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Supplementary material - Validation of a fast semi-analytic method for surface-wave propagation in layered media

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1 CONVERGENCE OF THE NUMERICAL MODEL FOR A TRUNCATED SHEAR-VELOCITY STRUCTURE

Wave propagation in a pure shear-velocity power-law structure is not numerically stable as the velocity would be zero at the surface. In order to stabilize he simulations we truncate the velocity model at the surface such that

$$v_s(z) = v_s(z_t) \quad \text{for } z < z_t, \tag{1}$$

where z_t is the truncation depth. Therefore, by using approximation (1), we create a constant velocity near-surface waveguide of thickness z_t that could impact surface-wave velocity amplitude spectrum for large z_t or high-frequency sources. To properly pick a truncation depth z_t that does not impact the velocity spectrum for a given spatial resolution Δx we run simple convergence tests. The first convergence test is performed by running simulations with decreasing depths z_t for a very fine spatial resolution $\Delta x \approx 0.25$ m. The second convergence test is performed by increasing the spatial resolution for a given depth z_t. To measure the error made by decreasing resolution and increasing truncation depths z_t we compute the L_2 error on the velocity spectrum over the frequency range $f = (0, 100)$ Hz between the various simulations and a reference solution provided by a simulation with a very fine mesh

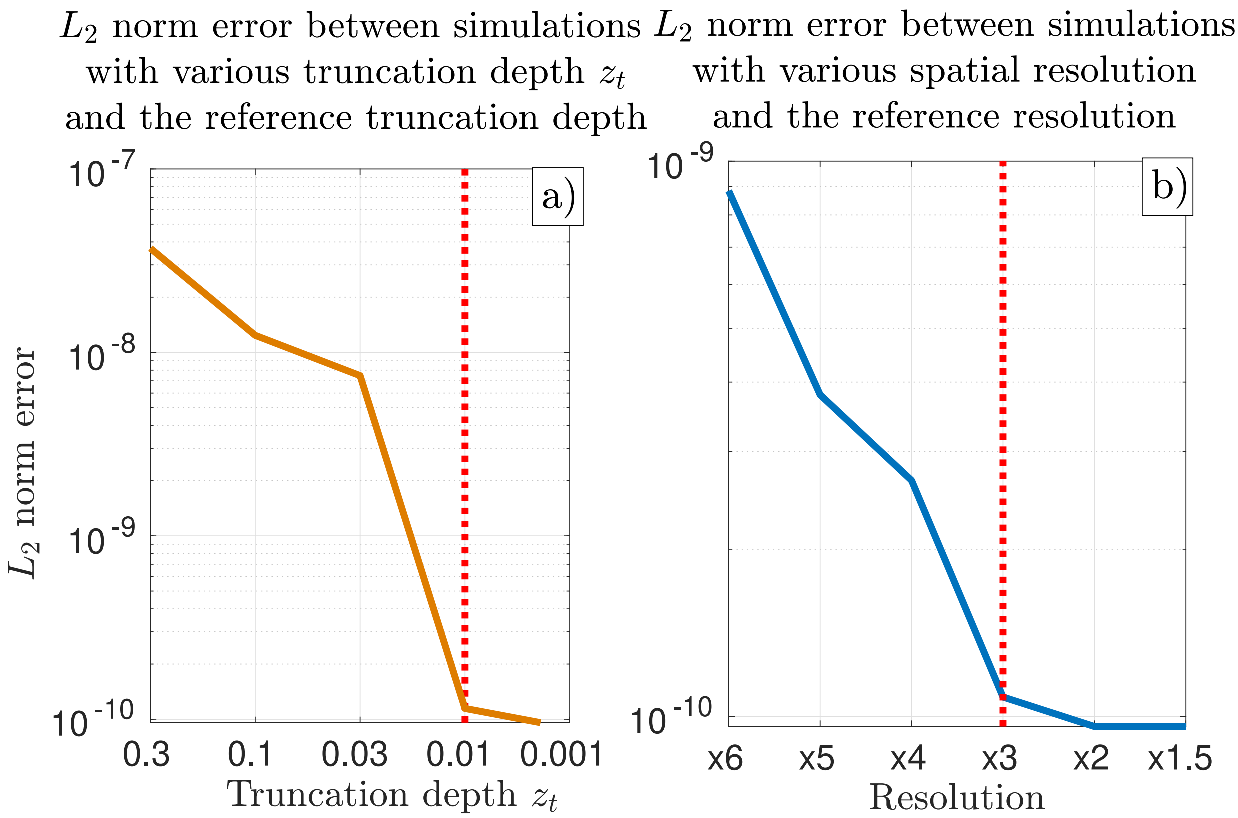

Figure 1. Panel a), L_2 errors between the velocity spectrum, over the frequency range $f = (0, 100)$ Hz, of a reference solution (with resolution $\Delta x \approx 0.25$ m and $z_t = 5 \times 10^{-4}$ m) and the numerical solutions for varying truncation depths and a given spatial resolution $\Delta x \approx 0.25$ m. Panel b), L_2 errors in the velocity spectrum between the reference and the numerical solutions with varying spatial resolutions for a truncation depth $z_t = 0.01$ m which corresponds to a depth for which the L_2 error converged in panel a).

