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Tick Size, Price Grids and Market Performance: Stable Matches as a Model of Market Dynamics and Equilibrium

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ABSTRACT

This paper reports experiments motivated by ongoing controversies regarding tick size in markets. The minimum tick size in a market dictates discrete values at which bids and asks can be tendered by market participants. All transaction prices must occur at these discrete values, which are established by the rules of each exchange. The simplicity of experiments helps to distinguish among competing models of complex real-world securities markets. We observe patterns predicted by a matching (cooperative game) model. Because a price grid damages the equilibrium of the competitive model, the matching model provides predictions where the competitive model cannot. Their predictions are the same when a competitive equilibrium exists. The experiment examines stable allocations, average prices, timing of order flow and price dynamics. Larger tick size invites more speculation, which in turn increases liquidity. However, increased speculation leads to inefficient trades that otherwise would not have occurred.

1. INTRODUCTION: THE POLICY ORIGIN[1]

The issue we study has its origin in the discrete nature of trading units, whether it is the unit of value or the "blocks" in which goods are transferred. The issue is brought into systematic focus by a controversy found in financial markets. Our focus is on very simple markets in which the issue can be studied experimentally. Our experimental approach is exploratory rather than a testing of theory because there is no refined or accepted theory to "test". However, our initial experiments suggest a theory that has not been applied previously to continuous markets. Subsequent experiments explore its features.

Policy debates on optimal tick size in financial markets have continued since the decimalization campaign in 1990s. According to Harris (1997)'s review on decimalization, the

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proponents for small tick sizes suggest that a small price increment would lead to smaller bid-ask spreads, hence encouraging price competition and decreasing cost of public traders. The SEC shared this viewpoint and since 2000, the U.S. equity markets have transformed from fractional ticks to penny increments.

More recently, however, arguments have emerged to re-increase tick sizes (Weild, Kim and Newport 2012), supported by claims that small tick sizes have repressed IPOs for small cap enterprises and that wider spreads, by increasing profit for market makers, would lead to more research and attract investors to small cap stocks, thus fostering a financial ecosystem for smaller companies. The SEC is skeptical. [2] Nevertheless, a pilot program of increasing tick size has been put into practice in 2016, despite the strong suspicion on its effectiveness from the commission.

Policy discussion and empirical studies in the academic literature lacks consensus regarding effects of the tick size. The smaller spread and improved price efficiency from small tick size has been reported (Harris 1994, 1997, 1999; Bessembinder 2003; Chakravarty, Wood and Van Ness 2004; Chung, Charoenwong and Ding 2004). However, research suggests that liquidity is harmed since the size of limit orders has become smaller after decimalization (Bacidore, Battalio and Jennings 2003). The effect on informed traders is unclear according to Gibson, Singh and Yerramilli (2003).[3]

Progress on theoretical foundations is found in the market microstructure literature but there is no generally accepted conclusion. A natural division of liquidity provider and liquidity

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[2] The Jumpstart Our Business Startups Act (the JOBS Act) of 2012 has begun to address the issue. The SEC, according to their report responding to the Act (Report to Congress on Decimalization 2012) and a subsequent report (Recommendation of the Investor Advisory Committee Decimalization and Tick Sizes), demonstrated its belief that decline in smaller company IPOs is caused by other exogenous factors rather than decimalization and increasing tick size will only benefit market makers at the cost of retail investors.

[3] Zhao and Chung (2006) test two contradicting hypotheses whether decimalization increases the probability of information-based trading due to smaller spread or reduces it because of undercutting, and shows that the first effect dominates. Although small tick size might have an effect on IPO of small cap companies, the decrease in IPO is mainly due to the increasing merge and acquisition of small companies into large corporations (Gao, Ritter and Zhu 2013). In the recent literature of high frequency trading, Yao and Ye (2016) find that an increase in tick size hinders price competition, generates rents for liquidity provision and encourages speed competition, resulting in higher proportion of liquidity provided by high frequency traders. Saar and Zhong (2015) document the same advantage enjoyed by HFT from large tick size. In addition, their empirical results show that the positive correlation between tick size and market volume only exists when the minimum price variation is binding, and otherwise the opposite relationship is found due to greater adverse selection. Moreover, the literature on stock splitting (Anshuman and Kalay 2002), exchange fees (Chao, Mao and Ye 2016) and dark pool suggest that the market automatically adapts to the optimal tick size itself (Dayri and Rosenbaum 2015), which brings into question the necessity of a uniform standard in tick size.

taker, represented by market makers and public traders respectively, has been inherited by subsequent theoretical studies ever since the fundamental work of Garman (1975) and Kyle (1985). The motivations for liquidity and immediacy as connected to market microstructures are explored by Demesetz (1968) and subsequent work by Garman (1975) and O'Hara (1995). Under that type of framework, small price variation lowers trading cost and leaves more surplus to the traders while large tick size increases rents enjoyed by the market maker and results in a higher incentive to provide liquidity. The separation of liquidity provider and taker simplifies the modeling of the market game. However, it is unable to explain some empirical controversies; for example, the insignificance of correlation between liquidity and tick size in some markets.

Our paper rests on laboratory experimental methods and with a return to the classical models of demand and supply, presumably based on fundamentals, departs from much of the existing literature. The competitive equilibrium model includes theories of price adjustment toward an equilibrium and conditions that define equilibrium. However, in the presence of a price grid, a competitive equilibrium need not exist and the lack of existence of equilibrium essentially prevents the straightforward application of the classical approach.

However, recent studies by Hatfield, Plott and Tanaka (2012, 2016) develop a model for markets where a classical equilibrium does not exist. The notion of competitive equilibrium is replaced by the concept of a stable outcome and the development of stable matchings is viewed as the process through which trading is organized. The matching model produces precise predictions of price and volume even when the competitive equilibrium does not exist due to the existence of price ceilings and floors. More substantially, with the notion of stable outcome as a generalization of competitive equilibrium, it's possible to study the market behavior in a disaggregate way and the principles that underlie market behavior as connected to traditional laboratory experimental results.

2. OVERVIEW OF PAPER

The paper is organized as follows. Two different experimental environments are studied. The first experimental environment studied in Series One, explores the impact of a price grid in the light of preferences that reflect no uncertainty about value to the participant. The classical method of preference inducement is used. The second set of experiments, Series Two, explores the case in which a security pays an uncertain, common dividend to all holders of the security.

The dividend is chosen randomly according to a public distribution. Each buyer receives an independently drawn signal of the dividend conditional on the actual dividend.

While the individual values are different between the two series, the theory used to produce predictions is the same. Section 3 outlines the experimental environment and market structure. Section 4 discusses the theoretical framework of assignments and stability (equilibrium) in the double auction market with a price grid. The section compares predictions from a matching model to predictions by classical competitive equilibrium. Section 5 introduces details of the experiments used to evaluate the models and provides some experimental background material. In section 6, we present the environment and parameters for Series One. Section 7 discusses the experimental results from Series One and compares the two competing hypotheses. Section 8 presents the parameters and discusses results from Series Two experiments. Section 9 addresses the information contained in Series Two prices. Section 10 is a summary of conclusions.

3. MARKET ORGANIZATION, PRICE GRIDS AND STABLE OUTCOMES AS MARKET EQUILIBRIA

The markets we study are organized by a computerized double auction with an order book implemented by traditional experimental economics methods. Preferences are induced with financial incentives and differ between the two types of experimental sessions discussed in Section 5. In series one preferences are constructed from the classical method in which demand and supplies are individually induced. In series two, a common and randomly determined common dividend is paid with each trader having different private information about the dividend

In both series, the market models are closely related to concepts of demand, supply and the resulting competitive equilibria. The specific market demand and supply parameters depend on the induced preferences and can differ from series to series and experiment to experiment. However, while the numerical predictions depend on specific parameters, the principles of competitive equilibrium and of stable matches do not.

Preferences are induced with financial incentives. Trading among agents takes place in a public market. In addition, each agent has a private market in which interactions with the experimenter take place. Subjects classified as “Sellers” acquire the units for sale by buying the units at fixed prices made available by the experimenter in the seller's private market. The seller keeps the difference between the price of a unit paid by the buyer to the seller and the price at which seller purchased the unit from the experimenter. Subjects classified as "Buyers" make money by buying in the market from sellers and selling acquired units at fixed prices made available by the experimenter in the buyer's private market. The buyer keeps the difference between the price of a unit paid to the seller and the price received for the unit from the experimenter.

A classification of agents according to their behavior will be useful. Fundamentalists are sellers who never attempt to buy in the public market and buyers who never attempt to sell in the public market. Speculators are defined conversely as sellers who put a buy order and as buyers who put a sell order in the public market. Sellers who buy in the public market make a profit on the transaction only if they can resell the unit in the public market at a higher price. Buyers who sell in the public market make a profit on the transaction only if they buy it back at a lower price. Thus, speculators are exposed to some risk of loss on the transaction.

Market supply and demand functions are derived from the induced preferences of individual sellers and buyers in a natural way. For purposes of exposition in the text presented here, the buyers and sellers have incentives for only one unit but the experimental incentives were based on multiple units.[4] The potential costs to the seller of acquiring units from the experimenter, the individual seller's marginal costs, are limit prices that can be configured into a market supply curve. That is, the marginal cost is the minimum price at which the seller can sell in the public market without making a loss. Let S denote the set of all sellers. Let c_i denote the marginal cost of seller i that the seller i finds in seller i's private market. For any price P, there is

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[4] Market demand and supply are derived from the induced preferences following standard procedures. The induced preferences are explained in detail in later sections but the key concepts are easily explained from a market aggregate perspective. The potential cost to the sellers of acquiring units from the experimenter can be configured into a market supply curve. That is if $\mathbf{C}(x)$ is the total cost of x units paid to the experimenter we represent the marginal cost , $\mathbf{C}'(x)$, as C(x) so $P = \mathbf{C}'(x)$ is the relationship between marginal cost to price and is used to derive a market supply function, $x = S(P)$. An example is the curve SS in Figure 1. If the potential value the buyers received from the experimenter from delivering units is $\mathbf{R}(x)$ then the marginal value, $\mathbf{R}'(x)$ and the equation $P= \mathbf{R}'(x)$ relates the buyer marginal value to price and can be configured into a market demand , $x=D(P)$ as shown in Figure 1 as DD.

a subset of sellers $S_P \subseteq S$ such that $S_P = \{i \in S: c_i \leq P\}$. The aggregation of sellers' limit prices is used to derive market supply function $x = S(P) = |S_P|$. Namely, the market supply at price P is the number of sellers with marginal cost less than or equal to P. An example is the curve SS in Figure 1.

Similarly, the orders placed in individual buyer's private markets are the value received from units acquired in the public market (the "utility" so to speak). It reflects a simple arbitrage opportunity in which the buyer buys in the public market and sells to the experimenter at the price found in the buyer's private market and keeps the profit. The marginal values found in the buyers private markets, can be configured into a market demand curve. Let B denote the set of all buyers. Let v_j be the marginal value that j finds in buyer j's private market. For any price P, there is a subset of buyers $D_P \subseteq B$ such that $D_P = \{j \in B : v_j \geq P\}$. The aggregation of buyers' preference is used to derive market demand function $x = D(P) = |D_P|$, as shown in Figure 1 as DD. Namely, the market demand at price P is the number of buyers with marginal value higher than or equal to P.

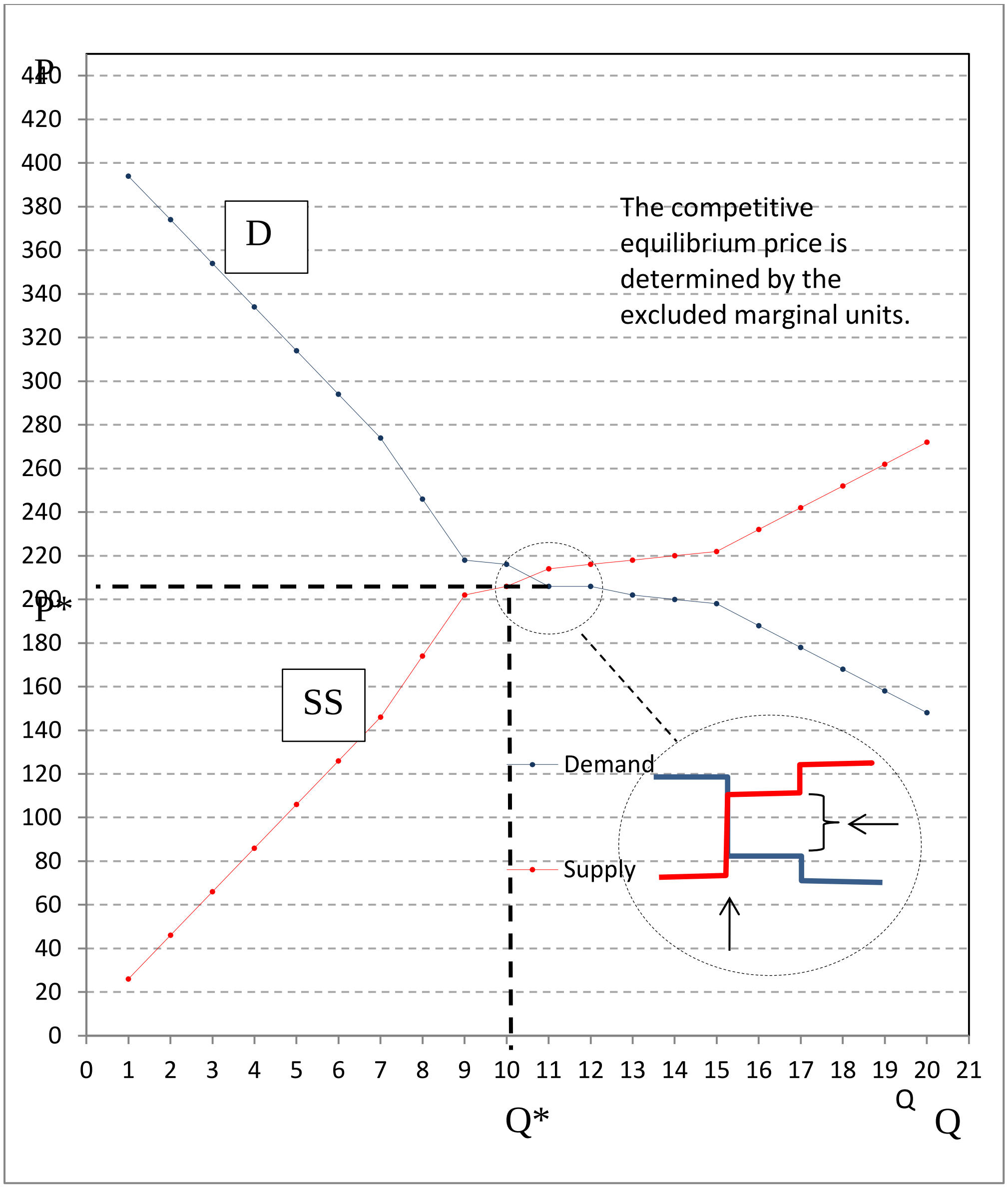


Figure 1: Market Demand and Supply with a Grid. In the absence of a grid, the competitive equilibrium prices are between the first excluded demand and supply units. The upper support of interval is the marginal cost of first excluded unit in the supply curve and the lower support is the marginal value of the first excluded unit in the demand curve. The competitive equilibrium quantity is set by the last included demand and supply units. The bold dashed lines signify the equilibrium based on the assumption that the curves are continuous.

