in the formulation of the problem. The trapezoidal rule which was used to carry out this summation has been previously used by many authors in problems of this type[2]. Since in this paper simple dipole excitation only is considered, it has been possible to quantitatively explore the extent to which the use of the trapezoidal rule is valid in lattice problems of the type described. The foregoing analysis clearly indicates the limitations of the method. These limitations will be more fully described in Section III.

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[2] L. Lewin, Advanced Theory of Waveguides (I. Cliffe and Sons, Ltd., London, 1951).

III. Dynamic Model of Triad Medium - Method 2.

In this section another formulation will be used to carry on a rigorous solution of the problem. The expression for the vector potential $\vec{A}$ may be expressed in terms of a Green's function and a source distribution function. The Green's function for the lattice structure being considered may be written as

$$G(\vec{r},\vec{r}_0) \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} = \frac{e^{-jk\left|\vec{r} - m_1 d\vec{a}_x - m_2 d\vec{a}_y - m_3 d\vec{a}_z\right|}}{\left|\vec{r} - m_1 d\vec{a}_x - m_2 d\vec{a}_y - m_3 d\vec{a}_z\right|}$$

$$= \frac{e^{-jk\left|\vec{r} - \vec{M}\right|}}{\left|\vec{r} - \vec{M}\right|}$$

where

$$\vec{M} = m_1 d\vec{a}_x + m_2 d\vec{a}_y + m_3 d\vec{a}_z \quad .$$

The definition of Green's function is similar to the conventional definition of a three-dimensional Green's function. The Floquet type phase variation between "sources" has been omitted in this definition and will be absorbed in the current distribution. Let $\vec{\mathcal{H}}$ represent a vector whose magnitude is equal to the propagation constant and whose sense is the direction of propagation. The current distribution on the $(m_1,m_2,m_3)^{th}$ dipole triad will be related by the factor

$$e^{-j(\vec{\mathcal{H}} \cdot \vec{M})}$$

to the current distribution at the center of the coordinate system. The vector potential is expressible in this way in terms of the current on the elements of the infinite unbounded three-dimensional array. The exact amplitude of each mode depends, of course, on the specified excitation. For dipolar fields, the current distribution is uniform and may be expressed in terms of dipole moments as follows:

$$\vec{A} = \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} \frac{1}{4\pi} \vec{i} \; G dv = \frac{j\omega}{4\pi} \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} G \vec{p} \, e^{-j(\vec{\mathcal{H}}\cdot\vec{M})}$$

$\vec{i}$ is the current distribution, dv is the volume element. The field at the point $\vec{r}$ is ,

$$E = \frac{p}{4\pi \epsilon_o} \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} \nabla \times \nabla \times \left[G \, \vec{a}_p \, e^{-j(\vec{\mathcal{H}}\cdot\vec{M})} \right] \tag{14}$$

where $\vec{a}_p$ is a unit vector in the direction of the dipole moment vector $\vec{p}$. The vector operators are taken with respect to the field point $\vec{r}$ but in view of the form of the operand, these operators may act upon the source point $\vec{M}$ without changing the value of the expression. If the observation point is chosen to be the origin of the coordinate system, the expression for the total field becomes (see Fig. 2)