$\Delta x \approx 0.25$ m and $z_t = 5 \times 10^{-4}$ m. Results are visible in Fig. 1 where we show in panel a) the test with varying truncation depths for a given resolution $\Delta x \approx 0.25$ m and in panel b) the tests with varying spatial resolutions for a truncation depth $z_t = 0.01$ m (corresponding to a depth for which the L_2 error converged in panel a)). Based on these results and on the trade-off between computational time and error we choose a resolution depth $z_t = 0.01$ m and resolution $\Delta x \approx 0.8$ m.

2 RAYLEIGH-WAVE QUALITY FACTOR COMPUTATION IN A HETEROGENEOUS MEDIUM

In a viscoelastic medium, when the compressional and shear quality factors are varying with depth and/or the velocity profile non-constant with constant, the Rayleigh-wave propagation becomes dispersive. Therefore, the quality factor will not be equal to the surface Q values anymore but will become frequency dependent. For a given piecewise constant body-wave quality factor model over N layers, the Rayleigh-wave quality factor can be expressed in terms of the shear and compressional quality factors such that (Anderson et al. 1965)

$$Q_R^{-1} = \sum_{i=1}^{N} (\frac{\beta_i}{v_c} \partial_{\beta_i} v_c Q_{s,i}^{-1} + \frac{\alpha_i}{v_c} \partial_{\alpha_i} v_c Q_{p,i}^{-1}), \tag{2}$$

where $(Q_{p,i}, Q_{s,i})_{i=1,N}$ are the compressional and shear quality factors of the i^{th} layer, $(\alpha_i)_{i=1,N} = v_{s,i}$ and $(\beta_i)_{i=1,N} = v_{s,i}$ are the compressional and shear velocities of the i^{th} layer and $(\partial_{c_{p,i}}, \partial_{c_{s,i}})_{i=1,N}$ are the partial derivatives along the compressional and shear velocities of the i^{th} layer. Finally, the partial derivatives can be written in terms of the Rayleigh-wave eigenfunctions (Lai & Rix 1998)

$$\begin{aligned} \partial_{\beta_i} v_c &= \tfrac{\rho_i \beta_i}{k^2 v_u I_1} \int_{z_{i-1}}^{z_i} \left((ke_2 - \partial_z e_1)^2 - 4ke_1 \partial_z e_2 \right) dz, \\ \partial_{\alpha_i} v_c &= \tfrac{\rho_i \alpha_i}{k^2 v_u I_1} \int_{z_{i-1}}^{z_i} \left((ke_1 - \partial_z e_2)^2 \right) dz, \end{aligned} \tag{3}$$

where v_c, v_u are the phase and group velocities, z_i is the depth of the i^{th} layer, ρ_i the density of the i^{th} layer, $k = \frac{2\pi f}{v_c}$ the angular wavenumber and $I_1^R = \frac{1}{2} \int_0^\infty \rho(z)(e_1(z)^2 + e_2(z)^2)dz$ with (e_1, e_2) the horizontal and vertical Rayleigh-wave displacement eigenfunctions.