The market is organized by a computerized double auction. Both buyers and sellers tender time stamped offers for multiple units at a stated price per unit (a limit order) that are placed in a public order book with price priority. In case of multiple orders with the same price, the first-in first-out principle is employed. A sell order tendered at a price below the highest priced buy order sells at the highest buy order price and a buy order tendered above the lowest priced sell order sells at the lowest sell order price. The classical competitive equilibrium consists of the price where the quantity demanded equals the quantity supplied which would correspond to (P*, Q*) in Figure 1 if the curves were continuous.

If tick sizes are imposed, then the price grids determined by the tick size are the only prices at which buy and sell orders can be tendered. Examples are the light dotted lines as shown in Figure 1. When the competitive equilibrium price P* is not on the grid, the competitive equilibrium does not exist. For example, in Figure 1 the competitive market price, P*, is not on one of the dotted grid lines. The grid prices above and below the competitive equilibrium price are effectively a ceiling and a floor that prevent convergence.

In the classical economic environment, the assignment/matching model is a natural generalization of competitive equilibrium that can be applied in the presence of price floors and ceilings, of which the price grid imposed by a tick size is a special case. We adapt the Hatfield, Plott and Tanaka (2012, 2016) method by modeling the trading activity as a two-sided matching in a continuous double auction with an open book.[5] Under the matching framework, bids and asks are interpreted as traders' attempts to form a match. A match happens when the buyer and seller have agreed on the price and quantity specified by the bids and asks and signed an exchange contract accordingly. The stable assignment/match is a situation when no individual wants to leave the match unilaterally or no pair of agents is willing to break their current match and form a new match with other agents given the opportunity.

4. THEORETICAL FRAMEWORK

A competitive equilibrium does not always exist when a price grid is present. Recent literature has used principles from matching and assignment theories to construct models of markets without competitive equilibria due to price floors and ceilings. We inherit that modeling

[5] Hatfield, Plott and Tanaka (2012) focus on single unit trades and use an assignment framework. Hatfield, Plott and Tanaka (2016) generalize the model to multiple unit trades and use a matching model framework.

strategy as well as solution concepts to build the theoretical foundation for a market with a price grid.

There is a finite set of buyers B and a finite set of sellers S trading the same good in a market. For purposes of model development, we assume that each buyer and seller deals with only a single unit. The fact that trading takes place in real time creates both practical and theoretical complexities in multiple unit matching models. At a practical level traders in today's markets, in contrast to long-term investors, tend to have very short horizons and typically focus on a single stock or bond at a time; i.e., they do not worry about non-diversified inventory positions because they typically divest them promptly. At a theoretical level the assumption of a single unit is more than just an expositional simplification. Models of stable matches are based on the cooperative game model in which all options are available in a static sense as is the information about all potential contracts. Given this broad set the choices are simultaneous. . Useful formal developments the multiple unit theory are found at HKNOW (2013) and HPT (2015).

4.1 Preferences

Multiple unit incentives in a continuous double auction environment create a challenge for assignment models, which tend to be static. All decisions take place simultaneously or in well-defined stages, By contrast a continuous market operates in real time with no schedule coordinating the timing of decisions. As a result individual decisions need not conform to the consistency predicted by static theory and might be interpreted as a failure of the rationality conditions of the model.

We adopt a modeling convention that addresses the problem by assigning each unit a value and treating the unit as an agent. The modeling challenge is that a contract executed later in time might be profitably substituted for an earlier contract suggesting that the agent is irrational for agreeing to the first of the two contracts. However such actions are also consistent with the fact that the early contract was made when information about a later contract was not available and hindsight provides an incentive to exercise options that were never available given the real time in which contracts evolve. We avoid the issue through the implementation of two postulates. The first assigns a value to the units and allows the unit to be assigned a contract if profitable. The second is to disallow the transfer of contracts among units. The combination facilitates the identification of stable matches. The loss is the ability to capture the dynamics of portfolio theory.

We depart from HPT (2016) and the associated model of stable matches by including an experimental fact of "sequence induced preferences" or "order dictated preferences". Values for buyers and sellers are assigned in an experimentally dictated order meaning that the value of a unit to the buyer or seller is dictated by the number of units consumed or produced by the particular agent prior to the consumption or production of the unit in question. That is, the value of the ith unit to a consumer is the marginal value created by the ith unit in an induced value or cost function. Since the marginal value decreases as units increase on the demand side and increases on the cost side the details must be acknowledged.

The second postulate is that a contract for the ith unit cannot be transferred to the jth. That is, a contract cannot be transferred from one unit to another. Substitution of contracts among units or individuals is not allowed. In the continuous double auction contracts take place in a sequence. The transfer of a contract from one unit to another would be an implicit change of the trading sequence and would separate the market from the real time in which information and contract opportunities take place. It is as if each unit is identified as an agent and the non-transferability of a contract is a natural constraint since it would take place outside the market.

With the exception of speculation the combined impact of the axioms is that each unit can be associated with its own value as determined by its order in the consumption/acquisition. Let the form of redemption value and cost functions be F(x) where x =|C| if C is a set of contracts. Now let f(k) = F(k) - F(k-1) so $F(x) = \Sigma_{k\in\{1,2,\ldots,x\}} f(k)$ and the value of unit k is f(k). If F(x) is interpreted as a utility function then f(k) is the marginal utility and if F(k) is the cost then f(k) is the marginal cost of the kth unit. The marginal value of a unit becomes the unit's own value and thus the unit can be treated as an individual with a value for a single unit.

Since the units have their own value there is no need to carry the identity of the (human) agent associated with the value. For theoretical purposes we can index the demand side units from high to low and the supply side units from low to high. So the unit index becomes the agent.

Let value for the one unit of buyer agent i be V(i) and cost of the one unit of seller agent j be C(j). Let p(i,j) be the price paid by i to j for the one unit supplied to i by j. A trade results in the utility for the buyer equal to u_i (p (i, j)) =V(i)- p(i,j) and the utility for the seller equal to

u_j (p (i, j)) = p(i,j) - C(j). The notation p(i,i) means that agent i does not engage in a trade (trades with self) and thus make no gains from the match, namely u_i (p (i, i))=0.

Allowing slight abuse of notation, we also allow *D*(p) to denote the set of buyers who have an induced demand values no lower than p; let *S*(p)denote the set of sellers, the agents who have an induced cost no higher than p. For a given price p, the market demand and supply functions are:

$$D(p) = |\mathcal{D}(p)| = |\{i \in \mathcal{D}: V(i) \geq p\}| \text{ and}$$

$$S(p) = |\mathcal{S}(p)| = |\{i \in \mathcal{S}: C(i) \leq p\}|$$

The classical definition competitive equilibrium (CE) is a mathematical property of the parameters. The CE price is a price p* and a quantity q*, such that D(p*) = q* = S(p*).

Given that linearity and integer values can be part of the analysis, more detailed definitions are needed for precision. The CE is an interval[6]. The upper support of the CE interval is the marginal cost associated with the first excluded unit and the lower support of the CE is the marginal redemption value of the first excluded unit. This interval will be called the CE if it is not constrained away by the price grid and the virtual competitive equilibrium, the VCE, if the price grid makes attainment impossible.

4.2 Matches and Assignments

A "match" (also called a "contract") specifies the parties and the terms of the trade. Denote a match as <(i,j),p(i,j)> , where (i,j) denotes the two agents, buyer i and seller j, and p(i,j) is the trading price paid by i to j. A match (contract) satisfies conditions of individual rationality, u_i (p (i, j)) =V(i) – p(i,j) > 0 and u_j (p (i, j)) = p(i,j) - C(j) > 0. An assignment in the market is a set of matches that satisfy the resource constraint of agents (one unit per person) and are contracted at feasible prices. Denote an assignment as G. The buyer agent i's utility

[6] The definition of equilibrium in the presence of integer units is a bit more complex. Let z be an index of buy orders (bids) ordered from high to low and sell orders (asks), ordered from low to high. Thus, z is an index of ordered pairs $\big(b(z), a(z)\big)$, where $b(z)$ is the bid, and $a(z)$ is the ask of the z^{th} pair. Let z^* be the smallest z for which $b(z+1) < a(z+1)$. Thus z^* is the index of the "last trade", the last accepted bid and the last accepted ask. The competitive equilibrium price is any $p^* > 0$ such that:
(i) For $z \leq z^*$, $b(z) \geq p^*$ and $a(z) \leq p^*$; and
(ii) $p^* \in [\max\{b(z^*+1), a(z^*)\}, \min\{b(z^*), a(z^*+1)\}$.

from the assignment G is u_i (p (i, j)) and the seller j's utility from the assignment G is u_j (p (i, j)) where <(i,j),p>∈G.

4.3 Price grid

Given a grid, the feasible prices in the market takes the form p=n·g where n is an integer and g is the size of the grid. If P is the set of admissible prices then **P** = {p: p = n·g for some n}. Suppose p* is the equilibrium price of a model. If there exists an integer n* such that p*= n*·g, then, abstracting from the dynamics, the situation is the same with the no-grid case in the sense that the competitive equilibrium (CE) price is feasible. However, if such n* does not exist then the CE price is not feasible and from the point of view of a model, the CE does not exist.

Let the set of feasible prices be ***P***. When the theoretical CE price is not on the grid, we denote it as the virtual competitive equilibrium (VCE). When CE price is not feasible we identify the VCE price as p* and we also identify two prices that are important, pU and pL. According to the definition of demand and supply set, we can find two price levels pU ∈ ***P*** and pL ∈ ***P*** as the lowest price and highest price, respectively such that:

$$pU\text{-}pL=g,\ \ D(pU) < S(pU) \text{ and } S(pL)<D(pL) \text{ and } pU > p^* >pL.$$

Namely at two feasible neighbor grids pU and pL, there is excess demand at pL and excess supply at pU. pU and pL are called the upper and lower bounding prices respectively.

Notice that if the grid is size 0 the pU and pL can be interpreted as the upper and lower supports of the interval of CE prices that define the CE.

4.4 Stable assignments

When there exists a price grid such that CE price is not feasible, we apply stable assignment to describe the equilibrium of the market. The stable assignment must satisfy two conditions:

S1: Non-blocking condition

An assignment G′ blocks an assignment G if there is a pair of individuals (α, β) such that u_α (p′(α, β)) > u_α (p (α, j)) and u_β (p′(α, β)) > u_β (p(i, β)).[7] An assignment G is not blocked if there is no other assignment G′ that blocks G.

S2: Individual rationality condition

An assignment G is individual rational if for all <(i,j),p(i,j)> ∈ G ,u_i (p(i, j)) > 0 and u_j (p (i, j)) >0

Note that if the model is based on individuals with single units rather than multiple units the individual rationality condition is different from those used in Hatfield et al. (2016). In a multiple-unit environment, we treat each unit from the same person as an independent agent. We impose the individual rationality constraint for each unit instead of individual subjects who typically trade multiple units. By treating each unit separately, we are able to rule out ex post regret by subjects. An example of ex post regret of trader with multiple items is as follows. A buyer has two items each with value of 100 and 50 and then bought them for 60 and 40, respectively. The total profit is 50, but by reneging on the contract for the cost 50 item and buying the value 100 item for 40, the profit increases to 60. In first glance, this looks like an unstable outcome, but note that when buying the cost 100 item, the opportunity to buy it for 40 presumably did not exist. Due to the dynamic nature of the environment the purchase of both items is not irrational behavior by the buyer. Furthermore, it is even possible that if the buyer had not purchased the value 100 item at 60, the future opportunity to purchase the value 50 item might not have arisen. [8]

4.5 Characterization of stable assignments

Our discussion will be confined to the conditions that are relevant to the limited conditions of our experiments. The demands and supply functions are linear and for purposes of characterizing the predictions, the individual demands and supplies are limited to one unit.

[8] Looking ahead at our data, 24% of the trades involve outcomes where an agent would like to change the contracts if switching between an agent’s own units were possible. If the sequence of actual trades is considered then these instances involve no instances of irrationality. Of course, if each item is treated as a single entity, there is no issue and the number of potential instances is reduced to zero. However, one can imagine a different issue even though it never occurs in our data. When a subject has multiple value/cost opportunities within the bounding grid, the prices are dictated by the bounds and when the bid or ask was submitted. One can imagine a series of actions that could result in the agent would prefer to revere the order in which the trades occurred.

The first proposition was established for very broad environments are just restated here for orientation.

Proposition 1. The Competitive Equilibrium is a stable assignment

Proof. Let the size of the grid be 0. In the proof of Observation 1 let the upper support of the CE play the role of pU and let the lower support of the CE play the role of pL. With the change of notation the proof of Proposition 1 follows as a corollary to Observation 1.

While the actual experimental markets involve multiple unit incentives for buyers and for sellers, Figure 2 illustrates the stable assignments under the conditions of one-unit capacities of traders and linear demand and supply functions. The VCE is represented in the figure and is bounded by pU and pL and these grids place an upper and lower bound on contract prices. That fact is summarized by observation 1.

Observation 1. Stable assignments cannot involve contracts above pU or below pL.

proof. Any assignment containing the contract <(i,j), p> with $i \in D(pU)$ and payment p > pU is blocked by the an assignment <(i,j′), pU > where $j' \in S$ (pU) because i prefers pU to p and because there exists an unmatched j′ due to the fact that S(pU)is larger than D(pU). A similar argument demonstrates that any assignment with p< pL is blocked.

The second observation, Observation 2, is not reflected directly in the figure because the demand and supply in the figure are not symmetric. Respectively they do not intersect pU and pL at the same quantity as required by the observation. However Observation 2 demonstrates explicitly that in the symmetric case where the VCE is located exactly between pU and pL all stable contracts will involve a buyer with value at or above pU and a seller with a cost at or below pL.

Observation 2. If demand and supply functions are mirror images and monotonic (respectively decreasing and increasing), the family of matched pairs from *D*(pU) and *S*(pL) at prices pU and pL is a stable assignment.

proof. Suppose the assignment is not stable then there must exist a blocking pair and a p* or individual rationality is not satisfied. Any blocking pair must involve either an i from *D*(pU) or j from *S*(pL) by the definition of blocking and stability. Suppose i* and j* are blocking with i* $\in$ *D*(pU) and j* $\in$ *S*: pL< C(j*)$\leq$ pU . p* cannot be at pU because i* would not be better off. p* cannot be at pL because the rationality of j* would be violated due to the fact that pL< C(j*)$\leq$pU. Since pL and pU are the only possible prices we have a contradiction that <(i*,j*) p*> is blocking. Individual rationality is satisfied since any subject in *D*(pU) prefers to be matched at pU or pL rather than unmatched and the same with *S*(pL).

The next observation addresses the case of asymmetry between market demand and supply as well as relative to the VCE. As is illustrated in the figure, when there is more demanded at pU than is supplied at pL not all buyers able to buy at pL will find a seller who can sell at pL. That is, there is limited supply at pL which forces buyers to contract at pU. The number of contracts that occur at pU and pL is dictated by relative slopes of the demand and supply functions.

Observation 3. The stable assignments always include buyer values at or above the upper bounding grid or seller costs that are at or below the lower bounding grid or both. Thus, the limitation of values at or outside the lower and upper price bounds influences the proportion of trades at the two bounding prices.

proof. Suppose A = {<(i,j) p>} is a stable assignment. If <(i*,j*) p*> $\in$ A then either (1) [i* $\in$ D: V(i) $\geq$ pU] or (2′) [j* $\in$ S:C(j)$\leq$ pL] or both.

