Effect of Inhomogeneous Heat Flow on the Enhancement of Heat Capacity in Helium-II by Counterflow near T_λ

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Abstract. In 2000 Harter *et al.* reported the first measurements of the enhancement of the heat capacity $\Delta C_Q \equiv C(Q)-C(Q=0)$ of helium-II transporting a heat flux density Q near T_λ. Surprisingly, their measured ΔC_Q was ~7-12 times larger than predicted, depending on which theory was assumed. In this report we present a candidate explanation for this discrepancy: unintended heat flux inhomogeneity. Because $C(Q)$ should diverge at a critical heat flux density Q_c, homogeneous heat flow is required for an accurate measurement. We present results from numerical analysis of the heat flow in the Harter *et al.* cell indicating that substantial inhomogeneity occurred. We determine the effect of the inhomogeneity on ΔC_Q and find rough agreement with the observed disparity between prediction and measurement.

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In order to evaluate the idea that unintended inhomogeneity of the heat flow in the Harter *et al.*[1] experiment might account for the discrepancy between measurement and predictions[2,3] of ΔC_Q, we must estimate the heat flow field $\boldsymbol{Q}(\boldsymbol{r})$ in the helium-II. It is not difficult to show[4] that thermal counterflow in helium-II can be solved simultaneously with the diffusive heat flow in the enclosing experimental cell using a standard finite-element solver[5], if the helium-II is nondissipative, nonvortical, nearly isothermal, and free of net mass flow ($\boldsymbol{J}=0$). These conditions should have been well-approximated in the Harter *et al.* experiment.

Such a numerical model has been constructed and solved for the Harter *et al.* cell. The model geometry is shown in Fig. 1. Not visible at this scale is the model for the Kapitza boundary resistance R_K: an artificial thin envelope of thickness δ=25 μm and thermal conductivity $\kappa_{RK}=\delta/R_K$ interposed between the helium and the cell walls.

For best accuracy, the helium-II diffusion coefficient should be modeled as $\kappa_{He}=\alpha(\rho_s/\rho_n)$, where α is a large constant required to reduce the variation of the scalar superfluid velocity potential function[4]. However, Harter *et al.* had a very short cell (0.64 mm) and did not approach closer to T_λ than $\sim 0.5\times 10^{-6}$ K, limiting the maximum variation of ρ_s/ρ_n over the height of their cell to ~10%. To reduce the number of required computations we have approximated ρ_s/ρ_n as constant and set $\kappa_{He}=10^6$ W/cmK. Test reductions of κ_{He} to 10^5 W/cmK changed calculated enhancements by only ~0.01%, verifying that κ_{He} is sufficiently large.

To within their measurement noise, Harter *et al.* found that $t^{-\alpha}\Delta C_Q$ was linear in $(Q/Q_c)^2$, where $t=(T_\lambda-T)/T_\lambda$ is the reduced temperature. Keeping only the

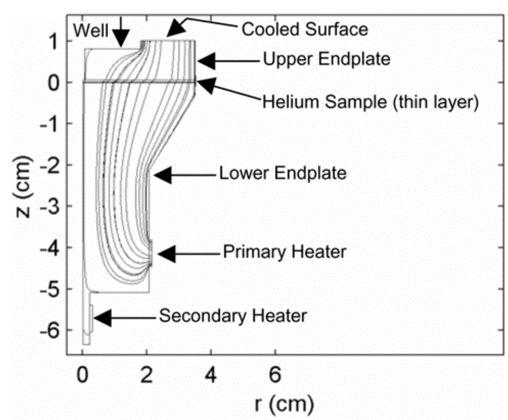

FIGURE 1. Cell model geometry with heat flow streamlines. The model is axisymmetric about r=0. Streamlines show heat flowing from the primary heater to the cooled surface. The principal cause of inhomogeneity of the heat flow in the helium is the "well" cut into the upper endplate to accommodate the diaphragm valve.

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lowest-order $(Q/Q_c)^2$ term of the expansion for ΔC_Q[2], and neglecting the variation of reduced temperature t over the height of the cell, the fractional enhancement of ΔC_Q by inhomogeneity is

$$E = \frac{\int t^{-\alpha}\left(Q(\mathbf{r})/Q_c\right)^2 d\mathbf{r}}{\int t^{-\alpha}\left(Q_{\text{nom}}/Q_c\right)^2 d\mathbf{r}} = \frac{\int Q^2(\mathbf{r})d\mathbf{r}}{Q_{\text{nom}}^2 V_{\text{helium}}} \qquad (1)$$

where the integrals are taken over the helium volume, and Q_{nom} is the "nominal" heat flux density (corresponding to that reported by Harter *et al.*) that would have been obtained for homogeneous heat flow.