The Rayleigh-wave quality factor, computed with eq. 2, in the case of the "Los Angeles Pasadena"model. Note that in order to compute Q_R through eq. (2), one needs first to discretize the velocity model into N layers and for the "Los Angeles Pasadena"model, we used $N = 199$ layers to properly capture the fine-scale velocity variations with depth. We observe that the higher the frequency the lower the Q_R value, as the body-wave quality factors are the smallest close to the surface. Also, the velocity jump in the "Los Angeles Pasadena"model introduced a discontinuity in the Rayleigh-wave quality factor around $f \approx 1.4$ Hz.

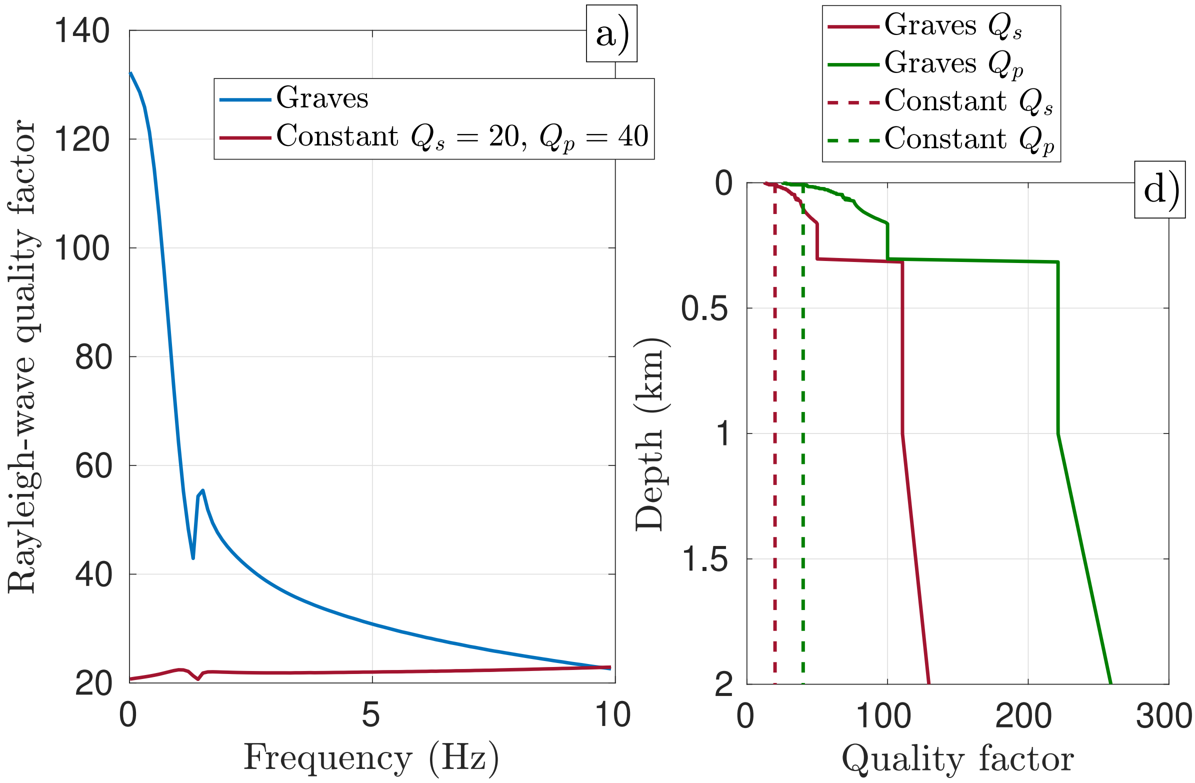

Figure 2. Panel *a*), the Rayleigh-wave quality factor with constant shear andcompressional quality factors Q_s and Q_p (red) and Rayleigh-wave quality factor with Graves model (blue) against frequency for the "Los Angeles Pasadena" model. Panel *b*), the corresponding shear (dark red) and compressional (green) quality factors against depth.

REFERENCES

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