Notice that the existence of the grid dictates that p*$\in$ {pU,pL}. If neither 1 nor 2 are satisfied then either V(i*) – p* < 0 or p* - C(j*)<0 so rationality is violated.

The matching model suggests that prices will bounce between pL and pU in proportions dictated by the demand and supply slopes as well as the range of potential volumes. Observation 4 provides a partial characterization. The support is easily inferred from figure 2.

Observation 4: Trading volume is in the range (Max{D(pU), S(pL)}, Min{D(pL), S(pU)}). Additional volume could reflect speculation.

Proof. Assume that the trading volume is smaller than Max{D(pU), S(pL)}. If D(pU)$\geq$S(pL), then there exists an i $\notin$ D(pU) that is unmatched. From observation 1, we know all matches are at either pU or pL. Furthermore, we know that there exists a j $\in$S(pU) that is unmatched since S(pU) > D(pU). Then i and j become a profitable match at pU. The argument is symmetric when D(pU)≤S(pL).

Now assume that the trading volume is larger than Min{D(pL), S(pU)}.

If D(pL) ≥ S(pU), then there exists a j $\notin$S(pU) that is matched. However, from observation 1, we know all matches at either pU or pL and this violates the individual rationality of j. The argument is symmetric when D(pL) ≤ S(pU).

Figure 2 illustrates how the stable outcome will appear in the experiment. Shown there is an equilibrium price, P*, bounded by the lower grid bound, *l*, and the upper grid bound, *h*. As can be seen, if a grid prevents the use of price levels between these two bounds the competitive equilibrium price does not exist from the point of view of a market model. That is, the CE can be defined as a mathematical property of the parameters but it does not exist as a property of a model of market behavior.

If price ceilings and/or floors exist presumably, the market price(s) will rest at the constraint(s). An excess supply or demand will necessarily exist. One hypothesis is that the excess demand or supply will be expressed as bids or asks and trades will take place. The exact volume is not clearly predicted by the model. The expectations and beliefs that accompany this process are not well specified by a generally accepted model.

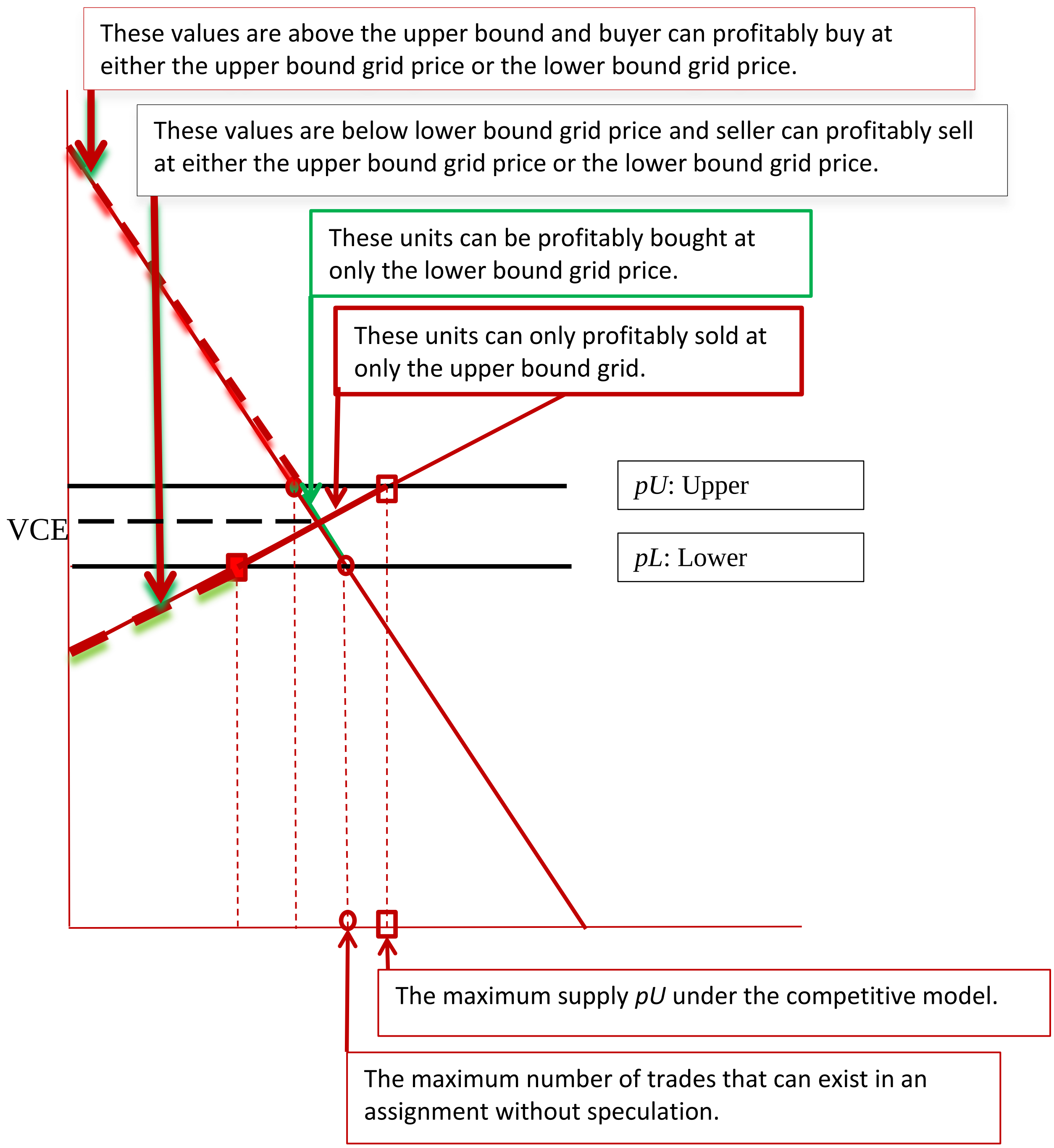


Figure 2: Assignment model allocation stable set.

5. EXPERIMENTAL ENVIRONMENT

The consequences of a grid imposed on a continuous market have not been explored experimentally and an accepted theory of market adjustment in the presence of a grid does not exist in the literature. However, peeks at the possible consequences occur here and there. The absence of a body of theory and experiment motivates the investigation of market conditions to ascertain if anyone has a disproportionate impact and, if so, to pursue the analysis in greater depth.

The grid is a form of price restriction that can induce surprising behavior. For example, the grid has similarities with price ceilings and floors that are known to induce special forms of behavior e.g., prices can equilibrate bounded away from bounding constraints (Hatfield, et.al. (GEB 2016), Isaac and Plott (AER 1981), Smith and Williams (Handbook p.46)); removal of bounding constraints can cause explosive behavior. The experimental design respects known patterns of "grid-like" institutions in the sense that it is structured to explore how markets with grids behave relative to well-established patterns of market behaviors.

All experiments were conducted on the internet. Experiments used an artificial currency called "francs". The conversion to dollars depended on the scaling of the francs, which differed across experiments. The parameters were set such that at equilibrium the subject would earn about $4.50 per period if the market was 100% efficient.

Two different series of experiments were conducted. The first series consisted of a total of seven experiments, all of which used very similar demand and supply functions and differed primarily according to the scaling of units, the size of the grid and when and if the grid was imposed. Induced values were for certain amounts known with certainty to the participant. The second series consisted of six experiments with common values, limited private information about the common value and whether or not a grid was imposed.

6: SERIES 1: CLASSICAL INDUCED, PRIVATE VALUES PREFERENCES

Following frequently used procedures, the preferences were financially induced separately for each of the several periods. Sellers acquire the units for sale by buying the units at fixed prices made available by the experimenter in the seller's private market. The seller keeps the difference between the price of a unit paid by the buyer to the seller and the price at which seller purchased the unit from the experimenter. Buyers make money by buying in the market from sellers and selling acquired units at fixed prices made available by the experimenter in the buyer's private market. The buyer keeps the difference between the price of a unit paid to the seller and the price received for the unit from the experimenter.

Market demands and supplies are derived from the induced preferences in a natural way. The potential cost to the seller of acquiring units from the experimenter, the individual seller's marginal costs, can be configured into a market supply curve.

Experiments 20160212, 20160224, 20160307 and 20160507 were performed with Caltech undergraduates as subjects. Experiments 20160325, 20160329 and 20160331 were conducted remotely with Purdue undergraduates as subjects. Differences in laboratory facilities, subtle difference in instruction, procedures, software and subjects could be responsible for subtle differences detected in the data.

Periods lasted from 8 minutes to 14 minutes depending on the number of participants. The number of participants had a natural impact on volume and the time required for orderly market adjustments. For an individual, the induced preferences were the same for all periods, except the first period, which was used for instruction and training. Preferences were not public and the fact that they were unchanging was not public.

The only difference among periods within an experiment was the existence of the grid and the size of the grid differed across experiments. In Series 1 the grid was 250 for some experiments and 20 for others.

Table 1 lists the size of the grid for each period of each experiment. As can be seen, some experiments start with a "large" grid for several periods after which the grid is reduced to 1. In other experiments, the experiment starts with a grid equal to 1 for several periods after which a larger grid is imposed. These manipulations are intended to provide insights about the impact of a grid and about grid size; they also open the possibility of observing any dramatic changes in equilibrium or equilibration due to experience with and without a grid, should such be an impact of the grid.

Individual preferences are always one of 8 "types". The letter B indicates one of four buyer types and the S indicates one of four sellers' types. The numbers associated with a "type", e.g. B1 or S2, identify parameters that are linear transformations of a common base. Thus, the parameters across experiments in the same series differ only by linear transformations (origin and scale) in the sense that all buyers and sellers in an experiment are related to the base by a common linear transformation. The scale is used for the selection of different grids across experiments. Period 1, always a practice period, has a special set of instructional parameters. The preference types for each of the experiments of Series 1 are listed in Table 1. The complete list of all parameters in all experiments is in the Appendix.

All parameters for experiments in series one are similar and as shown in Figure 1. Market demands and supplies were linear, given the integer constraint with "kinks". There is a slight difference in demand and supply slopes beyond the competitive equilibrium in selected experiments. This was done to increase the excess demand and supplies at grid prices.

The location of the competitive equilibrium price in the parameters was always near the middle between two grid prices. When the grid was greater than one the theoretical competitive equilibrium was always bounded away from a grid price and thus the CE did not exist.

Table 1: Experiments, periods, grids, types

Experiment	Periods	Grid	Type	B1	B3	B5	B7	S2	S4	S6	S8
20160212	[2,6] [1][7-11]	250 1		1	1	1	1	1	1	1	1
20160224	[2,6] [1][7-11]	250 1		2	2	1	1	1	1	1	0
20160307	[2,7] [1][8-12]	250 1		2	1	1	1	2	1	1	1
20160325	[2,6] [1][7-10]	250 1	Number	2	0	2	1	2	1	2	1
20160329	[2,7] [1][8-10]	250 1		3	3	3	3	3	3	1	3
20160331	[2,5] [1][6-10]	250 1		3	3	2	2	3	3	2	2
20160507	[1,6] [7-12]	250 1		2	2	2	2	2	2	2	2

7. SERIES ONE RESULTS

The experiments in the classical, private values environment address two broad classes of results. First, what is the impact of a price grid? Secondly, which model best captures market behavior in the presence of price grid? A third class of issues is related to the impact of a grid on the use of information but this type of question is best studied with the Series Two experiments.

7.1 Series 1 Private Value Results: Overview Example

An overview of the pattern of the data from an experiment can be found in Figure 3 below. Price is on the vertical axis and time is measured on the horizontal wtih vertical dotted lines that separate periods. The first period is a practicing period without grid. After that, we implement a price grid and the bids, asks and trades all happen at the grid. In the first several periods, bids and asks are

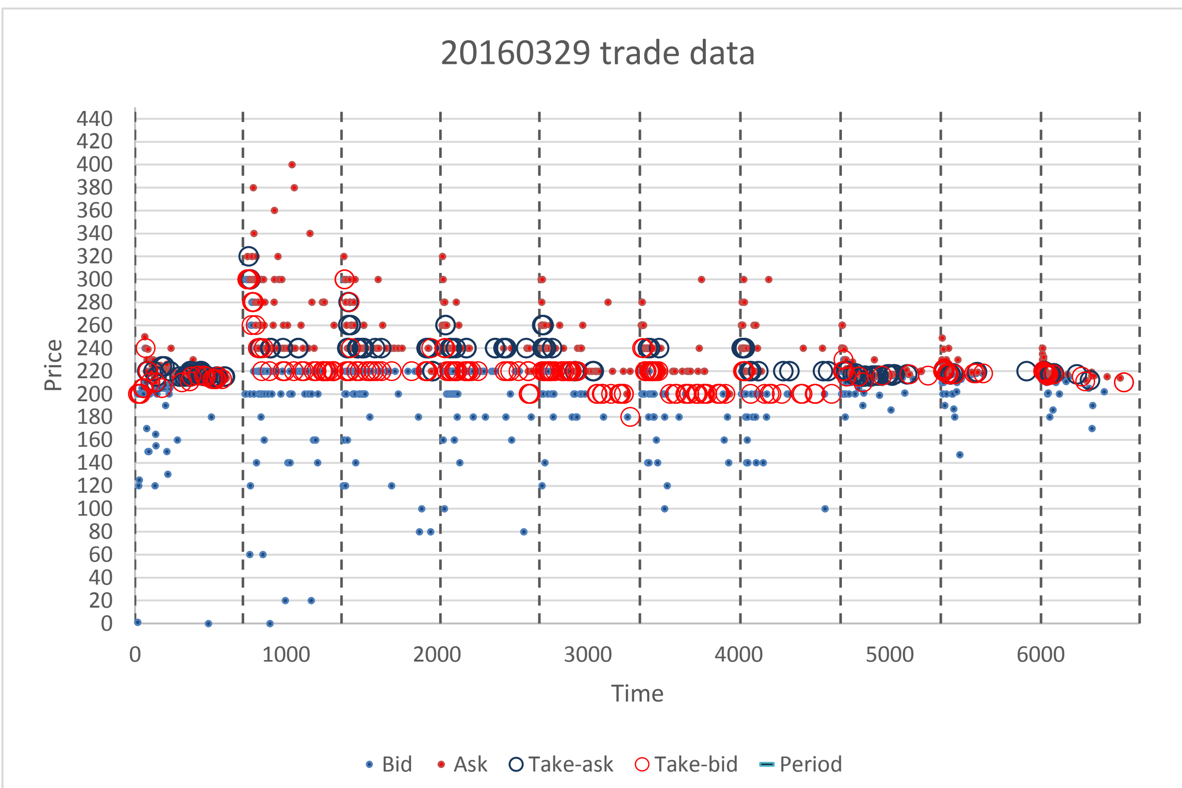


Figure 3: Data from Session 1 Experiment 20160329

distributed sparsely, while as time goes by they are more concentrated at the bounding grids around the CE price. As predicted by the assignment model, trading prices bounce between the two bounding grids as a feature of stable assignment. After the grid is removed in Period 8, the prices soon fall inside the bounding grids and a convergence to the CE price is observed.

7.2. Equilibration: Observed market equilibration is consistent with the stable assignment market model.

Equilibration involves the dynamic properties of individual contracts in contrast to equilibrium, which is based on the static concept of a stable assignment. The distinction is subtle. While an assignment is a collection of contracts, the contracts themselves have the capacity to dissolve and reform and in an empirical sense can be interpreted as stable or unstable. Thus, we focus on the empirical stability of contracts and properties that endure over time. We note that an assignment can only be empirically stable if and only if the associated contracts are empirically stable.