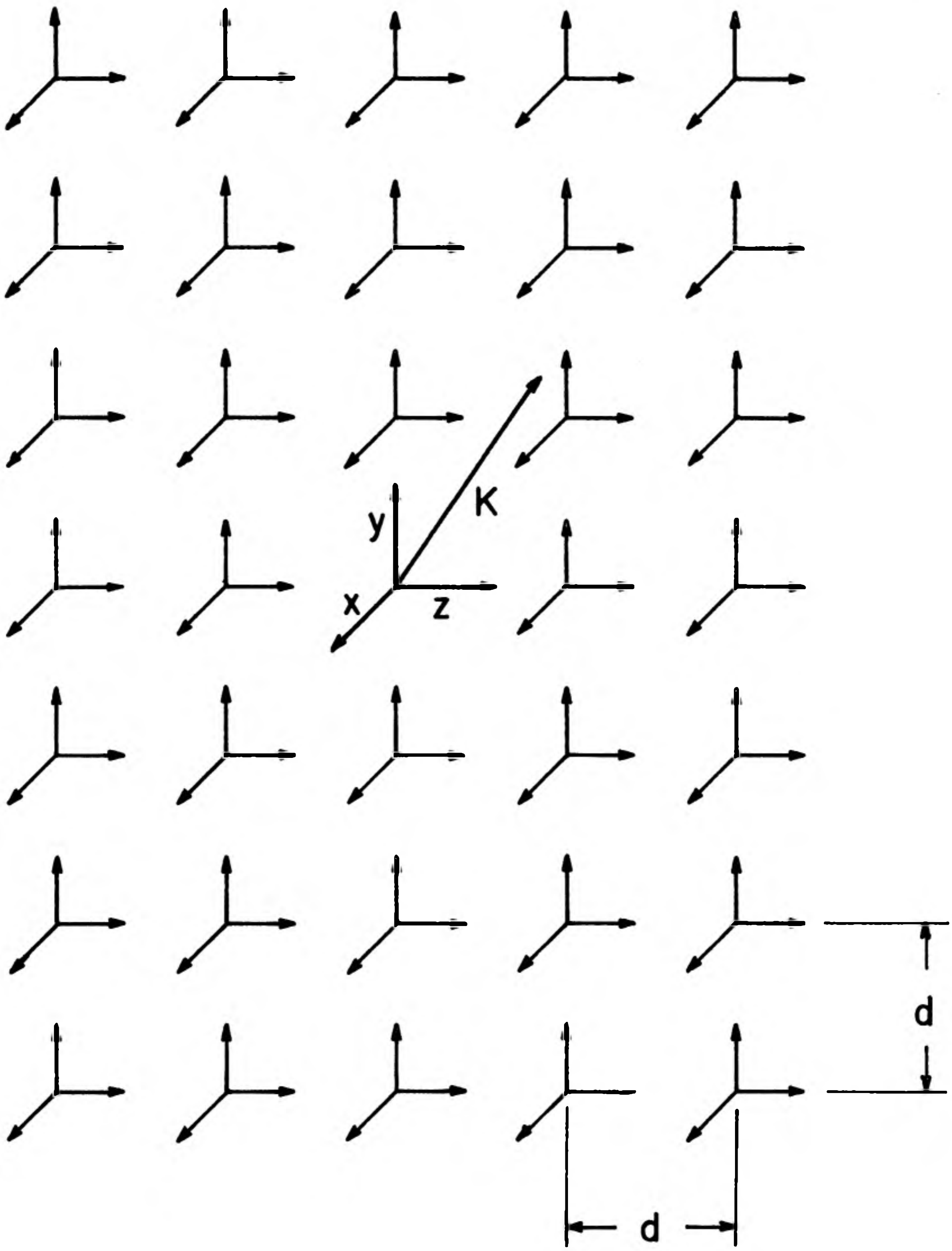
$$E = \frac{p}{4\pi \epsilon_o} \sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} \vec{F}(m_1,m_2,m_3) \tag{15}$$

where $\vec{F}(m_1,m_2,m_3)$ is defined by

$$\vec{F}(m_1,m_2,m_3) = \nabla \left\{ \nabla \cdot \left[f(m_1,m_2,m_3)\, \vec{a}_p \right] - \nabla^2 \left[f(m_1,m_2,m_3)\, \vec{a}_p \right] \right\}.$$

and

$$f(m_1,m_2,m_3) = \frac{e^{-j(kM + \vec{\mathcal{H}}\cdot\vec{M})}}{M} .$$



Infinite Region Containing Triad Elements in Cubic Lattice of side d .

The left hand side of Eq. (15) is a slowly converging series. A more tractable form can be obtained by using the Poisson summation formula[3]. The Poisson formula for an infinite triple sum may be written as

$$\sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} \sum_{m_3=-\infty}^{\infty} g(m_1,m_2,m_3) =$$

$$\sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \sum_{n_3=-\infty}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1,x_2,x_3)e^{-j2\pi(n_1x_1+n_2x_2+n_3x_3)} dx_1dx_2dx_3 \tag{16}$$

An essential restriction in the application of this formula is that continuity of the function is required. However, $f(m_1,m_2,m_3)$ is not continuous at $(m_1=0,\ m_2=0,\ m_3=0)$, but since this term is not included in the summation, the difficulty is avoided. Applying Eq. (16) and (A-9) of the appendix to Eq. (15) yields

$$\vec{E} = \frac{p}{4\pi\epsilon_o} \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \sum_{n_3=-\infty}^{\infty} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \vec{F}(x_1,x_2,x_3)e^{-j2\pi(n_1x_1+n_2x_2+n_3x_3)} dx_1dx_2dx_3$$

$$-\frac{p}{4\pi\epsilon_o}\left[\vec{L} + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\ dx_1dx_2dx_3\right. \tag{17}$$

$$\left. + 8\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\cos 2\pi n_1x_1 \cos 2\pi n_2x_2 \cos 2\pi n_3x_3\ dx_1dx_2dx_3\right]$$

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(3) J. L. Mordell, Proc. Cambridge Phil. Soc. 24, 412 (1929).

