and $W_c \ll L_n$, respectively. Therefore, T_s of the mesa type transistor can be reduced remarkably by doping trap impurities into the collector region or by reducing W_c by an epitaxial structure, and the reduction factor of T_s can be estimated quantitatively by (4) and (5), respectively.

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H. Yanai
T. Sugano
K. Tada
Dept. of Electronic Engrg.
University of Tokyo
Tokyo, Japan

Feedback in Cyclotron-Wave Frequency Multipliers*

Frequency multiplication at microwave frequencies by fast cyclotron-wave interaction through the use of multipole cavities has been studied by Cuccia,[1] Ashkin[2] and more recently by Dain and Thompson.[3] The general scheme of the operation is to introduce a beam carrying a fast cyclotron wave into a multipole cavity. The beam rotates like a metallic conductor around the inside of the cavity. Interaction between the beam (which is rotating at an angular frequency ω_c) and the slot field of the multipole cavity excites a π-mode oscillation. An output at angular frequency $n\omega_c$ can be coupled from the cavity. (The number of poles of the cavity equals $2n$.) As power is drawn from the cavity, the rotating beam loses its energy and its radius of rotation is decreased. Due to the fact that the interaction field in the neighborhood of the slot varies as $(r/a)^{n-1}$, where a is the radius to the vane tip, the field decreases very rapidly with the distance away from the slot. This places a limit on the amount of power that can be withdrawn from the cavity. For instance, in an 18-pole cavity multiplier, if half of the rotational energy of the beam is withdrawn, the radius of rotation reduces to $(a/\sqrt{2})$. At this radius, the interaction field has been reduced to $(1/\sqrt{2})^{9-1}$ (or $\frac{1}{16}$) of the slot field, which is far too weak for useful interaction. Cuccia[1] obtained a "transfer efficiency" of 1.5 per cent using an 8-pole cavity. In Dain and Thompson's[3] computation, 77 per cent of the rotational beam power is collected (and wasted) at the collector.

The present scheme proposes the use of feedback to enhance the conversion efficiency. The schematic diagram of the device is shown in Fig. 1.

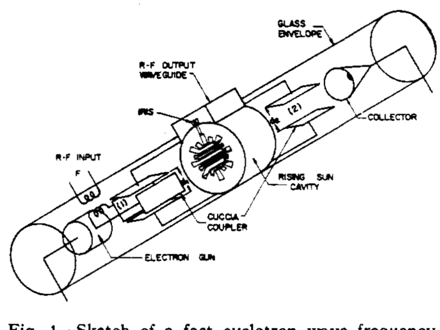

Fig. 1—Sketch of a fast cyclotron wave frequency multiplier using modified Cuccia coupler with feedback.

A Cuccia coupler [see (2) in Fig. 1] is used at the exit end of the beam to pick up its residual rotational energy and feed back to the input through a second Cuccia coupler arranged in space and time quadrature with respect to the input Cuccia coupler. This modified Cuccia coupler [see (1) in Fig. 2] excites the fast cyclotron wave through a circularly polarized wave. Part of the excitation power is supplied by the input signal and part by the feedback coupler. The power that would have been wasted is thus utilized. This means that for the same output power, the input power supplied by the signal source can be reduced. In other words the conversion efficiency of the device can be increased.

Let

- η_1 = input coupler efficiency (both couplers),
- η_2 = extraction coupler efficiency [Cuccia coupler (2)],
- η_3 = multipole cavity coupling efficiency,
- ζ = electronic conversion efficiency (due to left over rotational energy), and
- η = the over-all efficiency.

Then the over-all efficiency of a frequency multiplier with feedback is given by

$$\eta = \frac{\zeta \eta_1 \eta_3}{1 - (1 - \zeta)\eta_1 \eta_2}. \tag{1}$$

On the other hand, without feedback, *i.e.*, $\eta_2 = 0$, (1) reduces to

$$\eta = \zeta \eta_1 \eta_3. \tag{2}$$

The "transfer efficiency" in Cuccia's and the "output coupler efficiency" in Dain and Thompson's papers both correspond to $\zeta\eta_3$ by this definition.

In an ideal case inwhich $\eta_1 \simeq \eta_2 \simeq \eta_3 \simeq 100$ per cent, an overall efficiency of $\eta = 100$ per cent can be achieved irrespective of the inherent inefficient mechanism of the multipole cavity multiplier as expressed by ζ.