Empirically stable contracts are based on prices that repeatedly emerge and become implemented after previously formed contracts at those prices have ended by the natural progression of time.[9] At the end of a period, all contracts are automatically broken. Empirically stable contracts involve prices that become part of reformulated contracts and are implemented in another period. Empirically unstable contracts are those with prices that do not become part of new contracts. Instability of a contract occurs if one side of the contracting pair does not offer or declines the terms of the previous contract when offered. In operational terms, empirical instability of a contract is when only one side or neither side of a previously successful contract returns to the contract price (a buy offer or a sell offer of the terms found in previous contacts). If an offer is made the other side does not respond by taking the offer. Instability can be detected in terms of unaccepted bids and asks at previous contract prices.

Participants who find better partners do not return to previous contract levels and are not part of attempts to form contracts on previous terms. On the other hand, those who cannot find better opportunities return to the previous contract prices and try to establish contracts with

[9] The dynamics is similar to the process of "recontracting" postulated by Edgeworth where the recontracting takes place at designated times and need not involve the same agents.

offers (bids and asks) of previous terms. As offers fail to be accepted those making the offers initiate new offers that are worse for themselves but better for a possible counter-party.

The identification of empirical stability of contracts in the data follows the discussion above. Since only contract price is relevant, we classify the contracts according to whether contracts reform or unravel at the same price in next period. Specifically, if we find no bids or asks at the contract price in next period then the contract is denoted as a two-side unraveling contract. If we find only one side returns, namely only bids or asks at the price in the next period, the contract is denoted as a one-side unraveling contract. If both bids and asks occur in the next period, the contract is reformulated and denoted as an empirically stable contract.

According to the prediction of the assignment model, the stable set only contains contracts at the two bounding grids. Consider sellers and buyers who have assignments above the upper bounding gird. The next period the seller (or some seller) would return to that price with an ask but buyers, having observed contracts at lower prices, would not respond with an accepting bid (a "takeask"). The assignment at that price would not be stable (empirically) because one of the parties would like to return but the other does not. Similarly, a contract below the lower bounding grid would unravel. The next period the buyer (or some buyer) would bid at that price wanting to return to the partnership (contract) but there would be no take-bid coming from the sellers who can find a better deal. Contracts will unravel at non-bounding grids and appear at the bounding grids as a convergence process.

The phenomena are described in Result 1. As indicated by the example above, the one sided nature of the dynamics of an unstable contract helps provide an intuition about price movement. Instability is mostly one-side unraveling rather than two sided. Typically, one side finds a better contract and does not want to return while the other side was not able to find a favorable alternative and wants to return to the old contract. The summary statistics (Panel D of Table 2) show that it is actually the case. Two-side unraveling contracts account for only 4% of all the contracts across all experiments, compared to 23% for one-side unraveling contracts.

Result 1. (i) The non-bounding grid prices are empirically unstable and (ii) empirically stable contract prices converge to the bounding grids.

Support. To show that contracts at the non-bounding prices are unstable, we consider three different price groups (i) contracts at prices above the upper bounding grid, (ii) contracts at prices below the lower bounding grid and (iii) contracts at the bounding price grids. Also, to be complete, we check the quantity of each type of contract at the upper bounding grid, at the lower bounding grid, one or two grids above the upper bounding grid and one or two grids below the lower bounding grid. Very few contracts exist at farther grids.

The summary statistics (Table 2) show that there are fewer empirically stable contracts at non-bounding grids than at the two bounding grids. For contracts at the bounding grids, the percentage of empirically stable contracts is 84%, much greater than 56% for contracts at prices higher than the bounding grids and 16% for the contracts at prices lower than the bounding grids. In addition, the absolute number of empirically stable contracts is also larger at bounding grids than non-bounding grids (459 vs. 94), despite the fact that the bounding grids only contain two prices while the non-bounding grids could be at any other prices.

The pattern that stable contracts are more likely to be found at bounding grids persists when we check each price level separately. According to Table 2 Panel A, the percentage of stable contracts over all contracts at upper bounding grid and lower bounding grid both exceed 80%. While at one grid above the upper bound (Table 2 Panel B), the fraction falls to 68% and at one grid below the lower bound (Table 2 Panel C), it falls to only 29%. For two grids away from the bounding grids, only 50% contracts are stable at price greater than the upper bound (Table 2 Panel B) and only 11% contracts are stable at price lower than the lower bound (Table 2 Panel C).

Table 2: Summary statistics of contracts by type and price

Panel A: Price at bounding grids

	At upper grid		At lower grid		Total	
Contract type	Quantity	Percent (%)	Quantity	Percent (%)	Quantity	Percent (%)
Two-side unraveling	3	1%	3	1%	6	1%
One-side unraveling	43	15%	40	15%	83	15%
Stable	238	84%	221	84%	459	84%
Total	284	100%	264	100%	548	100%

Panel B: Price greater than bounding grids

	One grid greater	Two grids greater	Total

Contract type	Quantity	Percent (%)	Quantity	Percent (%)	Quantity	Percent (%)
Two-side unraveling	0	0%	1	8%	10	6%
One-side unraveling	35	32%	5	42%	59	38%
Stable	76	68%	6	50%	87	56%
Total	111	100%	12	100%	156	100%

Panel C: Price smaller than bounding grids						
	One grid smaller		Two grids smaller		Total	
Contract type	Quantity	Percent (%)	Quantity	Percent (%)	Quantity	Percent (%)
Two-side unraveling	0	0%	1	11%	8	19%
One-side unraveling	15	71%	7	78%	28	65%
Stable	6	29%	1	11%	7	16%
Total	21	100%	9	100%	43	100%

Panel D: All prices		
	Total	
Contract type	Quantity*	Percent (%)
Two-side unraveling	27	4%
One-side unraveling	170	23%
Stable	553	74%
Total	750	100%

*There are three contracts signed at prices in between the bounding grids due to system mistake. So total quantity of two-side unraveling contracts is 27=6+10+8+3

Contracts at non-bounding prices are unstable since they fail to reappear in next period. As a convergence process, empirically stable contract prices tend to be at the two bounding grids in later periods. To show the trend statistically, we apply the Ashenfelter-El-Gamal (AEIG) test[10] to analyze the effect of time:

$$y_{it} = B_{11}D_1\left(\frac{1}{t}\right) + B_{12}D_2\left(\frac{1}{t}\right) + \cdots + B_{1i}D_i\left(\frac{1}{t}\right) + \cdots + B_{1n}D_n\left(\frac{1}{t}\right) + B_2\left(\frac{t-1}{t}\right) + u$$

where i indicates the particular experiment, t represents time as measured by market periods in the experiment, D_i is a dummy variable that takes 1 for experiment i and 0 otherwise and B_{1i} is the origin of a possible convergence process. Notice that if $t = 1$ then value of the dependent

[10] Plott et al 1995 AER.

variable is equal to B_{1i} for experiment i. B_2 is the asymptote of the dependent variable. As t gets large the weight of B_{1i} is small because $\frac{1}{t}$ approaches zero while the weight of B_2 approaches 1. Notice that B_2 is common to all experiments. Finally, u is the random error term that is distributed normally with mean zero.

To make comparison across experiments and pool the data, we look at contracts at different price levels or belong to different price groups (above, below and at the bounding grids), without referring to the exact contract price, whose magnitude varies in different experiments. We also construct an indicator of stability of contracts for each price group, measured by the proportion of empirically stable contracts over all contracts in a given period. What we want to show is that, stability of contracts at non-bounding prices decreases over time, while stability of contracts at bounding grids increases or remains high.

Table 3 presents the estimated coefficients from the AEIG test. In panel A we group the contracts according to whether the price is at, greater than or lower than the bounding grids. As indicated by the estimation of coefficients, stability at the bounding grids stays at very high level. In all experiment, it starts at a minimum of 61.9% and converges to 90.8%, with all the estimated coefficients significant at 99% level. For the group of contracts with prices above the bounding grids, there is a decreasing trend of stability for 5 out of 7 experiments. For example, the estimated coefficient starts at 87.5% in experiment 2 and converges to 42.4% in the end. For the group of contracts with prices below the bounding grids, there is no obvious trend since the coefficients are not statistically significant, but stability stays low for all the experiments (the highest level is 35.8% in experiment 1 and it ends at 38.6%).

Table 3: Coefficient estimates for AEIG model

	B_11	B_12	B_13	B_14	B_15	B_16	B_17	B_2	n	R^2
Panel A: Stability at pooling prices										
Stability at the bounding grids	0.880***	0.826***	1.005***	0.891***	0.964***	0.747***	0.619***	0.908***	54	0.942
	(0.14)	(0.14)	(0.19)	(0.14)	(0.19)	(0.14)	(0.12)	(0.06)		
Stability above the bounding grids	0.83	0.875**	1.165***	0	0.559***	0.567	-0.6	0.424**	33	0.666
	(0.79)	(0.38)	(0.39)	(0.32)	(0.19)	(0.34)	(0.88)	(0.19)		
Stability below the	0.358	-0.386	0.241	-0.0236	-1.157	0		0.386	16	0.429

bounding grids	(0.26)	(0.85)	(0.27)	(0.23)	(1.81)	(0.40)		(0.28)		
Panel B: Stability at separate prices										
Stability at the upper bounding grid	1.060***	0.748***	1.058***	0.906***	1.062***	0.913***	1.015***	0.775***	29	0.955
	(0.19)	(0.19)	(0.19)	(0.19)	(0.19)	(0.19)	(0.19)	(0.08)		
Stability at the lower bounding grid	0.705***	0.908***	0.983***	0.880***	0.474	0.584***	0.650***	1.031***	27	0.976
	(0.15)	(0.15)	(0.15)	(0.15)	(0.41)	(0.15)	(0.15)	(0.07)		
Stability at the upper bounding grid + 1 grid	0.613	0.958**	1.107***	0	1.017**	0.867**	-0.835	0.590**	19	0.868
	(0.68)	(0.33)	(0.33)	(0.38)	(0.33)	(0.33)	(0.76)	(0.20)		
Stability at the lower bounding grid - 1 grid	0.451	-0.59	0.346	-1.77	-1.77	0		0.59	10	0.57
	(0.55)	(1.30)	(0.54)	(2.86)	(2.86)	(0.59)		(0.54)		
Stability at the upper bounding grid + 2 grids				0	1.133	-0.333		0.333	6	0.822
				(0.52)	(0.50)	(1.16)		(0.53)		
Stability at the lower bounding grid - 2 grids	0.333		0	0				-0.333	4	1
	0.00		0.00	0.00				0.00		

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

In panel B, we examine each price level separately. Consistent with panel A, at the two bounding grids, the measure of stability either stays high throughout all periods or displays an increasing trend. For contracts at a price one grid greater than the bounding grid, the stability decreases over time in 5 out of 7 experiments. As showed by the coefficients, for example in experiment 2, the proportion of stable contracts starts at 95.8% and converges to only 59.0%. The coefficients are generally insignificant and indistinguishable from 0 for the three other price levels (1 grid lower than the lower bounding grid and 2 grids away from the bounding grids).

Overall, the data support the dynamic movements stated as Result 1. The test results confirm that there is a trend that non-bounding grids are empirically unstable contract prices and empirically stable contract prices converge to the bounding grids.

We now establish the link between stable contracts and stable matches suggested by theory. Result 2 illustrates that the repeated contracts we observe until the end of the experiment (empirically stable contracts) are the stable matches predicted by theory.

Unstable assignments are revealed by unsuccessful attempts to reestablish them while stable assignments are visibly successful. The stability defined in Result 1 is based on whether a contract reforms or not. In fact, the theory developed in previous section also makes predictions about the set of stable outcomes. According to section 4, contracts in a stable match must have two properties. (1) Stable contracts must be priced at two bounding grids (no blocking pair) and (2) stable contracts must be profitable to both sides (individual rationality). Therefore, we can identify whether a contract is in the theoretical stable set using the same criteria.

The first criterion is easily checked, since we can directly compare the price of each contract with the bounding grids. The second criterion requires supplementary information, since the traders' profits are not specified in the contract. As buyers' values are determined by the prices, they sell to the experiments in the private market and sellers' costs are determined by the prices they buy from the experiments in the private market, we can track the redemption value and cost of each unit and associate them with each contract with the time of trade execution. In the end, a contract is deemed to be in the stable set if it is priced at the bounding grids, the redemption value is higher than the contract price, and the cost is lower than the contract price.

Besides the stable outcome, during the converging process, there will also be unstable matches that will eventually disappear. Also, since we don't restrict the direction of trading in the public market, participants could trade on both sides of the market in an attempt to speculate. When the trading prices are bouncing between two equilibrium grids, speculators in theory could make a profit if they buy at the lower bounding grid and sell at the higher bounding grid. Therefore, we don't necessarily anticipate that speculative contracts will completely disappear with time.

The primary research interest is to test the explanatory power of matching model in the market environment with price grids. The major prediction of the model is the stable outcome. If the theory is the proper principle in the market with grids, the stable contracts we observed should be elements in the stable outcome. Hence, we want to test whether the empirical stable contracts are consistent with the theoretical prediction. The finding is summarized in result 2.

Result 2. Empirically stable contracts converge into the stable outcome predicted by the theory.

Support: From the data, we observe mainly two types of contracts, stable and unraveling contracts, defined by whether they reform in next period or not. On the theoretical side, we define three sets of contracts, the stable set (stable outcome), unstable set and speculative set. Each contract in the data should have both an empirical identification and a theoretical identification. Result 2 states the prediction that the empirically stable contracts fall into the theoretically stable set as the outcome of convergence.

Below is a table (Table 4) that summarizes quantity of each type of contract pooling all experiments. Putting aside the speculative set, we find that more (empirical) unraveling contracts fall into the (theoretical) unstable set than the stable set (88 vs. 32) while more empirically stable contracts fall into the stable set (229 vs. 134). Alternatively, stated in the opposite way, the contracts predicted stable by the theory are more likely to reform in the experiment, while those predicted to be unstable are more likely to disappear.

Table 4: Empirical and theoretical stable contracts

		Theoretical			
		Unstable set	Stable set	Speculative set	Total
Empirical	Unraveling	88	32	87	207
	Stable	134	229	194	557
	Total	222	261	281	764

Combining the empirical and theoretical identifications, all contracts could be assigned into one of the six categories: unraveling contracts in the unstable set, unraveling contracts in the stable set, unraveling contracts in the speculative set, stable contracts in the unstable set, stable contracts in the stable set and stable contracts in the speculative set. What we care more about is the trend of change. In order to pool the data of all the experiments and compare across different periods, we use proportion instead of absolute number to measure the likelihood that a contract observed in the experiment falls into a set. Our primary interest is to see among all the stable contracts how many fall into the stable set. Hence, we calculate the percentage of stable contracts in the stable set over all the stable contracts. The Ashenfelter-El-Gamal test is applied to test the trend. To make a comparison, we also test the trend of changes of stable contracts in other sets. As showed by Table 5, the proportion of stable contracts in the stable set displays an increasing trend in all experiments except experiment 2 (160224). For example, it starts at 26.5% in experiment 1 (160212) and converges to 65.7% in the end. In experiment 2, it starts at 70.8%, a very high level and stays high until the end. Contrary to the stable set, stable contracts in the unstable set experience an obvious declining trend. It starts at 67.5% over all stable contracts in experiment 1 and ends up at 17.1%. The stable contracts in speculative set show a mixed trend. The percentage decreases over time in experiments 160325, 160329 and 160331, (4, 5 and 6, respectively) and increases but stays low in all other experiments. In conclusion, the contracts repeatedly observed in the experiments mostly come from the stable outcome predicted by the matching theory.