In Eq. (17) the term in the brackets is the result of the application of Poisson's summation formula for a finite term (see Appendix, Eq. (A-9)) to the term at ($m_1 = 0$, $m_2 = 0$, $m_3 = 0$). $\vec{L}$ represents the sum of the single term, single terms and double terms appearing in Eq. (A-9). In this application $\vec{L}$ is simply the sum of a finite number of terms.

$$\begin{aligned}\vec{L} = -\frac{1}{8}\Big[&\vec{F}(-1,-1,-1) + \vec{F}(1,-1,-1) + \vec{F}(1,-1,1) + \vec{F}(1,-1,-1)\\
&+ \vec{F}(-1,1,-1) + \vec{F}(-1,1,1) + \vec{F}(1,1,-1) + \vec{F}(1,1,1)\Big]\\
&- \frac{1}{4}\Big[\vec{F}(0,-1,-1) + \vec{F}(0,-1,1) + \vec{F}(0,1,-1) + \vec{F}(0,1,1)\\
&+ \vec{F}(-1,0,-1) + \vec{F}(1,0,-1) + \vec{F}(-1,0,1) + \vec{F}(1,0,1)\\
&+ \vec{F}(-1,-1,0) + \vec{F}(1,-1,0) + \vec{F}(-1,1,0) + \vec{F}(1,1,0)\Big]\\
&- \frac{1}{2}\Big[\vec{F}(0,0,-1) + \vec{F}(0,0,1) + \vec{F}(-1,0,0) + \vec{F}(1,0,0) + \vec{F}(0,-1,0) + \vec{F}(0,1,0)\Big] .\end{aligned} \tag{18}$$

The first integral in Eq.(17) is evaluated in the appendix (See Appendix equations (A-1) to (A-5)). Substituting this value gives

$$\begin{aligned}\vec{E} = \frac{p}{\epsilon_o d^3} &\frac{\vec{\mathcal{H}}(\vec{a}_p \cdot \vec{\mathcal{H}}) + k^2 \vec{a}_p}{\left|\vec{\mathcal{H}} - \frac{2\pi m_1}{d}\vec{a}_x - \frac{2\pi m_2}{d}\vec{a}_y - \frac{2\pi m_3}{d}\vec{a}_z\right|^2 - k^2}\\
&- \frac{p}{4\pi\epsilon_o}\Bigg[\vec{L} + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\, dx_1 dx_2 dx_3\\
&+ 8\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3) \times\\
&\cos 2\pi n_1 x_1 \cos 2\pi n_2 x_2 \cos 2\pi n_3 x_3\, dx_1 dx_2 dx_3\Bigg]\end{aligned} \tag{19}$$

The electric moment of an element of the triad medium is given by

$$\vec{p} = (\delta) \cdot \vec{E} \quad . \tag{20}$$

Solving Equations (19) and (20) simultaneously gives

$$\vec{a}_p = (\delta) \cdot \left\{ \frac{1}{\epsilon_o d^3} \frac{\vec{\mathcal{H}}(\vec{a}_p \cdot \vec{\mathcal{H}}) + k^2 \vec{a}_p}{\left| \vec{\mathcal{H}} - \frac{2\pi n_1}{d} \vec{a}_x - \frac{2\pi n_2}{d} \vec{a}_y - \frac{2\pi n_3}{d} \vec{a}_z \right| - k^2} \right.$$

$$- \frac{1}{4\pi \epsilon_o} \left[\vec{L} + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\, dx_1 dx_2 dx_3 \right. \tag{21}$$

$$\left. \left. + 8 \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3) \cos 2\pi n_1 x_1 \cos 2\pi n_2 x_2 \cos 2\pi n_3 x_3\, dx_1 dx_2 dx_3 \right] \right\}$$

This is the most general expression for the propagation constant of the triad medium. Using Eq. (A-11) of the Appendix, Eq. (20) becomes

$$\vec{a}_p = (\delta) \cdot \left\{ \frac{1}{\epsilon_o d^3} \frac{\vec{\mathcal{H}}(\vec{a}_p \cdot \vec{\mathcal{H}}) + k^2 \vec{a}_p}{\left| \vec{\mathcal{H}} - 2\pi n_1 \vec{b}_1 - 2\pi n_2 \vec{b}_2 - 2\pi n_3 \vec{b}_3 \right|^2 - k^2} \right.$$

$$- \frac{1}{4\pi \epsilon_o} \left[\vec{L} + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\, dx_1 dx_2 dx_3 \right.$$

$$- \frac{\vec{F}_{x_1^1 x_2^1 x_3^1}(a_1,a_2,a_3) - \vec{F}_{x_1^1 x_2^1 x_3^1}(b_1,b_2,b_3)}{1728} +$$

$$\left. \left. \frac{\vec{F}_{x_1^3 x_2^1 x_3^1}(a_1,a_2,a_3) - \vec{F}_{x_1^3 x_2^1 x_3^1}(b_1,b_2,b_3)}{26820} \cdots \right] \right\} \quad . \tag{22}$$

Equation (22) is expressed in terms of the reciprocal lattice vectors $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_3$.[4] The expression for $\vec{\mathcal{K}}$ in terms of the derivatives of $\vec{F}(x_1,x_2,x_3)$ is more convenient since in this case it is easier to evaluate the derivatives than the integrals. It is seen that Eq. (22) is not defined when $\vec{\mathcal{K}}$ is so chosen that

$$\left| \vec{\mathcal{K}} - 2\pi n_1 \vec{b}_1 - 2\pi n_2 \vec{b}_2 - 2\pi n_3 \vec{b}_3 \right|^2 = k^2 \quad .$$

This is, however, precisely the condition for Bragg reflections in a lattice. Since in the case under consideration there exists coupling between the triad elements, $\vec{\mathcal{K}}$ can never assume values which describe Bragg reflections; hence the series can always be considered to converge for the purpose of this paper.