The electronic conductance of a Cuccia coupler[4] is given by

$$G_{e2} = \frac{1}{8}\left(\frac{l_2}{d_2}\right)^2 \frac{I_0}{V_0}, \tag{3}$$

where I_0 is the beam current and V_0 is the beam voltage; l_2 and d_2 are the length and separation of the Cuccia coupler [(2) of Fig. 1] plates, respectively. For a modified Cuccia coupler used to excite a circularly polarized wave described in (1) Fig. 1, the electronic conductance seen by each coupler is found to be

$$G_{e1} = \frac{1}{4}\left(\frac{l_1}{d_1}\right)^2 \frac{I_0}{V_0}. \tag{4}$$

Thus if it is intended to feed back half of the rotational power to the input, the impedance matching can be accomplished by letting $G_{e2} = G_{e1}$. If $l_1 = l_2$, it is found that the separation of the plates of the extracting coupler d_2 should be $(1/\sqrt{2})d_1$, that of the input coupler.

C. Yeh
J. C. Lee
Electron Physics Lab.
The University of Michigan
Ann Arbor, Mich

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Power-Law Nature of Field-Effect Transistor Characteristics*

It is interesting to see in Richer and Middlebrook's communication[1] the experimental justification of a power-law relationship between drain current and gate voltage.

We have already proposed similar formulas for cylindrical structure field effect transistors (tecnetron).[2] Taking $n = 2$ in (2) of Richer and Middlebrook,[1] we obtain

$$g_m = \frac{2I_{do}}{V_p}\left(1 + \frac{V_g}{V_p}\right).$$

Our "parabolic approximation,"[2] using the same symbolism, gives exactly the same result (third line, p. 88A[2]). However, as it has been shown in our publication, this approximation is very rough in the case of cylindrical structures, giving results sometimes which are twice the exact value, as in the case of the maximum drain current. In fact, our graph 6, p. 7A[3] shows the variation of the slope as a function of the gate voltage for different approximations. It also indicates that the above formula is the worst of all the approximations in the case of cylindrical structure devices.

It would be interesting to know with what type of geometric structures Richer and Middlebrook have been experimenting and what changes in the relationships obtained experimentally are brought about by modifications in the geometric structure of the device.

* Received September 23, 1963.

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We have published[4,5] comparisons between characteristics of plane and cylindrical structure field effect transistors in the hypercritical region where the mobility is assumed to vary as $(E)^{-1/2}$, E being the axial electric field. We also have proposed a number of approximate relationships giving the electrical characteristics of field effect transistors.[2]

Other geometric structures have been studied. A paper on this subject will be published shortly.

JEAN LEMÉE
Control Systems Dept.
Westinghouse Electric Corp.
Research and Development Center
Pittsburgh, Pa.
A. V. J. MARTIN
Director, Electronique et Automatisme
et Electronique Medicale
Paris, France

Authors' Comment[6]

The details of the *theoretical* justification for our power-law approximation to field-effect transistor (FET) characteristics have not yet been published, but a summary of some of the results is given in two recent articles.[7,8] We consider a simple, planar FTE and show that its properties are essentially independent of the impurity profile. Thus, the theory obviates the solution of an analytically intractable device, such as one with a diffused junction, because such a device may be satisfactorily approximated by an analytically simple structure.

The normalized drain current is especially insensitive to the impurity profile, and beyond pinch-off may be approximated by a power law of the form

$$\frac{I_d}{I_{do}} = \left(1 + \frac{V_g}{V_p}\right)^n. \qquad (1)$$

The main advantage of this approximation is that it permits determination of the pinch-off voltage V_p from a straight-line plot of experimental quantities. If some weak conditions are imposed on the impurity profile, then the exponent n is restricted to the rather narrow range $2 \leq n \lesssim 2.25$.[8] Eq. (1) is not intended to be exact at any one point, but represents an approximation to the over-all shape of the actual transfer characteristics. Further, the worst percentage errors in (1) occur near $V_g = -V_p$ where $I_d \approx 0$, *i.e.*, where percentage errors are unimportant experimentally.

The theory embraces essentially all FET's that are produced domestically, because practical impurity profiles generally satisfy the necessary conditions, and because most structures are planar or approximately so. The devices for which experimental results were reported in our communication[1] have the following geometries:

Crystalonics 610 . . .	concentric
Motorola MM764 . .	planar
Texas Ins. TIX 691 . .	approximately planar
Fairchild FSP 401 . .	planar.