Table 5: Trend of stable contracts in theoretical sets

	B_11	B_12	B_13	B_14	B_15	B_16	B_17	B_2	n	R^2
Stable contracts in the stable set	0.265	0.708***	0.363*	0.0941	-0.132	0.138	0.0831**	0.657***	29	0.86
	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)	(0.21)	(0.04)	(0.10)		
Stable contracts in the unstable set	0.675***	0.311***	0.733***	0.147	0.308***	0.175	0.00176	0.171***	29	0.9
	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.02)	(0.05)		
Stable contracts in the speculative set	0.0673	-0.0135	-0.0877	0.766***	0.833***	0.693***	0.00884	0.157**	29	0.887
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	-0.0217	(0.06)		

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The focus now becomes the cases in which the competitive equilibrium price does exist. The next two results report features of contract prices as compared to competitive equilibrium prices. Given the discrete nature of incentives, the competitive equilibrium prices form a range of prices we will designate as the competitive equilibrium price range, CER. We will explore the relationship between average contract prices in a period and the CER. The first of the two experiments examines periods without grids, so the competitive equilibrium price range is a natural result. The second experiment has an imposed grid and contract prices cannot equal the competitive equilibrium prices. However, the range of prices can be computed from the parameters and will be called the Virtual Competitive Equilibrium - the VCE. For each period, we calculate the average price of completed trades, denoted Average Transaction Price (ATP). Then we ascertain whether the ATP lies within the interval of prices defined by the VCE.

Result 3 reveals that prices approach the interval (range) of competitive equilibrium prices. This property emerges when the grids do not exist, which is sometimes imposed when the market opens and when the grids are removed after having been in place for a number of periods. The second result, Result 4, tells us that that the ATP is close to the VCE when grids are in place.

Result 3. When the CE, the Competitive Equilibrium, exists (including an allowance for transactions costs) the CE prices emerge as contract prices. The emergence occurs when grids do not exist at the market opening and when previously existing grids are removed.

Support. The grids and units differ across experiments so the patterns are most easily seen in terms of the error of the model. If the average price is within the interval defined by the CE range, then the price converges perfectly and we denote the error as being zero. If the average price is not in the interval, then we calculate the difference between the average price and the competitive equilibrium price range and scale it across experiments by dividing it by the average price.

An Ashenfelter-El-Gamal test demonstrates that the difference between ATP and CE price range shrinks with time. As shown in table 6, coefficients for the starting period in experiments 1 to 6 are not statistically different from 0, meaning that when the grids are removed the ATP falls into the set of competitive equilibrium prices immediately. An exception is experiment 7, which had no grid imposed and that began with no information about the bounding grids. In that experiment, the AEIG test shows a clear declining trend of ATP from the CE ending at 1.12% in the last period, which is statistically indistinguishable from 0.

In addition to the deviation of ATP, we also present how the variation of prices changes over time. The measure we use is the relative standard deviation of price. As the AEIG test shows, for experiments 160212, 160224. 160307 and 160507 (respectively 1,2,3 and 7), there is a clear trend that the variation of prices decreases. Combining this with the fact that ATP is close to the CE, we conclude that contract prices converge to CE in general.

Table 6: Deviation of ATP from VCE

	B_11	B_12	B_13	B_14	B_15	B_16	B_17	B_2	n	R^2
|ATP-VCE|/ATP	0.0323	0.0198	0.0241	0.00167	-0.0026	0.00689	0.253***	0.0112	33	0.847
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)		

Relative standard deviation of price	0.0645*	0.131***	0.0764**	0.02	0.01	0.03	0.0859**	0.0337**	33	0.721
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.01)		

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The next result, Result 4, is focused on experiments in which the price grid is imposed and asks about the information content of the average transaction price, the ATC. Although a competitive equilibrium does not exist due to price restrictions, the Virtual Competitive Equilibrium does exist and is positioned where the CE would exist should there be no price grid. Result 4 has two parts. The first part states that the ATP converges close to the VCE. The second part demonstrates that the VCE is a better predictor of the ATP than is the midpoint of the bounding grid prices.

Result 4. When a price grid is in place (i) average transaction prices (ATP) converge to the VCE and (ii) the VCE is a better predictor of the ATP than is the midpoint of the bounding grid prices (ABP).

Support. Table 7 presents the deviations of ATP from VCE and ABP. We define the **Average Boundary Price (ABP)** to be the arithmetic average of the two bounding grid prices at the equilibrium. We compare the convergence of the ATP to the VCE with the convergence of the ATP to the ABP.

Table 7: Convergence of ATP with price grids

Period*	Experiment (indicated by date)							
	160212	160224	160307	160325	160329	160331	160507	Average
	Panel A: |ATP-VCE|/ATP							
1								
2	0.0%	0.0%	3.4%	0.0%	7.6%	4.0%	0.3%	2.2%
3	5.5%	0.9%	2.9%	0.0%	3.8%	0.0%	0.0%	1.9%
4	0.0%	0.0%	0.0%	2.4%	0.3%	0.0%	0.0%	0.4%
5	2.2%	1.5%	0.5%	1.8%	1.8%		0.0%	1.3%
6			0.0%		0.0%		0.0%	0.0%
	Panel B: |ABP-VCE|/ATP							

1								
2	1.5%	2.1%	7.1%	2.9%	2.5%	6.7%	0.9%	3.4%
3	7.2%	0.6%	6.7%	5.4%	1.5%	1.9%	0.9%	3.4%
4	0.0%	1.7%	1.9%	0.6%	5.2%	0.7%	0.4%	1.5%
5	4.1%	0.0%	2.3%	1.2%	8.4%		0.6%	2.8%
6			2.8%		6.1%		0.0%	3.0%

* Periods are relabeled by order of time. The number of periods varies across experiments. The first periods were practice periods are not included as part of the data

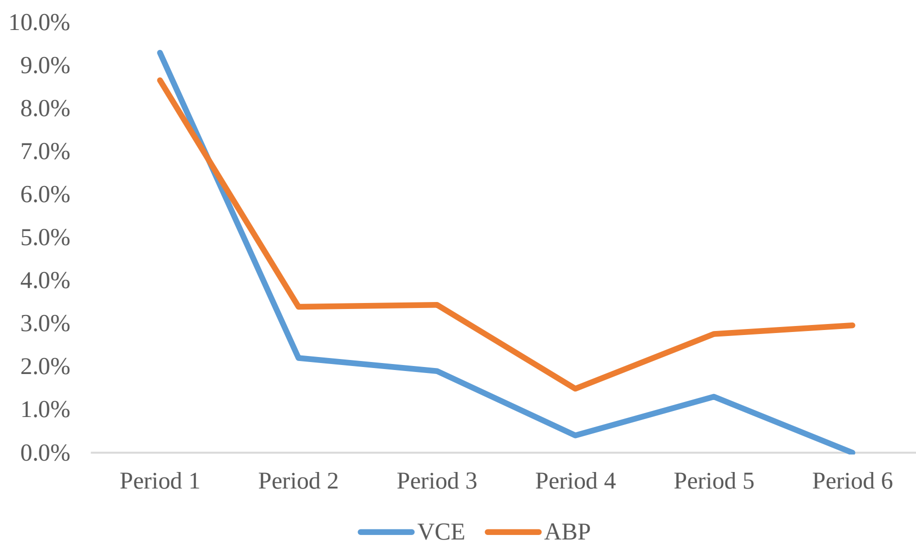


Figure 4: Convergence of average price

Figure 4 depicts the convergence of average price by pooling the data across different experiments. With the exception of period 2, the ATP is always closer to VCE than it is to the ABP. The difference between the ATP and VCE disappears by period 6.

7.3. Speculation, volume and liquidity

Results 1 through 4 establish that the major pricing features of the stable sets emerge and that those features are better predictors than the competitive model. The stable assignments also involve specific predictions about the set of agents that will emerge in the market matches that can be used as tools to understand market behaviors. However, all predictions become clouded by the possibility of speculation, liquidity supply and other market behaviors suggested by the literature and by policy controversies.

<u>Result 5.</u> Grid increases volume because speculation subsidizes trade through losses to speculators.

Support: The parameters change across experiments. For each separate experiment, since the number of participants and preference parameters may differ, the predicted volume of trades and other activities can be different. However, within each experiment, the parameters do not change whether or not a grid exists. This allows us to make a comparison of average trades and speculation in each experiment period by partitioning the data according to the existence of a grid.

Table 8 presents t-tests on the average contracts and speculation per trader in the markets with and without a price grid. In general, the sign of the differences confirm that grid attracts more volume and the increased volume arises partially because of speculation.

The fact that grid invites more speculation means that the grid serves as a central mechanism to attract excess demand and supply. Define those participants who always trade on the right side of the market given their induced values as fundamentalists. That is, the fundamentalists are sellers who never attempt to buy in the public market and buyers who never attempt to sell in the public market. By contrast, speculators are defined conversely as sellers who buy order or buyers who sell. According to table 8, when the grid exists there are more fundamentalists trading in the market and there are also more active speculative activities. Thus, there is increased overall trade volume when the grid exists. In general, a larger tick size attracts speculators by signaling a larger profit opportunity due to the price grid. Speculators bring more trade volume not only because they trade with both sides of the market, but also because they tend to hang onto inventory by the end of the experiment due to bad management. This introduces additional demand and supply, which arises despite speculators' losses.

Table 8: Relationship between grid and volume

	Without Grid			With Grid			Diff [w/o grid-w/ grid]	
	N	Mean	Confidence Interval	N	Mean	Confidence Interval	Mean	Confidence Interval
Average contracts	32	3.36 (0.13)	[3.10, 3.62]	37	3.59 (0.15)	[3.29, 3.89]	-0.23 (0.20)	[-0.62, 0.17]
Average speculation		0.48 (0.11)	[0.25, 0.71]		0.60 (0.13)	[0.33, 0.86]	-0.12 (0.18)	[-0.47, 0.24]

7.4. Efficiencies

A natural test of efficiency follows after the examination of volume. Since most of extra volume caused by grid comes from the inefficient supply and demand, the stable assignment model predicts the possibility of a decreased social welfare due to the introduction of the grid.

Result 6. The grid decreases efficiency (efficiency increases with period and decreases with grid)

Support. The underlying economic parameters are the same whether or not a grid is in place. Thus, from a technical point of view, total achievable surplus is identical with or without the grid. Similarly, the efficient allocation is included as an element of the set of stable matches when the grid is in place. However, because the efficient allocation is only one of the many equilibria when the grid is in place and there is no mechanism that naturally guides exchanges between specific partners, maximal surplus is achieved with grid due to sheer luck. Nevertheless, one could define a **minimal surplus** achievable when all subjects outside of the grid trade and nobody else does. This corresponds to subjects with two-colored dashed lines in figure 2.

Sometimes subjects carry a leftover inventory at the end of the experiment, which clearly decreases their profit. Therefore, when calculating the social surplus, we adjust the data assuming that any purchases from the experimenter that resulted in a leftover inventory have no effect on social surplus,

We perform a regression on adjusted surplus by date, the existence of grid, and different periods using the model:

Adjusted Surplus = $\beta_0 + \beta_{1*}$ Grid + $\beta_{period*}$ Period + β_{date*} Date.

The existence of grid is associated with loss in surplus at a significance level of 0.052 as can be seen from the second line of the table. Furthermore, each period is also associated with a gain in surplus at a significance level of 0.04 as can be seen from the third line of table 9.

Table 9: Factors affecting social surplus

Coefficients	Estimate	Std. Error	t value	Pr(>|t|)
GridY	-307.25	154.75	-1.986	0.0517 .
Period	55.95	27.09	2.066	0.0432 *
Date160224	-161.16	243.81	-0.661	0.5112

Date160307	3649.17	232.32	15.708	<2e-16 ***
Date160325	-10714.4	243.81	-43.947	<2e-16 ***
Date160329	-9222.93	244.5	-37.722	<2e-16 ***
Date160331	-9612.09	2344.32	-39.342	<2e-16 ***
Date160507	-10009.38	232.32	-43.473	<2e-16 ***
(Intercept)	13094.92	282.58	46.34	<2e-16 ***

We can see that the existence of grid does indeed decrease social surplus. The efficiency converges to 100% when there is no grid. When there is grid, the result is mixed. For Caltech subjects and experienced Purdue subjects, the efficiency is very close to the maximum. For inexperienced Purdue subjects, we observe a very low starting level of efficiency. Efficiency converges to around 95% with the grid when pooled across experiments. We can compare this with efficiency converging close to 100% in the absence of grid.

Other data that supports the idea that a grid leads to increased unwelcome liquidity is that the number of speculative trades is positively correlated with fundamental trades. When we perform a regression on the number of fundamental trades to speculations with the model:

$$\text{Fundamental trades} = \beta_0 + \beta_{1*}\ \text{speculation} + \beta_{2*}\ \text{Number of subjects}$$

We observe a significant positive correlation between the two at a significance level of 0.02 as can be seen from the first row of the table.

Table 10: Speculative contracts increase fundamental trades

Coefficients	Estimate	Std. Error	t value	Pr(>|t|)
Speculative contracts	0.10829	0.04526	2.393	0.0196 *
Subjects	1.2637	0.06852	18.443	<2e-16 ***
(Intercept)	1.83665	0.77188	2.379	0.0203 *

This suggests that as speculators engage in trades, there are outstanding bids and asks that attract otherwise inefficient units and traders into the market. Profit opportunities are created for units that would be excluded in an efficient market. Therefore, the total number of fundamental trades is positively correlated with speculative trades, which are in turn negatively correlated with social surplus.

8. SERIES TWO: SECURITIES WTIH UNCERTAIN PAYOFF AND ASYMMETRIC INFORMATION

The previous sections highlight the importance of assignment principles in the dynamics of price adjustment. This section expands the scope by allowing limited and heterogeneous information across agents. The asset now pays a common dividend about which each individual has only a small amount of information; indeed, information about the dividend is quite limited even after it is aggregated across agents.

It is well known that individuals with asymmetric information can improve their knowledge through market interactions. If and how that phenomena might survive the imposition of a grid are central questions. In unconstrained markets, the vehicles carrying information are known to be the bids, asks and contracts. The number and structure of the markets are known to be important. Speculation can play a role as information creates the opportunity for arbitrage and thus prices can carry motivations and beliefs unrelated to private information about states, such as beliefs about the behavior of others. The competitive equilibrium plays a central role in this process. However, much of that institutional and behavioral support might be called into question when grids are present. The questions posed here are to what extent does the information aggregation features of markets remain if a grid is imposed and whether matching theory holds tools for understanding what emerges.

8.1 Experimental design

In the experiments with uncertainty a total of six experimental sessions were conducted with eight subjects recruited from the Caltech undergraduate and graduate student populations. Each experimental session had twelve independent periods including a first practice period. Three experimental sessions were conducted with a grid imposed and three experimental sessions were conducted without a grid. In experiments with a grid, subjects were only allowed to trade in increments of 10.