<u>The Propagation Constant $\vec{\mathcal{K}}$ for the First Brillouin Zone $n_1 = n_2 = n_3 = 0$</u>

If the propagation constant $\vec{\mathcal{K}}$ is assumed to be perpendicular to the resultant dipole moment, for the first Brillouin zone $n_1 = n_2 = n_3 = 0$, Eq. (22) reduces to

$$\vec{a}_p = (\delta)\cdot\frac{k^2}{\epsilon_o\, d^3}\,\frac{\vec{a}_p}{\mathcal{K}^2 - k^2} + \frac{1}{4\pi\epsilon_o}\left[\vec{I} + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \vec{F}(x_1,x_2,x_3)\, dx_1 dx_2 dx_3\right] \tag{23}$$

To compare the expression in Eq. (23) with that obtained in Section II, the same approximation used in that section for the evaluation of the near fields will be used here. Therefore, in Eq. (23) representation of $\vec{F}(x_1,x_2,x_3)$, defined by Eq. (15), by the first term of its series expression yields

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4 L. Brillouin, Wave Propagation in Periodic Structures (McGraw-Hill Book Company, Inc., 1946).

$$1 = \frac{1}{\epsilon_o d^3} \frac{k^2}{\mathcal{H}^2 - k^2} + \frac{1}{4\pi \epsilon_o d^3} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \frac{\partial^2}{\partial x_1^2} \frac{1}{\sqrt{x_1^2+x_2^2+x_3^2}} \, dx_1 dx_2 dx_3 \tag{24}$$

where $\vec{a}_p$ is taken parallel to x_1-axis $\vec{L}$ defined by Eq. (18) reduces to 0 . Carrying on the integration yields

$$k_{exx} - 1 = \frac{N/\epsilon_o}{1 - N/3\epsilon_o}$$

with the notation of Section I, the dielectric tensor may be written as

$$(k_e) = (1) - \frac{N}{\epsilon_o} (T) \tag{25}$$

This is identical with the results of Section I and II and very clearly indicates the limitations of the method given in those sections. Considering the expression

$$\sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_3=1}^{\infty} \frac{k^2}{\left|\vec{\mathcal{H}} - \frac{2\pi n_1}{d} \vec{a}_x - \frac{2\pi n_2}{d} \vec{a}_y - \frac{2\pi n_3}{d} \vec{a}_z\right|^2 - k^2}$$

which may be written in terms of the wavelength λ as

$$\sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_3=1}^{\infty} \frac{1}{\left|\frac{\vec{\mathcal{H}}}{k} - \frac{\lambda n_1}{d} \vec{a}_x - \frac{\lambda n_2}{d} \vec{a}_y - \frac{\lambda n_3}{d} \vec{a}_z\right|^2 - 1} .$$

It is seen that for large values of λ/d , Eq. (25) is a very good approximation since the contribution of terms n_1, n_2 and $n_3 \neq 0$ may be considered negligible. On the other hand, for d of the order of λ, the contribution of the terms when n_1, n_2 and $n_3 \neq 0$ in Eq. (21) or (22) is important and must be considered. Therefore it can be seen that the Clausius-Mosotti relation is a very good approximation for large values of λ/d . For general values of the ratio λ/d on the other hand, the complete expression derived in this section must be used.

The analysis given in this section is most general. It is assumed that the geometry of the elements of the triad medium is so restricted that they will scatter only dipole fields. This restriction is imposed to guarantee isotropy. The expression given by Eq. (25) shows that the dielectric constant of the lattice may be less than or greater than unity for negative or positive polarizabilities respectively. The elements of the polarizability tensor will be positive if the dipoles are operated at a frequency below their resonant frequency and will be negative if operated above.

To obtain a dielectric constant of unity, the lattice medium must be embedded in a dielectric binder of appropriate dielectric constant. By evaluating the dielectric parameters of the triad medium by any of the three aforementioned methods, the expression for the dielectric constant corresponding to Eq. (25) for the case in which the lattice is embedded in a uniform dielectric with dielectric constant k_m, is given by

