Note on a supposed determination of the lunar diurnal tide in the ionosphere

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Since Newton founded and Laplace developed the gravitational theory of the tides, the somewhat concealed tidal influence on the atmosphere, as well as the obvious oceanic tide, has been much studied.[2] Later these studies were extended to the ionosphere, first by Appleton and Weekes.[3] Hitherto only the *semidiurnal* lunar tide has been found in the atmosphere; the *diurnal* tide predicted by theory, reversed fortnightly when the moon crosses the equator, has never yet been detected in meteorological data; all claims to have done so have proved fallacious.

In the ionosphere the semidiurnal tide is far larger than the oceanic or lower-atmospheric tides; hence the hope of detecting the lunar diurnal tide is brighter there than elsewhere. The purpose of this note is to show, however, that a recent announcement by Jones and Jones[4] of its detection in the F layer at College, Alaska, is not well-founded; this does not imply that it may not exist there and be determinable.

The diurnal and semidiurnal components of the lunar tide potential have a common factor, and otherwise are proportional to

$$\sin 2l \sin^2 D, \quad \text{(diurnal)},$$
$$\cos^2 l \cos^2 D, \quad \text{(semidiurnal)},$$

where l denotes the terrestrial latitude and D the lunar declination. At College, Alaska, where $l = 64°51'$, $\sin 2l = 0.77$ and $\cos^2 l = 0.18$; thus the latitude of the station is favorable to the detection of the diurnal tide, as stated by Jones and Jones.[4]

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[2] A comprehensive account of atmospheric tides and oscillations is to be included in the forthcoming *Compendium of meteorology*, to be published by the American Meteorological Society. See also M. V. Wilkes, *Oscillations of the Earth's atmosphere*, Cambridge, Cambridge University Press, 1949.

[3] E. V. Appleton and K. Weekes, "On lunar tides in the upper atmosphere," *Proc. roy. Soc. London*, **A171**, 171–187, 1939.

[4] M. W. Jones and J. G. Jones, "Tidal effects in the ionospheric F-layer," *J. Meteor.*, **7**, 14–20, 1950.

The material for their paper, which contains interesting and valuable results in addition to the one here criticized, consists of hourly data for the *thickness* of the F layer, taken to be $2(h^M - h^m)$, where h^M is the height of maximum electron density, and h^m is the virtual height of the F layer. Their most detailed results are obtained from a quiet winter period (when the E and F layers are only slightly developed at Alaska), 9 December 1947 to 1 January 1948 (24 days). From this material they obtain the following solar diurnal and semidiurnal variations (in km):

$$S_1 = 18.5 \sin (t + 269°), \text{ radius of probable error circle 1.9 km},$$
$$S_2 = 7.1 \sin (2t + 328°), \text{ radius of probable error circle 0.9 km},$$

where t denotes solar hour angle reckoned from midnight.

The probable errors here given are not indicated in the cited paper; I have calculated them from the 24 daily dial points for each harmonic, shown in Jones and Jones' fig. 8. The large ratios of the amplitudes to their probable errors show that the 24 days afford good determinations of S_1 and S_2.

In seeking the *lunar* diurnal and semidiurnal variations, the solar daily variation was removed before retabulation and measurement of lunar hourly values. Jones and Jones' fig. 9 gives the mean lunar daily variation for the 24 days, from which the authors calculated its lunar *diurnal* component as $3.1 \sin (t' + 25°)$, where t' denotes the lunar hour angle reckoned from lower culmination. This is given as the diurnal tidal component, disregarding the fact that the mean is taken for days of positive and negative lunar declination without distinction, though the diurnal tide has opposite signs in the two cases. In their fig. 10, however, the authors give mean curves for the two sets of days separately, and state that "it appears certain from these graphs that the lunar diurnal tide extracted is that predicted by the equilibrium theory." This is far from being the case.

On examination of the *Nautical Almanac*, I find that the first 12 of the 24 days were days of negative lunar declination; the mean value of $\sin 2D$ on these days was -0.61. The mean value of $\sin 2D$ on the remaining 12 days, of positive lunar declination, was 0.55. The mean lunar diurnal component of fig. 9 should therefore contain only a small residue of the diurnal tides, corresponding to a very small negative declination of the moon. The component found, $3.1 \sin (t' + 25°)$, must therefore be almost wholly non-tidal, an accidental effect of irregularities in the F-layer variations; the probable error of S_1, 1.9 km, also applies to this determination, and it is not in any way remarkable that accidental causes should give a spurious lunar diurnal variation exceeding the probable error by little more than 50 per cent.

I have determined the diurnal components of the two curves given in Jones and Jones' fig. 10, in which the 24 days are divided, as they should be for this purpose, according to the sign of the lunar declination. The results are

$$5.5 \sin (t' - 16°), \quad D \text{ positive (12 days)}, \qquad (1)$$

$$3.2 \sin (t' + 87°), \quad D \text{ negative (12 days)}. \qquad (2)$$

The mean of these is 2.8 sin $(t' + 18°)$, agreeing reasonably well with the value 3.1 sin $(t' + 25°)$ obtained by the authors from fig. 9, and with 3.0 sin $(t' + 25°)$, obtained by analyzing the sequences of ordinates of the points in fig. 9.

According to tidal theory, the two components (1, 2) should be practically equal in amplitude (as sin $2D$ has practically the same numerical mean value in each case), and should have opposite phases. Neither requirement is even approximately fulfilled; the phase difference, instead of being 180 deg, is only 103 deg. This confirms the view that these variations are of accidental origin. This conclusion is supported also by the estimated probable error, 2.7 km ($2^{\frac{1}{2}}$ times 1.9 km, the estimated probable error determined from 24 days' data as compared with the 12 days here used); any determination is to be considered as uncertain unless its amplitude is three or more times the probable error.

In the latter part of their paper the authors give solar and lunar component daily variations derived from six months' data, October 1948 to March 1949. For S_1 and S_2 the phases agree remarkably well with those for December 1947, and the amplitudes do not differ more than might be expected to arise from a seasonal variation.

The supposed lunar diurnal variation for these six months is 2.3 sin $(t' + 70°)$; it is derived, as before, from the mean of all days, whether of positive or negative lunar declination, and can therefore only be ascribed to accidental error. One expects the accidental variation in a six-monthly mean to be less, as is found, than that for a one-monthly mean, but the magnitude (2.3 km) is still surprisingly large. The data given in the paper do not permit examination of the variation on days of positive and negative declination separately.

From the six months' data, the lunar semidiurnal variation is found (in a correct manner) to be 0.3 sin $(2t' + 135)$, but no probable error is given by which to assess the reliability of this result; the corresponding result (not given by Jones and Jones) for the 24 days from 9 December 1947, obtained from fig. 9 by measurement and analysis, is 0.23 sin $(2t' - 11°)$, with the probable error 0.9 km (the same as for S_2). Hence both these results must be regarded as merely accidental.

It is to be hoped that these valuable data will be reexamined, with more material and improved methods, because the lunar tidal influence on the ionosphere in the high latitude of Alaska is a matter of great interest. It would seem best to study first the height, and the value, of the maximum electron density, rather than a differential quantity like the thickness of the layer.

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