Table 11 Session 2: Experiments with common value

Experimental sessions with grid	Number of periods	Experimental sessions with No grid	Number of periods
Session 1: 20170804	12	Session 4: 20170811	12 same as 1
Session 2: 20170807	12	Session 5: 20170810	12 same as 2
Session 3: 20170808	12	Session 6: 20170809	12 same as 3

Prior to the opening of a period, each subject was given private information about the probability of the dividend that period. Each subject started with four units of the commodity in each period. Negative inventories, short sales, were allowed only up to -1. In addition to the initial endowment of securities, the subjects also started with a 4,000 francs loan, which was repaid at the end of each period. Each unit of the commodity paid a single dividend per unit to the holder that was the same across all holders. The dividend paid per unit at the end of the period, was randomly determined each period from a uniform distribution between 100 and 400. After each period ended, the value of the dividend was revealed. For comparison across experiments, the values of the dividend and the corresponding private signals were identical across sessions 1 and 4, sessions 2 and 5, and sessions 3 and 6.

The information about the dividend was determined by the same process for all experiments. Each subject received a private signal drawn from a truncated normal distribution (σ=60) centered around the dividend. Each participant was given a personalized table with the Bayesian posterior odds calculated for each bin of value. Subjects' ability to perform probability updating is not a variable. The draws and the information are contained in an appendix.

Figure 5 illustrates the data and order flow produced in a market with a grid. Several features are of interest and will be addressed later. Sell orders are red and buy orders are blue. Contracts are the large circles. Periods are separated by the vertical lines. A grid is in place and is shown as the horizontal lines that become the locations of all market activity. The figure reveals that contracts are signed at more widely scattered prices in the early stage of each period. A closer look at the data reveals that the ratio between number of orders to the number of contracts changes within a period. Bids and asks come in the market as participants attempt to trade. In the early stage when subjects have limited knowledge, more options have to be provided as a process of search for someone willing to take bid or ask and sign the contract. In the later stages, if an equilibrium emerges, traders can avoid meaningless attempts and issue bids and asks that are more likely to be taken. As a result, if an equilibration emerges in a market, we expect to see higher offer-trade ratio at an early stage of each period and a decline of the ratio as time goes by. The tighter patterns of contracts near the end of the periods give an impression of market equilibration.

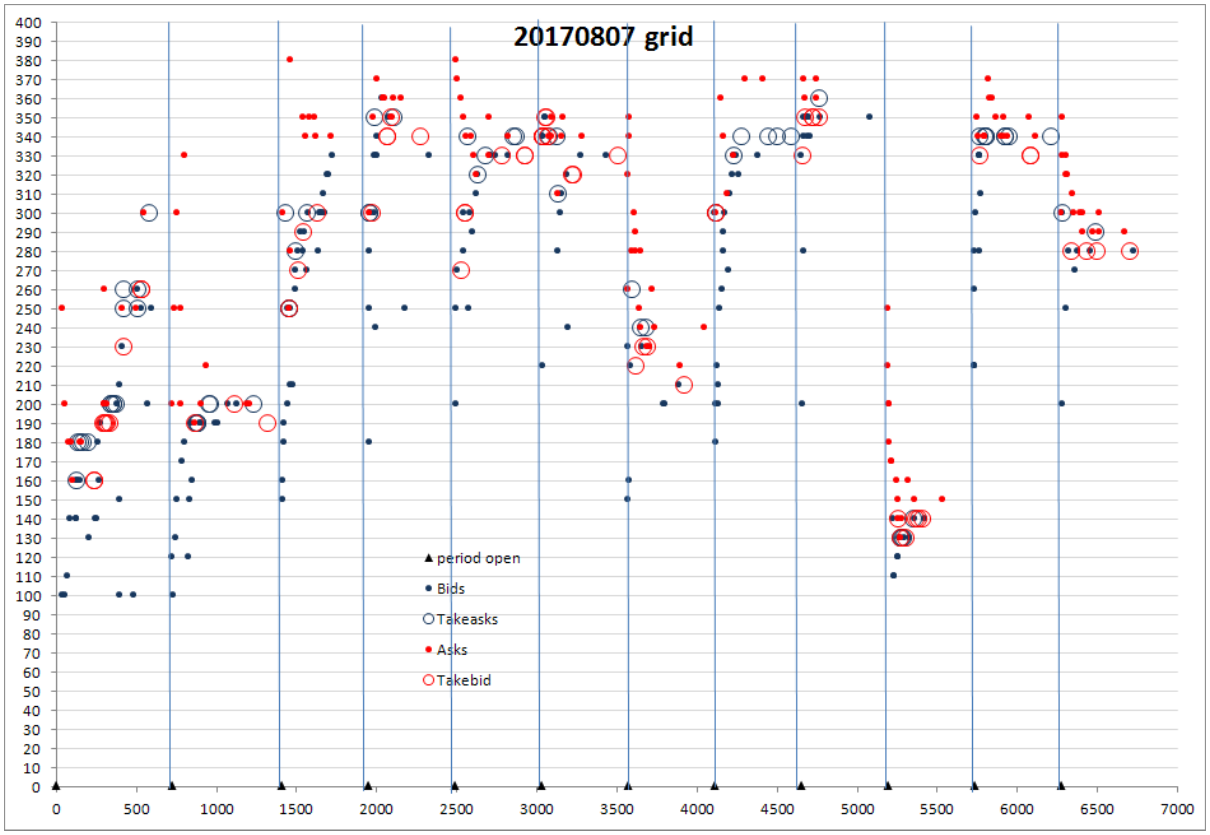


Figure 5: Data from Session 2 Experiment

8.2 Private information equilibrium: decision theoretic market demand and supply functions

Under conditions of certain preferences, the stable matches are directly related to the competitive equilibrium if no grid exists or the focus is on the bounding grids that are the nearest grids on the two sides on the VCE if grids exist. Under conditions of uncertainty, candidates for the CE and VCE are unknown but theory suggests they can be deduced from the private information held by participants. If participants have no other source of information, the expected dividend can be used as a measure of limit prices. Those with relatively high signals would be the demanders with the magnitude of signal used to rank buyers from high to low thereby creating a demand function with classical shape. Those with relatively low signals having limit prices ranked from low to high creates a supply function with classical shape. The candidate CE or VCE would be at the intersection of these two functions. Of course, such an analysis must reflect a realization that in market participants have access to any information contained in the flow of bids and asks so in the last analysis all conclusions must be measured against that possibility.

The next result demonstrates that the market agents self-divide into a market demand function and market supply function based on the contacts they make. That is, the high private signals create demanders and low private signals create suppliers and the median private signals are held by traders at the intersection of the curves. According to the model, they are the agents whose private signals and thus expected values place their limit prices at the CE or the VCE.

Result 7: Private signal values separate agents into demanders and suppliers that create a stable match**.** Preferences and trading are influenced by private information. Receivers of large signals as buyers match up with receivers of small signals as sellers in the market. The median signals serve as the CE of VCE of the private information equilibrium model.

Support. The support follows two steps. The first is to demonstrate that participants' buying and selling behavior is predicted by the private signals. The second step uses the bounding grids to further identify possible relations to assignment equilibria.

We use linear regression to test the relation between a subject's signal and her side of the market. We calculate each subject's net purchase in each period and we rank the subjects according to their signal. The subject with highest signal is ranked as 1, second highest as 2 and so on. Our first hypothesis is that, the higher the rank of signal, the more likely the subject is going to purchase. We run the test separately for the first three experiments with grid (column 1) and last three without grid (column 4). The coefficients are significant and negative.

A further step to validate the assignment model is to look at the two bounding prices in the equilibrium. We use the static theory that tells us to look for bounding prices for the market equilibrium (the VCE that we assume exists). Because of the uncertainty, (expectations and thus preferences are changing with experience) the market price does not converge to the same two bounding prices in all periods. We only focus on candidates for equilibrium prices that can be associated with bounding grid prices. Then we check the identity of buyers at the upper bounding grid and sellers at the lower bounding grid. If the traders' valuation of the security is correlated with their signal, receivers of high signals will be high value subjects who are going to bid at upper bounding grid and receivers of low signals will be low value subjects who are going to ask at lower bounding grid.

We test the relationship between subjects' total purchase at the upper bounding grid with their identity (column 2). Similarly, we test the relationship between their total sales at the lower bounding grid with their identity and corresponding private information (column 3). The coefficients are significant and have the expected sign. Subjects with high signals are more likely to buy at the upper grid and those with low signals are more likely to sell at the lower grid. In a word, the equilibrium bounding grids are the natural separator that divides the subjects into buyers and sellers based on their private signal.

Table 12: Value separating property of stable match

	(1)	(2)	(3)	(4)
	Experiment with grid			Experiments without grid
VARIABLES	Net Purchase	Total purchase at high bound	Total sell at low bound	Net purchase
Rank of Expected Value	-0.950***	-0.200***	0.211***	-0.411***
	-0.0663	-0.0411	-0.0351	-0.0973
Constant	4.276***	1.540***	-0.256	1.847***
	-0.335	-0.208	-0.177	-0.491
Observations	264	192	192	264
R-squared	0.44	0.111	0.159	0.064

Standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1

8.3 Stable Assignments as the equilibrium

We examine the data for evidence that the contracts settle at a stable assignment equilibrium. Two features are prominently suggested by the assignment model as and the results reported in the previous section: the emergence of contracts only at adjacent grids and a convergence process evident in bids and asks. The fact of asymmetric information creates the possibility of information aggregation associated with order flow motivates models in which information about the common value and thus demand and supply are endogenous. As suggested in the section above a good candidate is the private information equilibrium and the literature suggests the rational expectations equilibrium that would be a consequence of more refined information aggregation.

Private information equilibrium in this context refers to the state where trades are made near the median of the private signals of the individuals. Rational expectations equilibrium corresponds to the hypothetical scenario of where people would trade if all the private signals were public. In this context, it corresponds to the maximum likelihood estimate from the aggregated signals. If the grid is in place, the equilibrium would be a VCE and the contract data would be at the bounding grids. The next result establishes those properties in the data.

Result 8: In the common value environment with asymmetric information, the contract prices exhibit properties of a stable assignment equilibrium. The properties of market activity exhibit equilibration over time.

(i) Contract prices exhibit a reduction in variance.

(ii) The ratio of order flow (bids and asks) relative to contract prices fall and approach 1 (bids and asks are placed to trade quickly).

(iii) If grids exist then contract prices near the end of a period occupy only two, adjacent grids.

Support. Our data confirms the emergence of equilibration. As a common property of the price convergence process, the moving standard deviation of trading prices should decline through time. For each period, we take every four neighboring prices and calculate their standard deviation. We show that contract prices exhibit a reduction in variance using A-El-G test. In the table below, the test results are presented for all six experiments, separately depending on the existence of grid.

Table 13: Trend of Moving S.D.

Panel A: Experiments with Grid

	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12	B_2	n	R^2
20170804	15.76***	4.363	-2.898	21.13***	26.86***	4.217	4.114	22.27***	0	5.363	-0.0701			
	(5.80)	(5.76)	(5.76)	(5.72)	(5.72)	(5.69)	(5.70)	(5.80)	(7.05)	(5.84)	(5.84)			
20170807	-0.166	26.79***	28.39***	27.36***	2.819	15.65***	18.51***	7.963	0.409	-0.661	7.264	7.048***	224	0.802
	(5.80)	(5.80)	(5.80)	(5.74)	(5.71)	(5.92)	(5.84)	(6.05)	(5.76)	(5.74)	(5.92)	(0.77)		
20170808	89.97***	7.572	31.24***	2.455	1.57	20.08***	1.815	24.64***	5.901	37.69***	13.51**			
	(5.71)	(5.76)	(5.84)	(5.80)	(5.80)	(5.72)	(5.80)	(5.71)	(5.74)	(5.84)	(5.76)			

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Panel B: Experiments without Grid

	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12	B_2	n	R^2
20170809	31.18***	4.167	7.976	-1.679	3.445	41.68***	4.284	10.4	8.517	17.65***	0.0216			
	(6.30)	(6.35)	(6.32)	(6.36)	(6.35)	(6.39)	(6.60)	(6.46)	(6.52)	(6.35)	(6.35)			
20170810	-3.357	3.902	39.70***	12.12*	1.404	4.996	10.11	20.99***	30.21***	13.81**	-1.018	6.196***	324	0.697
	(6.29)	(6.46)	(6.75)	(6.42)	(6.39)	(6.60)	(6.75)	(6.46)	(6.39)	(6.75)	(6.46)	(0.64)		
20170811	43.26***	4.173	10.33	21.56***	22.86***	17.29***	17.64***	38.06***	8.152	59.39***	34.24***			
	(6.31)	(6.27)	(6.27)	(6.32)	(6.33)	(6.39)	(6.33)	(6.35)	(6.31)	(6.29)	(6.36)			

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Fifteen out of 33 periods in experiments with price grids (Panel A) and 16 out of 33 periods in experiments without grids (Panel B) exhibit a significant declining trend of moving variance of trading prices. Because of the price grid, the ending level of moving standard deviation is higher in Panel A (B_2=7.048) than in Panel B (B_2=6.196). However, the fact that nearly half of the periods are consistent with a declining trend confirms the equilibration process whether or not there is a grid. Moreover, for the remaining periods, even though the A-El-G test does not illustrate a clear trend, the starting level of moving deviation is so low that it cannot be distinguished from the ending level. In those periods, price variation is small from the beginning, which is also consistent with existence of equilibrium.

Besides the moving price variation, another way to show the equilibration process is by looking at the offer-trade ratio over time. In our markets, traders make bids and asks before the other side takes bids and asks and makes a trade. Therefore, the number of bids and asks during a certain interval measures the level of disagreement in the market. When there is limited information derived from market activity, traders make offers according to private information that varies for each participant. As the model suggests, at the beginning of each experiment, there should be large number of bids and asks coming into the market before the first trade is made. As time goes by, and information expands the equilibrium price converges, traders agree on that price and there will be less offers to make before a trade happens. We construct a measure of offer-trade ratio to illustrate the equilibration process.

We divide a period into several intervals according to the time each trade happens. Specifically, we define interval as interval 1 for the period before the first trade happens, equals to 2 for the period between the first and second trade and so on. Then we calculate the offer trade ratio for each interval. The number of offers is the number of bids plus the number of asks in that interval. The number of trades is the number of take-bids and take-asks happen after that interval but before the next. Since trades may happen at the same time, the number of trades between two intervals can be greater than one. For example, in the three experiments with price grid (Panel A), there are 293 intervals in total, of which 270 are followed by only 1 trade.