$$(k_e) - (k_m) = -\frac{N}{\epsilon_o}(T) \quad . \tag{26}$$

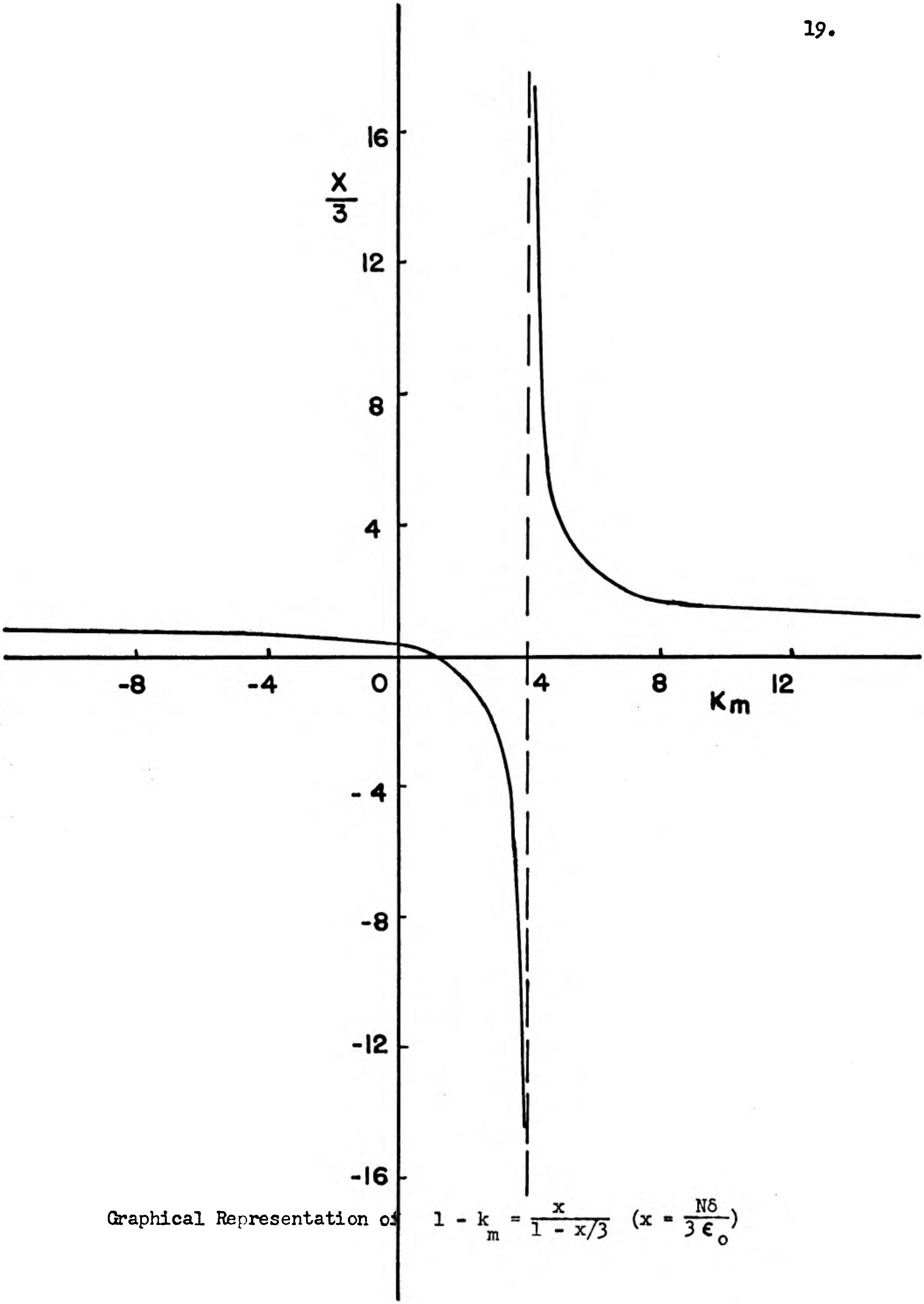
For the composite dielectric tensor to be a unity tensor, the following relation must be satisfied

$$(1) - (k_m) = -\frac{N}{\epsilon_o}(T) \quad . \tag{27}$$

The corresponding scalar equation is

$$1 - k_{mxx} = \frac{n\delta/\epsilon_o}{1 - N\delta/3\epsilon_o}$$

with similar equations of k_{myy} and k_{mzz}. This relation is plotted in Fig. 3. Since ordinary embedding dielectric materials have a dielec tric constant greater than one, only the portion of the curve to the right of $k_m = x$ is of interest. There are two distinct regions of



Graphical Representation of $1 - k_m = \frac{x}{1 - x/3}$ $(x = \frac{N\delta}{3\epsilon_o})$

importance. For the first region, $1 < k_m < 4$, the polarizability is negative and therefore the medium has to be operated at a frequency below the resonant frequency of the triad elements. For the region $k_m > 4$, the polarizability is positive and the medium must be operated above the resonant frequency.

For small anisotropies ~~Since~~ the magnitudes of the dipole moments are equal along the three principal axes of the triad elements, the directional cosines of $\vec{a}_p$ are equal and therefore the expression for the propagation constant given by Eq. (21) is isotropic with respect to the coordinate axes.