In solutions of the linear heat flow equation for a given mixed boundary condition, the distribution of heat flux $\boldsymbol{Q}(\boldsymbol{r})$ is unaffected if all conductivities are scaled by the same factor. We have deliberately set κ_{He} so high that it is effectively infinite, thus $\boldsymbol{Q}(\boldsymbol{r})$ can depend only on the ratio of R_{K} to the endplate thermal conductivity κ_{Cu}. The calculations confirm this scaling: values of E agree to within ~0.1% or better for scenarios where the product $R_{\text{K}}\kappa_{Cu}$ is equal.

Although it was impossible to deduce an accurate R_{K} from the Harter *et al*. data, extensive measurements[6] exist of the value and reproducibility of R_{K} for Cu surfaces and helium-II near T_λ. Those measurements, together with others made by us at the University of New Mexico and Caltech, show that an estimate of R_{K}=1.0±0.2 cm^{2}K/W should be very reliable. We determined κ_{Cu} from published fits of κ_{Cu}(RRR)[7] and measurements of the RRR of several "core samples" cut from the bottom endplate of the Harter *et al*. cell by electrical discharge machining. These samples yielded RRR=240-260, thus κ_{Cu} =6.9-7.4 W/cmK.

The calculated E is shown in Fig. 2. Using R_{K}=1 cm^{2}K/W and κ_{Cu}=7.2 W/cm·K yields E=3.0, compared to the observed anomalous enhancement of ~7-12. Given the complexity and approximations involved in this post-experiment analysis, this level of agreement seems quite good.

Also shown in Fig. 2 are calculated maximum values of Q_{nom}/Q_c. Harter *et al*. found that above a maximum Q_{nom}/Q_c~0.3 (their "β" point) additional thermal resistance appeared between the bottom and top endplates. They proposed that this happened when the coherence length grew to exceed the surface roughness of the bottom endplate, effectively decreasing the bottom endplate area and increasing the apparent Kapitza resistance. Our present analysis provides another candidate explanation: the β point might be occurring when the maximum value of $|Q(\boldsymbol{r})|$

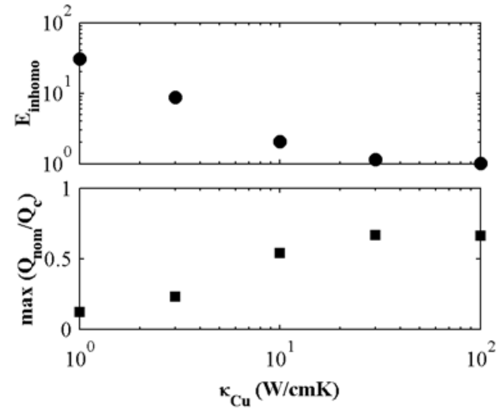

FIGURE 2. Calculated values of E and the β point over the expected range of κ_{Cu}, assuming R_{K}=1 cm^{2}K/W.

becomes comparable to Q_c, causing a local breakdown of superflow. In this case max(Q_{nom}/Q_c) should equal the calculated Q_{nom}/max($|Q(\boldsymbol{r})|$), which for our estimated values of R_{K} and κ_{Cu} is 0.43, in reasonable agreement with observation. A recent reanalysis of some of the Harter *et al*. data by one of us (ARC) has shown that this new explanation may fit the data better than the correlation length argument.

In summary, numerical estimates of $\boldsymbol{Q}(\boldsymbol{r})$ in the Harter *et al*. cell indicate significant inhomogeneity which might explain both the anomalously large ΔC_Q and the β point. There is a clear need for new measurements of ΔC_Q in a new cell with homogenous heat flow. Such a cell has been prepared at the University of New Mexico, and the cooldown is presently underway at Caltech. Work supported by NASA Fundamental Physics Discipline NAG3-1763 (STPB), NAG3-2900 (ARC, RAML, DLG), and JPL #960494 (RVD). RVD acknowledges support at Caltech as a Moore Distinguished Scholar.

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