Table 14: Trend of Offer Trade Ratio

Panel A: Experiments with Grid

	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12	B_2	n	R^2
20170804	9.650***	11.68***	5.466**	4.663*	5.073**	5.185**	3.963	4.677*	8.045***	6.379**	14.26***			
	(2.58)	(2.57)	(2.58)	(2.56)	(2.56)	(2.56)	(2.56)	(2.59)	(2.67)	(2.58)	(2.59)			
20170807	11.07***	4.990*	3.627	5.258**	1.098	7.240***	4.122	3.765	9.219***	8.089***	5.736**	1.207***	293	0.577
	(2.57)	(2.57)	(2.57)	(2.57)	(2.56)	(2.59)	(2.58)	(2.63)	(2.57)	(2.57)	(2.59)	(0.29)		
20170808	3.491	15.83***	7.287***	13.30***	2.323	4.649*	6.393**	5.162**	3.299	2.847	4.692*			
	(2.56)	(2.57)	(2.58)	(2.61)	(2.58)	(2.56)	(2.58)	(2.57)	(2.57)	(2.59)	(2.57)			

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Panel B: Experiments without Grid

	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12	B_2	n	R^2
20170809	3.783*	6.553***	6.017***	5.514**	5.217**	4.198*	6.424***	6.933***	3.075	4.297*	3.175			
	(2.29)	(2.31)	(2.30)	(2.31)	(2.32)	(2.31)	(2.38)	(2.32)	(2.33)	(2.31)	(2.31)			
20170810	5.431**	6.102***	6.864***	6.426***	5.779**	6.581***	10.91***	6.172***	7.692***	7.010***	9.787***	1.908***	401	0.555
	(2.29)	(2.32)	(2.36)	(2.32)	(2.32)	(2.34)	(2.36)	(2.32)	(2.31)	(2.36)	(2.32)	(0.20)		
20170811	4.743**	8.919***	2.967	3.259	3.565	6.477***	4.611**	4.624**	1.445	4.221*	8.105***			
	(2.30)	(2.29)	(2.29)	(2.30)	(2.30)	(2.31)	(2.31)	(2.31)	(2.30)	(2.29)	(2.31)			

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

We run the Ashenfelter- El-Gamal (A-El-G) test to show a declining trend in the offer trade ratio. We conduct the test separately for experiments with and without a grid. As shown by the table above, in all the periods of the experiments with a grid, the ratio starts at higher level than 1.207, the ending level predicted by the model. In 24 out of 33 periods, the coefficients are significant for the estimated starting level. Similarly, in 32 out of 33 periods of the experiments without grid the ratio starts at higher level than 1.908, the ending level predicted by the model. In 28 out of 33 periods, the coefficients are significant for the estimated starting level. Combining all the results, the A-El-G tests prove that the declining trend of the offer-trade ratio is a general feature of an experimental market. This supports the argument that the market converges to some equilibrium.

Next, we check whether the assignment model predictions are consistent with our observation from the data. Specifically, we examine for those experiments with a price grid whether bounding grids emerge as the market prices in late stage of each period. To do so, we split all trades evenly into two groups according to the time of contracting. Trades in the second half are considered “late” trades and the prices for those trades are prices in the late stage. Note that those trades might still show varying price of more than two grids.

We calculate the number of equilibrium prices as well as their distance. If the period has no more than two equilibrium prices and their distance is within one grid, the period satisfies the prediction of the assignment model. Below we check all the periods with a price grid:

Table 15: Number of equilibrium prices and whether they are adjacent prices

Experiment	Period	2	3	4	5	6	7	8	9	10	11	12
20170804	number of equilibrium prices	2	2	2	2	2	2	2	3	1	2	1
	adjacent price	Y	Y	Y	Y	Y	Y	Y		Y	Y	Y
20170807	number of equilibrium prices	2	4	2	2	4	3	1	2	2	2	2
	adjacent price	Y		Y	Y			Y	Y	Y	Y	Y
20170808	number of equilibrium prices	3	3	2	1	3	2	2	2	3	3	3
	adjacent price			Y	Y		Y	Y	Y			

Nineteen out of 33 periods have two prices in the group of second half of trades. Four out of 33 periods have only one price in the group of second half of trades. For all 19 periods with two prices at the end, the two prices are adjacent. The sign of converging to adjacent prices implies that not only information aggregation happens, but also the assignment works well in predicting the pattern of equilibrium prices. The bounding grids are similar to those we discovered in the first series of experiments. Due to the price grid, agents are unable to contract at competitive equilibrium prices. The equilibrium outcome is actually a set of matches and the two adjacent contract prices ensure that the matching is stable.

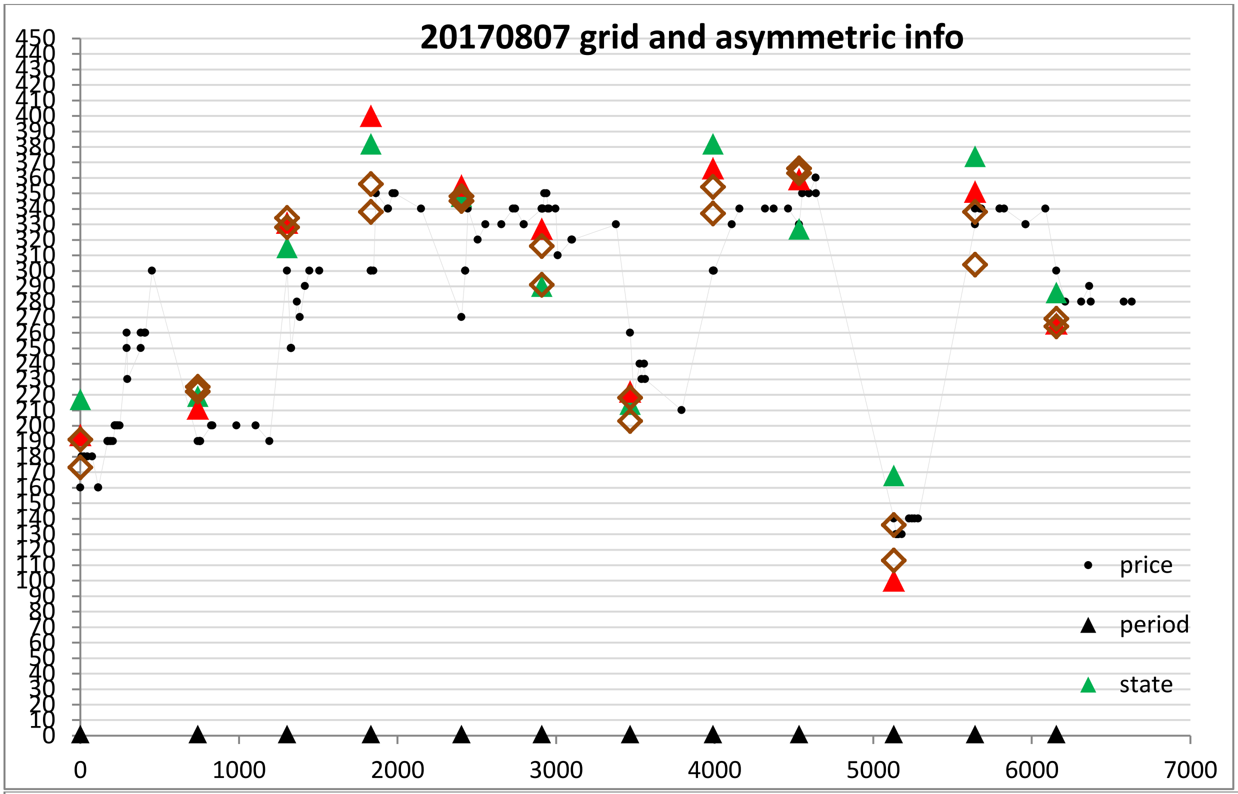
9 THE INFORMATION CONTAINED IN PRICES

9.1 Information aggregation

Two models of information are tested. The classical rational expectations equilibrium holds that all information available to the market will be reflected in market price. In this case, the information available is summarized by the MLE based on the pooling of all signals. By contrast, the private market equilibrium holds that demands and supply values are created from private information alone. In this case, the price would reflect the median of market signals.

Figure 6 contains the time series for all trades in experiment 20170807, which operated with a grid, and experiment 2070810 that had no grid. The states and information was identical for respective periods in the two experiments. The equilibrium predicted by the private information model assumes each individual applies the expected utility model and the information contained in his or her own signal to estimate the value of a unit of the security that, in turn, serves as the individual demand for a unit. In this model, individuals do not acquire information about the dividend from the buying and selling activities of others. Furthermore, in the model, individuals buy or sell only one unit. The equilibrium is a price range so the figure contains the upper and lower bound prices of the range. The MLE model assumes that individuals are able to extract the full posterior probabilities of the dividend given the sample of all signals pooled. Market equilibrium prices are based on such assumptions.

The figure contains a reliable impression of the statistics reported in the result. The variance of contract prices becomes reduced over time and moves in the direction of the private information equilibrium. The same pattern exists for the case where the grid exists and the case where the grid does not exist. Of course, exceptions can be found.


20170807 grid and asymmetric info
price
period
state
0
1000
2000
3000
4000
5000
6000
7000

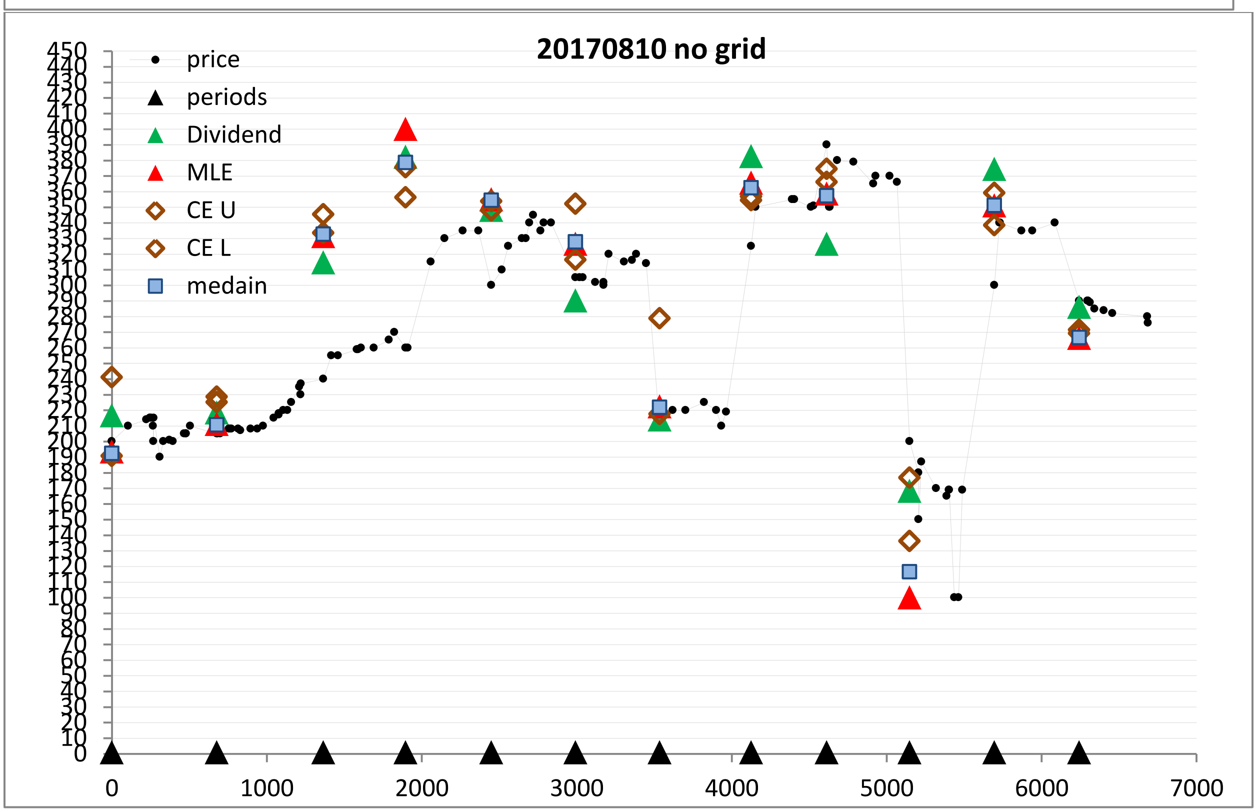

20170810 no grid
price
periods
Dividend
MLE
CE U
CE L
medain
0
1000
2000
3000
4000
5000
6000
7000

Figure 6. Experiments differ according to the existence of a grid. Payoffs and information are identical. Shown are the time series of contract prices. Aggregated information for each period include maximum likelihood, median signals and actual dividend

Result 9: The private information market equilibrium is accurate with or without a grid.

Support. There are four possible candidates for which the convergence might occur. They are the median private signal (private information equilibrium), the median of individual maximum likelihood estimates, the full maximum likelihood estimate based on the aggregated private signals, and the true value of the dividend. We calculate the second moment of the difference between the price at which the trades equilibrate towards and the four possible candidates and compare them. This shows that for both with the grid and without the grid, the price convergence is closest to the median signal, i.e., the private information equilibrium.

	Median Signal	Median MLE	Full MLE	True Value
Grid	21.60	23.29	25.51	27.67
No Grid	29.27	29.82	32.1	31.86

Table 16: The second moment of the difference between converging prices and four candidates. For each period, let the converging price be p_i and the median signal be $p_{i,ms}$. Then the value on the table is $\sqrt{\sum_i (p_i - p_{i,ms})^2}$.

We could use an alternative measure to test which of the equilibrium is most accurate. Under the grid condition, 64.4% of total trades happened in the neighborhood of the median signal as opposed to 57.6% of total trades occurring in the neighborhood of the rational expectations equilibrium, the full MLE.[11]

[11]One possible explanation as to why private information equilibrium outperforms rational expectations equilibrium has to do with the distribution we used for the parameters. By setting up a lower bound and an upper bound and using a truncated normal distribution, the maximum likelihood estimate is skewed towards extreme endpoints especially if the value of the dividend is not near the center.

9.2 The Average Trading Price

A natural question to pose is about the information content of average contract prices. The issue is especially interesting in the case of a grid where marginal information cannot be contained in prices or in order flow. The next result says that average contract price contains key information when the grid exists and when it does not exist. The study of values induced with certainty demonstrated that average price is close to the VCE but here VCE is unknown because preferences based on beliefs are unknown. Based upon the assumption that average price approximates the VCE we can ask about the implicit information aggregation and assess market equilibration models based on the aggregation of information contained in market activity.

Under the information aggregation setting, due to the uncertainty of individual signals, the price discovery process takes longer. Therefore, instead of using the average price of all trades, we focus on the final five trades where convergence occurs. We treat the median signal as a private information equilibrium since market demand/supply can be constructed from the limit prices implied by expected value maximization and use the difference between average trade price and the median signal as a proxy for how close the trades are to the underlying value.

Result 10. The average trading price moves toward the CE when it exists and the VCE when the CE does not exist.

Support. The first observation is the relation between the median of the private information signals and the time during period as contained in Table 16.

Table 17. Gap between average trade prices and the median private signal (i.e. the private information equilibrium)

|Average trade price - median signal|	All trades	First five trades	Last five trades
All Periods	22	27.9	18.6
Early periods (2-6)	24.7	32.1	19.9
Latter periods (7-12)	19.7	24.4	17.5

There are two key observations to be made here. First, we see that the discrepancy is larger for the earlier periods compared to the latter periods. Add the number of observations to the table to make clear exactly what has gone into the count. Convergence toward aggregated information available or the closely positioned private information equilibrium takes place across periods. This suggests a base rate fallacy where the subjects put too much weight on their own signals compared to others'. Second, the first five trades exhibit a much larger difference with the median signal. That is, convergence takes place within a period as well.

Furthermore, we can break it down into cases with grid and without grid.

Table 18. Gap between average trade prices and the median private signal (private information equilibrium) with/without grid

Average trade price minus median signal	Grid			No Grid		
	All trades	First five trades	Last five trades	All trades	First five trades	Last five trades
All Periods	21.6	25.9	17.1	22.4	29.9	20.1
Early periods (2-6)	21.9	28.6	15.9	27.6	35.6	23.9
Latter periods (7-12)	21.3	23.6	18.0	18.1	25.2	17.0

Notice that in all cases the movement is greater in the no grid case, which suggests that the information reflected in the average trade price, is not reduced by the existence of a grid. The ability of the average price in a market to reflect information aggregation beyond the private information equilibrium is not influenced by the grid. However, in assessing the information content of average price about the underlying value of the dividend, the convergence of prices cannot be neglected.

The two observations still hold true with the exception of the gap shrinking from early to latter periods when there is a grid imposed. More importantly, the pooled difference is actually smaller with the existence of a grid even though it is statistically insignificant. This implies that the average price of last trades does in fact carry price information, which permeates the large grid size.