In certain applications of isotropic artificial dielectrics such as in radome design, it is important to obtain dielectrics with an index of refraction of unity. In order for this to be true, not only must the dielectric constant be unity, but the relative permeability must also be equal to 1 . The relative permeability may be made equal to unity by properly restricting the geometry of the elements so that magnetic dipole fields are not induced. Consequently in order to have a reflectionless and also isotropic dielectric, the geometry of the triad elements must be so restricted that neither higher order multipoles (electric or magnetic) nor magnetic dipoles are excited. Work is continuing on the realizability of the triad elements for such arrays.

Conclusion

Three different approaches have been used to evaluate the expression for the dielectric parameters of the triad medium:

(a) A molecular analogy with a consideration of static dipole interaction leading to the Clausius-Mosotti relations.

(b) An analysis based on the summation of scattered time-varying fields in which it is demonstrated that the use of the trapezoidal summation approximation which is so frequently used in these problems, completely removes retardation effects. The results obtained in this case could much more easily have been obtained directly from static considerations.

(c) An exact solution valid for all values of spacing to wave-length ratio. In this case, the importance of the retardation effect in the general result for the propagation constant of the composite medium is clearly seen.

This investigation has direct application in the design of isotropic materials for the control and direction of microwave energy. By varying the frequency of operation above and below the resonant frequency of the triad elements, the effective dielectric constant can be made less than or greater than unity respectively. If the triad medium is embedded in a uniform dielectric, it is possible to simulate materials with dielectric constant of unity. Such material may be useful in applications in which a reflectionless dielectric is required.

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APPENDIX

a) Evaluation of the Expression S .

In order to evaluate

$$S = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \vec{F}(x_1,x_2,x_3)\, e^{-j\,2\pi(n_1x_1 + n_2x_2 + n_3x_3)}\, dx_1\, dx_2\, dx_3$$

the change of variables $x_1 d = y_1$, $x_2 d = y_2$ and $x_3 d = y_3$ is made. Hence the expression for S becomes

$$S = \frac{1}{d^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ \nabla\left[\nabla\cdot\left(e^{-jk(y_1^2+y_2^2+y_3^2)^{1/2}}\, e^{-j\vec{\mathcal{K}}\cdot(y_1\vec{a}_x+y_2\vec{a}_y+y_3\vec{a}_z)}\vec{a}_p\right)\right.\right.$$
$$\left.\left. -\nabla^2\left(e^{-jk(y_1^2+y_2^2+y_3^2)^{1/2}}\, e^{-j\vec{\mathcal{K}}\cdot(y_1\vec{a}_x+y_2\vec{a}_y+y_3\vec{a}_z)}\vec{a}_p\right)\right\} e^{-j\frac{2\pi}{d}(n_1y_1+n_2y_2+n_3y_3)}\, dy_1\, dy_2\, dy_3 \tag{A-1}$$

Consider the expression

$$\frac{e^{-jk(y_1^2+y_2^2+y_3^2)^{1/2}}}{(y_1^2+y_2^2+y_3^2)^{1/2}}$$

the Fourier transform of this expression with respect to y_1 is

$$\int_{-\infty}^{\infty} \frac{e^{-jk(y_1^2+y_2^2+y_3^2)^{1/2}}}{(y_1^2+y_2^2+y_3^2)^{1/2}}\, e^{-j\mathcal{K}_x y_1}\, dy_1 =$$

$$-\pi j\, H_o^{(2)}\left[(k^2-\mathcal{K}_x^2)^{1/2}\,(y_2^2+y_3^2)^{1/2}\right] \tag{A-2}$$

Taking the Fourier transform of the right hand side of Eq. (A-2) with respect to y_2 gives

$$\int_{-\infty}^{\infty} -\pi j\, H_o^{(2)} \left[(k^2 - \mathcal{K}_x^2)^{1/2} \, (y_2^2 + y_3^2)^{1/2} \right] e^{-j\mathcal{K}_y y_2} \, dy_2$$

$$= 2\pi \, \frac{e^{-j(\mathcal{K}_x^2 + \mathcal{K}_y^2 - k^2)^{1/2} \, |y_3|}}{(\mathcal{K}_x^2 + \mathcal{K}_y^2 - k^2)^{1/2}} \quad . \tag{A-3}$$

The Fourier transform expression on the right hand side of (A-3) with respect to y_3 yields

$$\int_{-\infty}^{\infty} 2\pi \, \frac{e^{-j(\mathcal{K}_x^{\,2} + \mathcal{K}_y^2 - k^2)^{1/2}}}{(\mathcal{K}_x^2 + \mathcal{K}_y^2 - k^2)^{1/2}} \, e^{-j\mathcal{K}_2 y_3} dy_3 = 4\pi \, \frac{1}{\mathcal{K}_x^{\,2} + \mathcal{K}_1^2 + \mathcal{K}_2^2 - k^2} \tag{A-4}$$

With the use of well-known relations of operational calculus the expression for S reduces to

$$S = \frac{4\pi}{d^3} \, \frac{\vec{\mathcal{K}}(\vec{a}_p \cdot \vec{\mathcal{K}}) + k^2 \vec{a}_p}{\left| \vec{\mathcal{K}} - \frac{2\pi m_1}{d} \vec{a}_x - \frac{2\pi m_2}{d} \vec{a}_y - \frac{2\pi m_3}{d} \vec{a}_z \right|^2 - k^2} \quad . \tag{A-5}$$

b) Poisson Summation Formula for a Finite Sum .