The functional relations for an FET with concentric geometry are identical to those for a planar FET. At present, the theory does not include cylindrical FET's but undoubtedly the method of approach summarized in Richer[7] could be applied to those devices.

I. RICHER
R. D. MIDDLEBROOK
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, Calif.

[4] A. V. J. Martin and J. LeMée, "First Order Analysis of Hypercritical Field Operation and Compared Performances of Field Effect Devices," Coll Int. sur les dispositifs à semiconducteurs, UNESCO, Paris, France, vol. 1, p. 516.

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Diocotron Gain Reduction and Space Charge Smoothing in Crossed-Field Guns*

Until recently it was commonly believed that noise in crossed-field devices is increased as the operation of the device is changed from a temperature-limited operation to a space-charge-limited operation. It is only recently that both the theoretical[1] and experimental[2] results have indicated that the space-charge-limitation causes, in fact, a considerable reduction in the current fluctuations, which in turn reduces the noise content in the beam. The velocity fluctuations, however, may vary, depending upon the type of beam focusing system used in the gun region, and Kino gun[3] operation seems to be quite reasonable in yielding a laminar flow. Comparison of the noise characteristics of a device under the two operating conditions has also become questionable particularly when the trajectories and velocity components under the two different operating conditions require two completely different gun designs.[4]

It has been reported that the reduction in noise figure, possibly due to space-charge smoothing, applies more to larger values of beam currents[2] which are obtained by increasing the values of V_a (the gun-anode voltage) and B (the magnetic field) in such a manner that (V_a/B^2) remains constant so that the trajectories in the gun region are unaltered. There are, in fact, several factors causing this extra reduction in noise. From the expression for the diocotron gain in the gun region developed in the following, it is shown, by operating at a larger value of gun anode voltage, that the value of the required cathode current density is increased, which reduces the diocotron gain (or excess noise) in the gun region. It appears from this information that the noise reduction for larger values of beam currents is due mainly to a reduction of excess noise in the gun region and not to increased space-charge smoothing. An increase in the value of the cathode current density also increases the electron plasma frequency at the potential minimum and, as observed in the one-dimensional calculations, reduces further the current fluctuations at the operating frequency by an increased space-charge smoothing. However, the two-dimensional calculations[1] have not revealed such a dependence on the electron plasma frequency at the potential minimum for the various portions of the beam studied beyond the potential minimum.

The expression for α, the diocotron gain per unit length as derived by Gould,[5] is given by

$$\alpha = \frac{\pi I_0 f}{\epsilon_0 h B u_0^3} \quad \text{nepers/unit length,} \qquad (1)$$

where I_0 = beam current, f = operating frequency, ϵ_0 = permittivity of free space, h = width of the beam in the direction of the magnetic field, B = magnetic field intensity and u_0 = beam velocity. This expression is valid for a beam drifting between two parallel conducting planes. For the case of the Kino gun,[3] this formula for the diocotron gain may also be used piecewise when multiplied by a factor F which takes into account the contribution due to nonparallel conducting planes in the gun region (between the π and 2π planes of the Kino gun $F \simeq 1$); thus the dependence of the diocotron gain on the various parameters in the gun region (assuming thin beams) can be determined. By using the definitions of Φ [the normalized voltage, $\phi = (\eta J_y^2/\epsilon_0^2 \omega_c^4)\Phi$], and Z [the normalized distance, $z = (\eta J_y/\epsilon_0 \omega_c^3)Z$]; as used in the Kino gun model, G the over-all diocotron gain in the gun region is given by

$$G = \int_a^b \alpha(Z)dZ = \frac{\pi\epsilon_0 \eta f}{(2)^{3/2}} \frac{wB^2}{J_y} \int_a^b \frac{F}{\Phi^{3/2}} dZ, \quad (2)$$

where w = width of the cathode, η = absolute ratio of electron charge to its mass, J_y = cathode current density, Φ and Z are functions of $\omega_c T$ only, and the planes a and b are two arbitrary planes along the beam. The values of a and b are specified by the values of $\omega_c T$. Thus the diocotron gain in the Kino gun model is proportional to w and B^2 and inversely proportional to J_y. The expression for J_y in a Kino gun is given by[6]

$$J_y = f\left[\frac{V_a}{B^2}\right] V_a^{3/2}, \qquad (3)$$

where V_a = gun anode voltage. While maintaining (V_a/B^2) = constant, the value of J_y

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