Below, in Figure 7, we plot (median signal-average contract price of first five trades) on the horizontal axis and (average price of last five trades-average price of first five trades) on the vertical axis. A strong positive relation suggests that if the first five trades are too low compared to the median signal, then the trade prices are going to go up and vice versa. Ideally, we would expect the points to be around y=x, so there is a bit of under-correction. The figure reports the data from the grid and the no grid periods. In both cases, the average contract price moves toward the median signal. Average transaction price carries information about the median of signals.

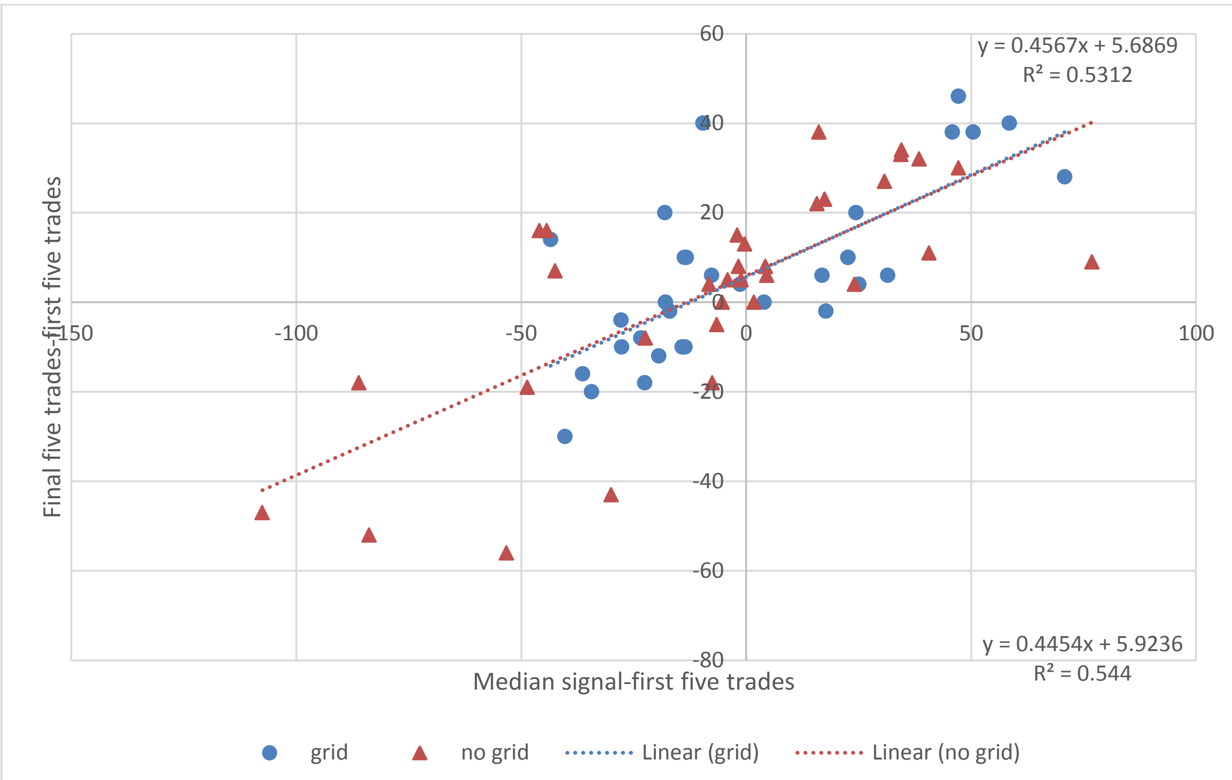


Figure 7. Price Adjustments to Signals. Median signal deviation from last five trades' v median signal deviation from first five trades. Price movement is toward the median of the signals stopped

10. SUMMARY OF CONCLUSIONS

Grids and tick sizes have not been studied experimentally. The advantages of an experimental approach are made apparent by a lack of consensus in the policy related literature and a disconnection from the classical theory of markets, which has a successful history unmasking basic principles of market behavior. Traditional analysis pivots on the competitive equilibrium, its game theory variants and the dynamics it suggests about disequilibrium adjustments. When a tick-related price grid is imposed on a market, the fundamental tool, the competitive equilibrium, need not exist. No consensus exists about appropriate theory. Thus, the question backs into a broad question of what theories might apply. Our exploratory approach suggests experiments that might point to deeper understanding of these real-world phenomena.

Our study begins with experiments involving large tick sizes, a stationary market and no uncertainty about values. The main discovery is that the appropriate model appears to be stable assignments from matching theory. With that background model in mind, we study suggestions from related literature and examine issues raised in the policy related literature.

Principles taken from the stable matching model make precise predictions for cases when a competitive equilibrium exists and cases when its existence is destroyed by the imposition of a grid. Result 1 and Result 2 demonstrate the power of a stable assignment model as a predictor of the experimental outcomes. Result 4 demonstrates that the average of the bouncing of prices created by the non-existence of the competitive equilibrium actually approximates the competitive equilibrium price. Result 5 and Result 6 address issues of speculation, market depth and liquidity. In these experiments, the imposition of a grid increases liquidity and volume but it does so at the expense of market efficiency and brings losses for those seduced into speculation.

Order flow in these stationary markets bears similarity to that in markets without a tick size grid. Namely, the direction of order flow reflects excess demand/supply. It remains to be determined if such patterns continue to hold as parameters change or substantial asymmetric information is introduced.

A second series of experiments examine a market trading a security with an uncertain (dividend) payoff. The results are similar to the certainty case in the sense that grid impeded contracts still converge to the bounding prices of a virtual competitive equilibrium (VCE.) The VCE that emerges is a private information equilibrium. That is, it assumes that reservation prices are based on private information as opposed to information based on the aggregation of all information that would reflect the maximum likelihood of aggregated information. Interestingly, while the prices do not approximate the maximum likelihood of the security payoff, prices do carry key information. The average trading price is a close approximation of the VCE determined by the median of the information distribution.

Appendices

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Appendix AlTable 1-

Parameters Series 1 :(160212; 160224; 160307)[periods;grid]

160212 1

Type(no.)	1st	2nd	3rd	4th
B1 (1)	260	225	165	155
B3 (1)	255	200	160	155
B5 (1)	260	250	175	160
B7 (1)	235	215	195	160
S2 (1)	120	155	215	220
S4 (1)	125	180	220	225
S6 (1)	120	130	205	220
S8 (1)	145	165	185	225

	pd2-6	gd250	ps7-11	gd1	
type	1st	2nd	3rd	4th	5th
B1 (1)	2500	2100	1700	1300	900
B3 (1)	2400	2000	1600	1200	800
B5 (1)	2300	1900	1500	1100	700
B7 (1)	2200	1800	1400	1000	600
S2 (1)	250	650	1050	1450	1850
S4 (1)	350	750	1150	1550	1950
S6 (1)	450	850	1250	1650	2050
S8 (1)	550	950	1350	1750	2150

160224 1

Type(no.)	1st	2nd	3rd	4th
B1 (1)	260	225	165	155
B3 (1)	255	200	160	155
B5 (1)	260	250	175	160
B7 (1)	235	215	195	160
S2 (1)	120	155	215	220
S4 (1)	125	180	220	225
S6 (1)	120	130	205	220
S8 (0)	145	165	185	225

	pd2-6	gd50	pd7-11	gd1	
type	1st	2nd	3rd	4th	5th
B1 (2)	2500	2100	1700	1300	900
B3 (1)	2400	2000	1600	1200	800
B5 (1)	2300	1900	1500	1100	700
B7 (1)	2200	1800	1400	1000	600
S2 (1)	250	650	1050	1450	1850
S4 (1)	350	750	1150	1550	1950
S6 (1)	450	850	1250	1650	2050
S8 (0)	550	950	1350	1750	2150

160307 1

Type(no.)	1st	2nd	3rd	4th
B1 (2)	260	225	165	155
B3 (2)	255	200	160	155
B5 (1)	260	250	175	160
B7 (1)	235	215	195	160
S2 (1)	120	155	215	220
S4 (1)	125	180	220	225
S6 (1)	120	130	205	220
S8 (1)	145	165	185	225

	pd2-6	gd250	pd7-11	gd1	
type	1st	2nd	3rd	4th	5th
B1 (2)	2500	2100	1700	1300	900
B3 (2)	2400	2000	1600	1200	800
B5 (1)	2300	1900	1500	1100	700
B7 (1)	2200	1800	1400	1000	600
S2 (1)	250	650	1050	1450	1850
S4 (1)	350	750	1150	1550	1950
S6 (1)	450	850	1250	1650	2050
S8 (1)	550	950	1350	1750	2150

Appendix A:Table 2

Parameters Series 2

169325 | | | | | | pd1-6 | Grid20 | pd7-10 | grid1

type	1st	2nd	3rd	4th						
B1 (2)	260	225	165	155	B1 (2)	394	246	206	202	158
B3 (1)	255	200	160	155	B3 (1)	374	274	206	200	168
B5 (1)	260	250	175	160	B5 (2)	354	314	216	198	178
B7 (1)	235	215	195	160	B7 (1)	334	294	218	188	148
S2 (2)	120	155	215	220	S2 (2)	26	174	214	218	262
S4 (1)	125	180	220	225	S4 (1)	46	146	216	220	252
S6 (2)	120	130	205	220	S6 (2)	66	106	206	222	242
S8 (1)	145	165	185	225	S8 (1)	86	126	202	232	272

160329 pd1 gd1 | pd2-8 gd20 pd9-11 gd1

type	1st	2nd	3rd	4th						
B1 (3)	260	225	165	155	B1 (3)	394	246	206	202	158
B3 (3)	255	200	160	155	B3 (3)	374	274	206	200	168
B5 (3)	260	250	175	160	B5 (3)	354	314	216	198	178
B7 (3)	235	215	195	160	B7 (3)	334	294	218	188	148
S2 (3)	120	155	215	220	S2 (3)	26	174	214	218	262
S4 (3)	125	180	220	225	S4 (3)	46	146	216	220	252
S6 (2)	120	130	205	220	S6 (2)	66	106	206	222	242
S8 (2)	145	165	185	225	S8 (2)	86	126	202	232	272

160331 Pd1,6 gd1 | pd2-5 gd20

type	1st	2nd	3rd	4th						
B1 (3)	160	125	65	55	B1 (3)	394	246	206	202	158
B3 (3)	155	100	60	55	B3 (3)	374	274	206	200	168
B5 (2)	160	150	75	60	B5 (2)	354	314	216	198	178
B7 (2)	135	115	95	60	B7 (2)	334	294	218	188	148
S2 (3)	20	55	115	120	S2 (3)	26	174	214	218	262
S4 (3)	25	80	120	125	S4 (3)	46	146	216	220	252
S6 (2)	20	30	105	120	S6 (2)	66	106	206	222	242
S8 (2)	45	65	85	125	S8 (2)	86	126	202	232	272

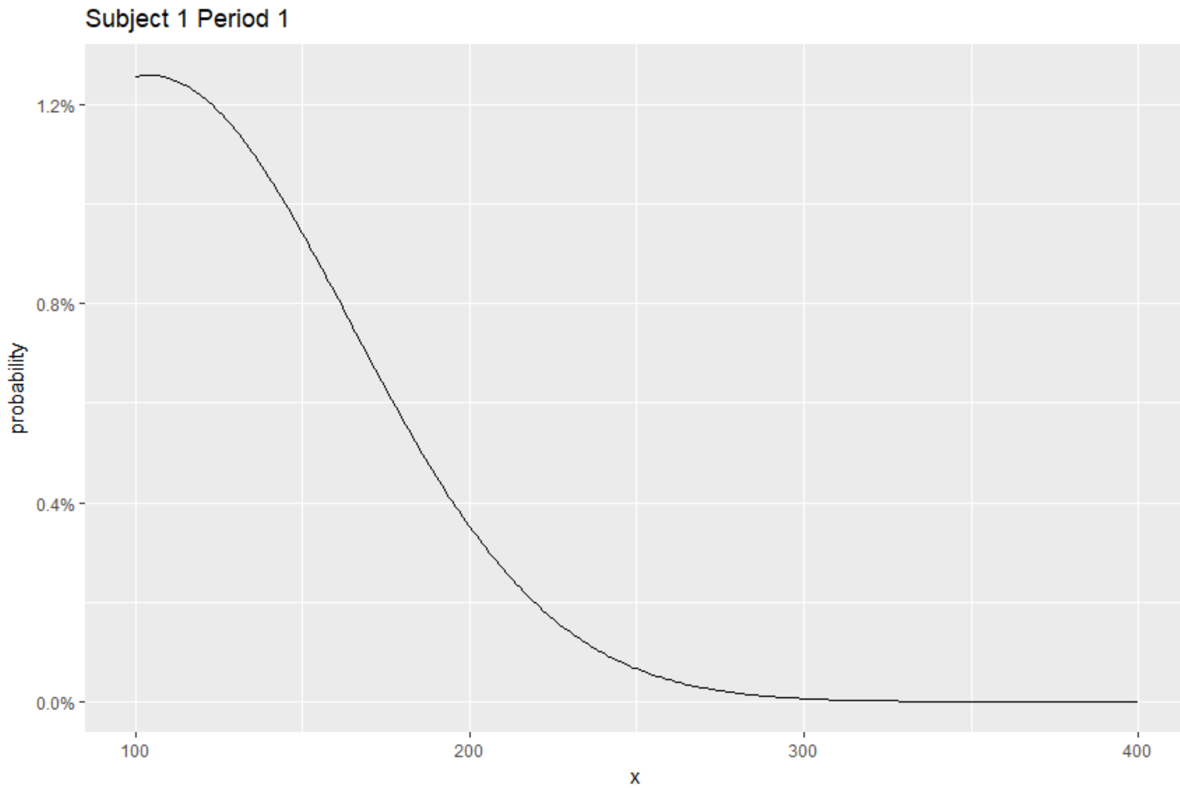
160507	pd1-6	gd1	pd7-11	gd20	
type	1st	2nd	3rd	4th	5th
B1 (2)	494	346	306	302	258
B3 (1)	474	374	306	300	268
B5 (2)	454	414	316	298	278
B7 (2)	434	394	318	288	248
S2 (2)	126	274	314	318	362
S4 (2)	146	246	316	320	352
S6 (2)	166	206	306	322	342
S8 (2)	186	226	302	332	372

Appendix B: Example Information provided to subjects

Most likely state: 104.3

Standard deviation of the most likely state: 56.3

Bin	Period 1	Bin	Period 1
[100,110]	12.57%	[250,260]	0.54%
[110,120]	12.37%	[260,270]	0.35%
[120,130]	11.84%	[270,280]	0.22%
[130,140]	11.03%	[280,290]	0.14%
[140,150]	9.99%	[290,300]	0.08%
[150,160]	8.80%	[300,310]	0.05%
[160,170]	7.54%	[310,320]	0.03%
[170,180]	6.28%	[320,330]	0.01%
[180,190]	5.09%	[330,340]	0.01%
[190,200]	4.02%	[340,350]	0.00%
[200,210]	3.08%	[350,360]	0.00%
[210,220]	2.30%	[360,370]	0.00%
[220,230]	1.67%	[370,380]	0.00%
[230,240]	1.18%	[380,390]	0.00%
[240,250]	0.81%	[390,400]	0.00%


Subject 1 Period 1
1.2%
0.8%
0.4%
0.0%
probability
100
200
300
400
x