A relation similar to Eq. (16) will be desired for a finite sum. For simplicity, the derivation will be carried through in one dimension. The derivation for the three-dimensional case is similar and the result only will be stated.

If in the interval $m \leq x \leq m+1$, $f(y)$ meets the requirements for representation as a Fourier series, the following equations give the value of that representation both in the open interval and at the end points of the interval $m \leq y \leq m+1$

$$f(y) = \sum_{n=-\infty}^{\infty} \int_{m}^{m+1} f(x)\, e^{-j\,2\pi n(y-x)}\, dx \qquad m<y<m+1$$

and

$$\frac{f(m) + f(m+1)}{2} = \sum_{n=-\infty}^{\infty} \int_{m}^{m+1} f(x)\, e^{-j\,2\pi nx}\, dx \qquad y = m,\ m+1 \quad . \tag{A-6}$$

If both sides are summed up from $m = a+1$ to $m = b-1$ and if $-\frac{f(a)+f(b)}{2}$ is added to both sides, the Poisson transformation for a finite sum is obtained.

$$\sum_{m=a+1}^{b-1} f(m) = -\frac{f(a)+f(b)}{2} + \sum_{n=-\infty}^{\infty} \int_{a}^{b} f(x)\, e^{-j\,2\pi nx}\, dx$$

$$= -\frac{f(a)+f(b)}{2} + \int_{a}^{b} f(x)\, dx + 2\sum_{n=1}^{\infty} \int_{a}^{b} f(x) \cos 2\pi nx\, dx \quad . \tag{A-7}$$

The first two terms on the right-hand side of Eq. (A-7) are the results of approximating a summation by an integration using the trapezoidal rule; hence the last term of Eq. (A-7) can be considered as the correction to the trapezoidal rule. In some cases this trapezoidal part is quite accurate in summing the series, thus transferring difficulty from summing $f(m)$ to integrating the same function. Integrating the last integral in Eq. (A-7) P times yields

$$\sum_{a+1}^{b-1} f(m) = -\frac{f(a)+f(b)}{2} + \int_{a}^{b} f(x)\, dx$$

$$+ 2\sum_{r=1}^{P} (-1)^r \frac{\delta(2r)}{(2\pi)^{2r}} \left[f^{(2r-1)}(a) - f^{(2r-1)}(b) \right] \tag{A-8}$$

$$+ \frac{2(-1)^{r+1}}{(2\pi)^{2r}} \sum_{n=1}^{\infty} \frac{1}{(n)^{2r}} \int_{a}^{b} f(x) \cos 2\pi nx\, dx$$

where

$$\delta(2r) = \sum_{n=1}^{\infty} \frac{1}{n^{2r}}$$

is the Rieman Zeta function. This derivation can be extended to the three-dimensional case, giving

$$\sum_{a_1+1}^{b_1-1} \sum_{a_2+1}^{b_2-1} \sum_{a_3+1}^{b_3-1} f(m_1,m_2,m_3) =$$

$$- \frac{1}{8} \Big[f(a_1,a_2,a_3) + f(b_1,a_2,a_3) + f(b_1,a_2,b_3) + f(b_1,a_2,a_3)$$

$$+ f(a_1,b_2,a_3) + f(a_1,b_2,b_3) + f(b_1,b_2,a_3) + f(b_1,b_2,b_3) \Big]$$

$$- \frac{1}{4} \sum_{m_1=a_1+1}^{b_1-1} \Big[f(m_1,a_2,a_3) + f(m_1,a_2,b_3) + f(m_1,b_2,a_3) + f(m_1,b_2,b_3) \Big]$$

$$- \frac{1}{4} \sum_{m_2=a_2+1}^{b_2-1} \Big[f(a_1,m_2,a_3) + f(b_1,m_2,a_3) + f(a_1,m_2,b_3) + f(b_1,m_2,b_3) \Big]$$

$$- \frac{1}{4} \sum_{m_3=a_3+1}^{b_3-1} \Big[f(a_1,a_2,m_3) + f(b_1,a_2,b_3) + f(a_1,b_2,m_3) + f(b_1,b_2,m_3) \Big]$$

$$- \frac{1}{2} \sum_{m_1=a_1+1}^{b_1-1} \sum_{m_2=a_2+1}^{b_2-1} \Big[f(m_1,m_2,a_3) + f(m_1,m_2,b_3) \Big]$$

$$- \frac{1}{2} \sum_{a_2+1}^{b_2-1} \sum_{a_3+1}^{b_3-1} \Big[f(a_1,m_2,m_3) + f(b_1,m_2,m_3) \Big]$$

$$- \frac{1}{2} \sum_{a_1+1}^{b_1-1} \sum_{a_3+1}^{b_1-1} \Big[f(m_1,a_2,m_3) + f(m_1,b_2,m_3) \Big] + \int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{b_3} f(x_1,x_2,x_3)\, dx_1\, dx_2\, dx_3$$

$$+\, 8 \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_3=1}^{\infty} \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} f(x_1,x_2,x_3) \cos 2\pi n_1 x_1 \cos 2\pi n_2 x_2 \cos 2\pi n_3 x_3 \, dx_1 dx_2 dx_3 \tag{A-9}$$

Changing Eq. (A-9) into a form similar to Eq. (A-8) results in

$$\sum_{a_1+1}^{b_1-1} \sum_{a_2+1}^{b_2-1} \sum_{a_3+1}^{b_3-1} f(m_1,m_2,m_3) =$$

$$L \;+ \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} f(x_1,x_2,x_3)\, dx_1 dx_2 dx_3 \;+$$

$$\sum_{r=1}^{N} \sum_{s=1}^{P} \sum_{t=1}^{Q} \frac{(-1)^{r+s+t}}{(2\pi n_1)^{2r}(2\pi n_2)^{2s}(2\pi n_3)^{2t}} \;\cdot$$

$$\left[f_{x_1^{2r-1}\, x_2^{2s-1}\, x_3^{2t-1}}(a_1,a_2,a_3) - f_{x_1^{2r-1}\, x_2^{2s-1}\, x_3^{2t-1}}(b_1,b_2,b_3) \right]$$

$$\underset{x_1 x_2 x_3}{P} \sum_{r=1}^{N} \sum_{s=1}^{P} \frac{(-1)^{r+s}\,(-1)^{t+1}}{(2\pi n_1)^{2r}(2\pi n_2)^{2s}(2\pi n_3)^{2t}}$$

$$\int_{a_3}^{b_3} \left[f_{x_1^{2r-1}\, x_2^{2s-1}\, x_3^{2t}}(a_1,a_2,x_3) - f_{x_1^{2r-1}\, x_2^{2s-1}\, x_3^{2t}}(b_1,b_2,b_3) \right] \cos 2\pi n_3 x_3 \, dx_3$$

$$+ \;\underset{x_1 x_2 x_3}{P} \sum_{r=1}^{N} \frac{(-1)^{r}\,(-1)^{s+1}\,(-1)^{t+1}}{(2\pi n_1)^{2r}(2\pi n_2)^{2s}(2\pi n_3)^{2t}} \;\times$$

$$\int_{a_2}^{x_2}\int_{a_3}^{x_3} \left[f_{x_1^{2r-1}\, x_2^{2s}\, x_3^{2t}}(a_1,x_2,x_3) - f_{x_1^{2r-1}\, x_2^{2s}\, x_3^{2t}}(b_1,x_2,x_3) \right] \cos 2\pi n_2 x_2 \cos 2\pi n_3 x_3 \, dx_2 dx_3$$

$$+ \frac{(-1)^{r+1}(-1)^{s+1}(-1)^{t+1}}{(2\pi m_1)^{2r}(2\pi m_2)^{2s}(2\pi m_s)^{2t}} \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3}$$

(A-10)

$$f_{x_1^{2r}\, x_2^{2s}\, x_3^{2t}}(x_1,x_2,x_3) \cos 2\pi m_1 x_1 \cos 2\pi m_2 x_2 \cos 2\pi m_3 x_3 \; dx_1 dx_2 dx_3$$

where L represents the sum of the single terms, single sums and double sums. In Eq. (A-10) $\underset{x_1 x_2 x_3}{P}$ indicates circular permutation of (x,y,z).

Unless the derivatives increase very rapidly the transformed series in Eq. (A-10) may be written as

$$\sum_{a_1+1}^{b_1-1}\sum_{a_2+1}^{b_2-1}\sum_{a_3+1}^{b_3-1} f(m_1,m_2,m_3) =$$

$$L + \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} f(x_1,x_2,x_3)\, dx_1 dx_2 dx_3 +$$

$$- \frac{f_{x_1^1 x_2^1 x_3^1}(a_1,a_2,a_3) - f_{x_1^1 x_2^1 x_3^1}(b_1,b_2,b_3)}{1728}$$

$$+ \frac{f_{x_1^3 x_2^1 x_3^1}(a_1,a_2,a_3) - f_{x_1^1 x_2^1 x_3^1}(b_1,b_2,b_3)}{26820} +$$

$$\frac{f_{x_1^1 x_2^3 x_3^1}(a_1,a_2,a_3) - f_{x_1^1 x_2^3 x_2^1}(b_1,b_2,b_3)}{26820}$$

$$\frac{f_{x_1^1 x_2^1 x_3^3}(a_1,a_2,a_3) - f_{x_1^1 x_2^1 x_3^3}(b_1,b_2,b_3)}{26820} \cdots \quad . \tag{A-11}$$

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